REPRESENTATIONS AND IDENTIFICATIONS OF STRUCTURAL AND MOTION STATE CHARACTERISTICS OF MECHANISMS WITH VARIABLE TOPOLOGIES

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ABSTRACT

A mechanism that encounters a certain changes in its topological structure during operation is called a *mechanism with variable topologies* (MVT). This paper is developed for the structural and motion state representations and identifications of MVTs. For representing the topological structures of MVTs, a set of methods including graph and matrix representations is proposed. For representing the motion state characteristics of MVTs, the idea of finite-state machines is employed via the state tables and state graphs. And, two new concepts, the *topological homomorphism* and *motion homomorphism*, are proposed for the identifications of structural and motion state characteristics of MVTs. The results of this work provide a logical foundation for the topological analysis and synthesis of mechanisms with variable topologies.

Keywords: mechanisms with variable topologies, structural representations, motion state representations, topological homomorphism, motion homomorphism

REPRÉSENTATIONS ET DÉTERMINATIONS DES CARACTÉRISTIQUES LIÉES À LA STRUCTURE ET À L'ÉTAT DE MOUVEMENT DE MÉCANISMES À TOPOLOGIES VARIABLES

RÉSUMÉ

Un mécanisme dont la structure topologique subit certains changement durant une opération est appelé un *mécanisme à topologies variables* (MVT). Cet article met en valeur les représentations et les déterminations liées à la structure et à l'état de mouvement des MVT. Pour représenter les structures topologiques des MVT, une série de méthodes, y compris les représentations graphiques et matricielles, est proposée. Pour représenter les caractéristiques liées à l'état de mouvement, l'idée de machines d'états finis est employée par l'entremise de tables et de graphiques d'état. Par ailleurs, deux nouveaux concepts sont proposés, l'homomorphisme topologique et l'homomorphisme de mouvement, aux fins de déterminations des caractéristiques liées à la structure et à l'état de mouvement des MVT. Les résultats de ce travail fournissent une base logique pour l'analyse et la synthèse topologiques de mécanismes à topologies variables.

Mots-clés : mécanismes à topologies variables, représentations liées à la structure, représentations liées à l'état de mouvement, homomorphisme topologique et homomorphisme de mouvement.

1. INTRODUCTION

Mechanisms are graceful arts. Based on the ingenious, inspirational, and/or experienced thinking, mechanisms are designated to fulfill human being's requirements around daily life. For certain applications, mechanisms are elegantly designated such that their topological structures are changed during operation, for example, legged walking machines, mechanical push-button stopper locks, and metamorphic mechanisms [1,2]. Such mechanisms, which have multiple topological structures over the operation, are so-called the *mechanisms with variable topologies* (MVTs). The MVTs achieve complicated tasks with refined solving techniques. Their metamorphic structures also lead the studies relevant to the structural characteristics of mechanisms into an unexplored territory.

Prior to the tasks of topological synthesis and analysis, the structural representations of mechanisms are always the first problem to engineers. Since the late 1870s, many noticeable methods have been proposed, such as symbolic, graph, and matrix representations [3-10], for representing the topological structures of mechanisms with single topology. However, these methods are not quite feasible for MVTs because the MVTs own different topological structures respectively in different topology states. For the purpose of describing MVTs in a compact form, Yan and Liu [11,12] pioneered to present a topological representation for MVTs based on mechanism topology matrix [13] and graph representation by using a concept namely the *joint code*. By means of this representation, the topological structures of MVTs can be represented in terms of topologically changeable revolute, prismatic, and cam joints. However, this representation is unusable for the other MVTs which contain kinematic pairs other than the present three kinds. And, for convenience of computer programming, a mathematically considerable representation is indeed needed. Moreover, after the mechanism representation, it is worth to pay attentions on the topological identifications of MVTs.

On the other hand, since the MVT is changeable with its topologies, the relationships between the input scheme and topology states become an interesting topic for the motion planning of changing topology. This particular property also constitutes the great distinguished characteristics of MVTs compared to the general mechanisms with single topology. For addressing this charming property, Yan and Liu [14] adopted the concept of finite-state machines to represent the transitions between topology states under the defined input scheme. Although the problem of representing topology transformation is solved, the transformation characteristics are, however, not discussed via their study.

Therefore, this paper is contributed to the structural and motion state representations of MVTs accompanying with the identifications of their derived characteristics. A set of structural representations for general MVTs is provided at first. And, the concept of *topological homomorphism* is introduced to identify the complexity of topological structures of MVTs. Then, the motion state representations of MVTs are provided. Accordingly, the motion state characteristics of MVTs are identified via the concept of *motion homomorphism*. Based on this proposed approach, it is believed that the representations and identifications of structural and motion state characteristics of MVTs can be well completed.

2. MECHANISMS WITH VARIABLE TOPOLOGIES

As denoted by F. Reuleaux [3], a *mechanism* is "a combination of resistant bodies so arranged that by their means the mechanical force of nature can be compelled to do work accompanied by certain determinate motion." The types and numbers of such resistant bodies (mechanical members) and joints as well as the incidences between them characterize the *topological structure* of the mechanism. A mechanism which encounters a certain changes in its topological structure during operation is called a *mechanism with variable topologies* (MVT). Generally, the topological variation of MVTs is achieved by the topological variation of kinematic joints. In effect, the topological representation of MVTs can be properly developed based on the topological representation of kinematic joints with variable topologies [15].

For example, a Geneva mechanism with variable topologies is given in Fig. 1. The purpose of this mechanism is to convert the continuous rotation into the intermittent rotation. This mechanism comprises of a crank (C_r) with a proximal pin, a slider (S_l) with one V-like side, a 4-slot wheel (W_h) with a proximal rectangular block, and a frame (F_r). On the front part of the mechanism, the crank is driven to rotate clockwise and guides the wheel exhibiting the intermittent rotation. On the back-distal end of the crank, a trapezoidal link shape is designated to guide the slider rising or returning. At the distal end of the wheel, a rectangular block is designated for matching with the slider. On the back part of the mechanism, the slider is equipped with two tensile springs to be imposed forces. A V-like link shape of slider is specified to engage the rectangular block such that the wheel can be retained on a desired position. Since the purpose of the springs is to provide an auxiliary input force, they are disregarded in this figure. Accordingly, the process of varying topology of this mechanism is expressed as the following four cyclic stages:

Stage *A*

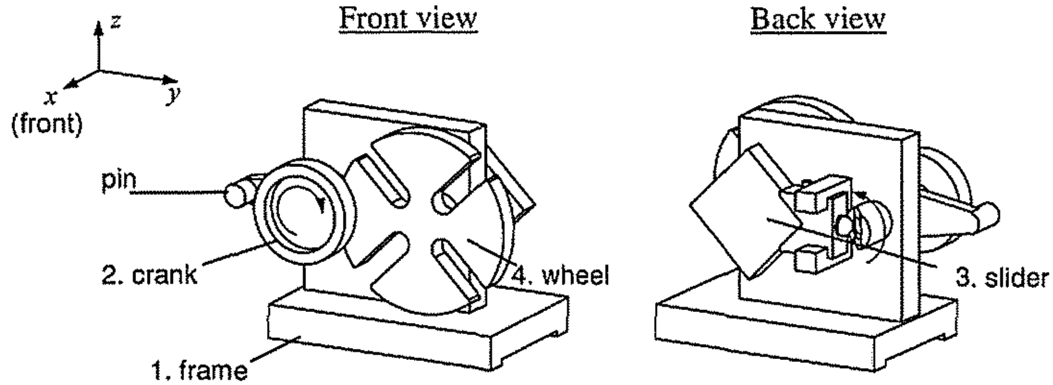
In the front part of the mechanism, the pin attached to the end of crank is not engaged into the wheel yet. So there is no physical contact between the crank and the wheel. In the back part, the slider belongs to the lowest location and it is guided in a dwell motion by the crank end. Since the slider is staying at the lowest location, the wheel is clamped by the slider by means of the conjugated rectangular and V-like link shapes.

Stage *B*

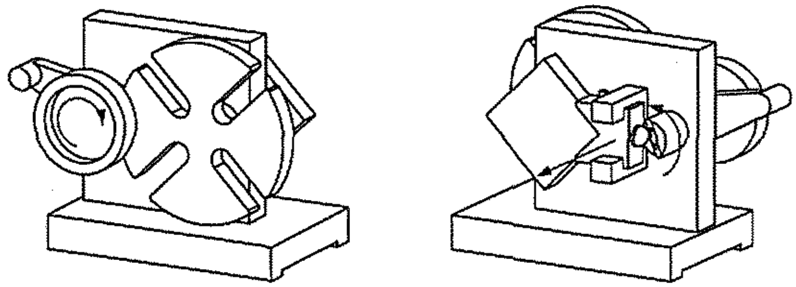
In the front part of the mechanism, the pin of the crank is still unengaged into the wheel. In the back part, the slider is lifted with a rising motion by the trapezoidal crank end. Since the slider has not reached the highest location, the wheel retains being clamped.

Stage *C*

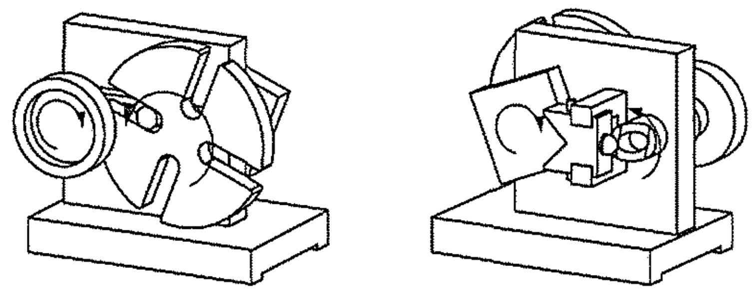
In the front part of the mechanism, the pin of the crank is engaged into the wheel. At the same time, the slider reaches the highest location and keeps a dwell motion so that the wheel is not clamped anymore. Since the wheel is free, it is guided to rotate counterclockwise by the pin of the crank.



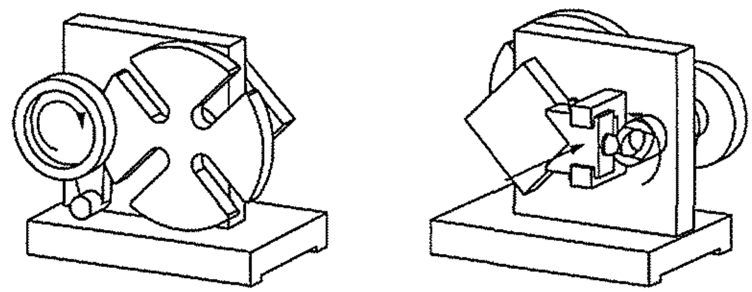
(a) Stage *A*



(b) Stage *B*



(c) Stage *C*



(d) Stage *D*

Figure 1 A four-bar Geneva mechanism

Stage *D*

In the front part of the mechanism, the pin of the crank is disengaged from the wheel. At the same time, the slider is pulled down with a return motion by the spring force. Hence the wheel is clamped again by the slider so that it returns being fixed. Then, stage *A* will be repeated and the cyclic period *A-B-C-D-A* goes on successively.

In conclusion to the above illustration, it is obvious that the types of some kinematic joints among this mechanism are changed over the four stages. From which, the topological structure of such mechanisms is changed.

3. A PROCEDURE FOR THE REPRESENTATIONS AND IDENTIFICATIONS OF MVTS

A procedure for the representations and identifications of structural and motion state characteristics of MVTs is as shown in Fig. 2. It is organized as the unified graphs, directionality topology matrices, and hexadecimal topology matrices for structural representations with topological homomorphism identification, and the state graphs and state tables for motion state representations with motion homomorphism identification. In what follows, each step will be explained and the Geneva mechanism shown in Fig. 1 is selected as the illustrative example.

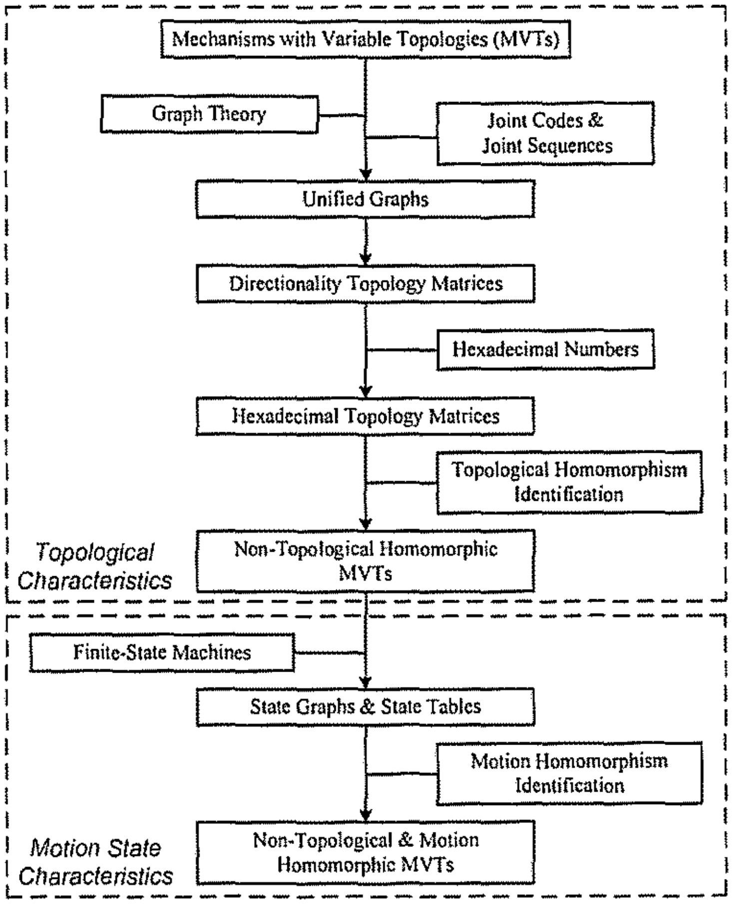


Figure 2 A procedure for the representations and identifications of structural and motion state characteristics of MVTs

4. GRAPH REPRESENTATIONS

Based on *joint codes* and *joint sequences* [15], the graph representations of MVTs can be concluded as the *graphs* and *unified graphs*. They are introduced as follows.

Joint Codes and Joint Sequences

At each topology stage of MVTs, a kinematic joint is concurrently possessing one topology structure. This topological structure indicates that what type of kinematic pair the joint has and what pair orientations the joint characterizes. Here we define such characterization as the *joint code*, J_{β}^{α}, in which α and β respectively represent the type and the corresponding orientations of the kinematic pair embedded in this joint. Table 1 shows the joint codes of commonly used kinematic pairs with the associated orientations. In addition, two kinematic pairs namely *fixed pair*, J_{v}^{X}, and *separated pair* [11,12], J_{v}^{D}, that are frequently appeared in joints with variable topologies, are introduced into this manner; and here v denotes the arbitrary orientation. If there is no relative motion between pair elements, a fixed pair is represented for this contact. Similarly, if there is no physical contact between pair elements, a separated pair is imaged for this contact. These two pairs are commonly appeared in most MVTs while two mechanical members are temporarily fasten or separated. Based upon above definitions, all frequently appeared kinematic pairs in MVTs are concluded.

During the MVT operation, a kinematic joint can undergo a series variation of types and/or motion orientations of the kinematic pairs. This variation process can be recorded as a string composing of joint codes of all topology states. Such a string is called the *joint sequence*, $J(\alpha,\beta)$, and is expressed as:

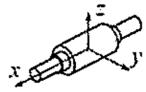
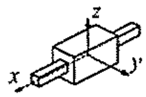
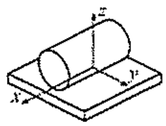
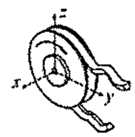
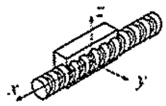
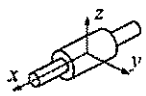
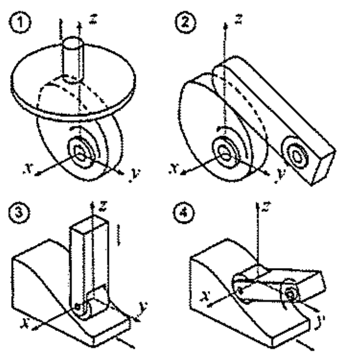
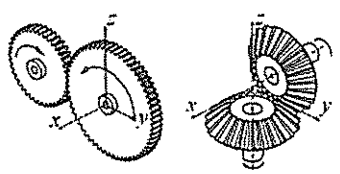
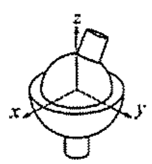
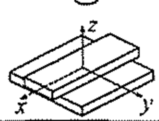
$$J(\alpha,\beta)=J_{\beta_1,\beta_2,\ldots,\beta_n}^{\alpha_1,\alpha_2,\ldots,\alpha_n} \tag{1}$$

in which α_n and β_n respectively represent the type and the representative orientations of joint codes of the kinematic pair at the nth topology state. For example, the kinematic pairs of the joint incident to the wheel and slider in Fig. 1 are fixed, primatic, separated, and primatic ones step-by-step. Hence its corresponding joint sequence is $J_{v,x,v,x}^{X,P,D,P}$.

Graphs

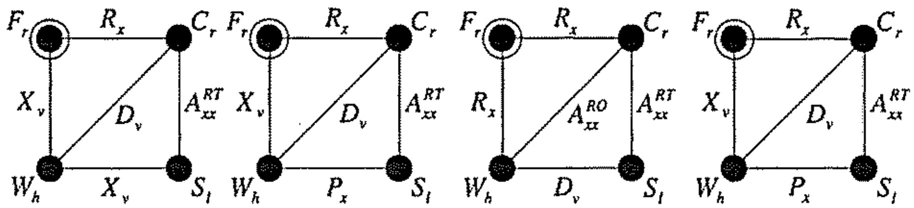
Since the topologies of joints can be expressed as the joint codes and joint sequences, the topological structures of MVTs can be represented in a more abstract form called the *graph representations*. In MVTs, the topological structure at each topology state has one corresponding graph for the individual topology. In a graph representation, the vertices denote links and the edges denote joints of a mechanism. The fixed link is denoted by two small concentric circles. The edge connection between vertices corresponds to the pair connection between links. To distinguish the topological variation of joint types and orientations, each

Table 1 Joint codes of commonly used kinematic pairs

Pair (Joint) type	Schematic diagram	DOF	Joint code	Representative orientations
Revolute		1	J_x^R	Direction of rotation axis.
Prismatic		1	J_x^P	Direction of the translation motion.
Rolling		1	J_x^O	Direction of rotation axis, coincident to the contact line or point.
Wrapping		1	J_x^W	Direction of rotation axis of the pulley or sprocket.
Helical		1	J_x^H	Direction of the screw axis.
Cylindrical		2	J_x^C	Direction of rotation axis.
Cam		2	$J_{xz}^{A_{RT}}, J_{xx}^{A_{RO}}$ $J_{xz}^{A_{TT}}, J_{yx}^{A_{TO}}$	Combination of: (1) (R,T)*: Directions of rotation axis of the cam and translation of the follower. (2) (R,O): Directions of rotation and oscillation axes of the cam and the follower. (3) (T,T): Directions of translation of the cam and the follower. (4) (T,O): Directions of translation of the cam and rotation axis of the follower.
Gear		2	J_{xx}^G, J_{xz}^G	Combination of the directions of rotation axes of gears.
Spherical		3	J_v^S	Defined as v since the spherical motion can be represented by any direction.
Flat		3	J_z^F	Direction of the normal vector of the contact surface.

* The alphabets denoted in the parentheses represent the motion types of the cam and the follower, respectively. And, R denotes the rotation, T the translation, and O the oscillation.

edge is labeled as α_β * according to its corresponding joint code J^α_β. For example, the graphs of the Geneva mechanism shown in Fig. 1 are depicted in Fig. 3. It clearly shows that although the four graphs have the same connectivity, the distinct joint codes are labeled to their edges. Note that because two separated mechanical elements are imaginarily treated as disconnected "pairing," the separated pair J^D_v is considered in the graphs. And, via the virtual connection of two separated mechanical members, the graph structures of all topology states are unified.



(a) (b) (c) (d)

Figure 3 Graphs of the Geneva mechanism at four topology stages

Unified Graphs

Since the graphs of MVTs at all topology states are unified, we can incorporate them into a more compact form called the *unified graph*. In the unified graph, the vertices and edges as well as their connectivities are identical to that of each individual topology state. But the labels of edges are replaced by the joint sequences. By this way, the overall topological variation of MVTs is described by simply one graph. For the graphs of the Geneva mechanism shown in Fig. 3, the corresponding unified graph is depicted in Fig. 4.

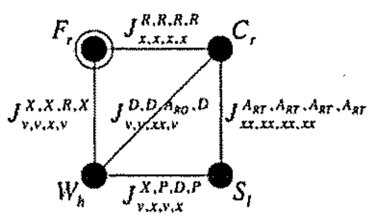


Figure 4 Unified graph of the Geneva mechanism

5. MATRIX REPRESENTATIONS

Since the graph representations provide simpler graphical drawings of MVTs, the matrix representations suggest the mathematically considerable manners for indicating their topological

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* α^t_β for cam pair, in which t defines the motion types of cam and follower.

structures. It includes the *directionality topology matrices* and the *hexadecimal topology matrices*.

Directionality Topology Matrices

The topological structure of an MVTs is represented by a unified graph and this graph can be expressed as a matrix. Based on the mechanism topology matrix [13], the *directionality topology matrix* (M_{DT}) is developed for representing the topological structures of MVTs. The directionality topology matrix of an MVT with N links is an $N \times N$ matrix. The diagonal element m_{ii} represents the type of mechanical member i, the upper right non-diagonal element m_{ij} represents the joint sequence incident to members i and j, and the lower left non-diagonal element m_{ji} denotes the label of the joint incident to members j and i. If members i and j are never adjacent, $m_{ij} = m_{ji} = 0$. For the Geneva mechanism shown in Fig. 1, its directionality topology matrix M_{DT} is:

$$M_{DT} = \begin{bmatrix} F_r & J^{R,R,R,R}_{x,x,x,x} & 0 & J^{X,X,R,X}_{v,v,x,v} \\ a & C_r & J^{A_{RT},A_{RT},A_{RT},A_{RT}}_{xx,xx,xx,xx} & J^{D,D,A_{RO},D}_{v,v,xx,v} \\ 0 & b & S_l & J^{X,P,D,P}_{v,x,v,x} \\ d & e & c & W_h \end{bmatrix} \tag{2}$$

Hexadecimal Topology Matrices

For convenience of computer programming, the joint codes and joint sequences can be transformed into the numerical numbers such that the directionality topology matrix is mathematically considerable. Here we employ a hexadecimal system to digitize the joint codes and joint sequences. Table 2 and 3 give the suggested numbers corresponding to the kinematic types and orientations in joint codes. Accordingly, the symbolic kinematic types and orientations of the joint sequences can be replaced by the corresponding numbers. For example, $J^{R,A_{RO},P,X}_{x,xx,y,v}$ is transformed into J^{2C31}_{1420}.

When the digitized joint sequences are obtained, we can adopt them into the directionality topology matrix. The numbers of a joint sequence are separated as two parts: the upper (kinematic types) and the lower (kinematic orientations). The upper numbers are put into M_{DT} in which the corresponding joint sequence locates. The lower numbers are put into M_{DT} to which the corresponding joint label belongs. And, for the mechanism with n topology states, the diagonal elements and each zero in M_{DT} are all replaced by n zeros, respectively. Hence the resulting matrix, namely the *hexadecimal topology matrix* (M_{HT}), is obtained. For the directionality topology matrix given in Eq. (2), its corresponding hexadecimal topology matrix M_{HT} is:

$$M_{HT} = \begin{bmatrix} 0000 & 2222 & 0000 & 1121 \\ 1111 & 0000 & BBBB & 00C0 \\ 0000 & 4444 & 0000 & 1303 \\ 0010 & 0040 & 0101 & 0000 \end{bmatrix} \tag{3}$$

Table 2 Hexadecimal numbers of kinematic types of joints

Kinematic type	*Corresponding number*	*Kinematic type*	*Corresponding number*
D	0	G	8
X	1	S	9
R	2	F	A
P	3	A_{RT}	B
H	4	A_{RO}	C
W	5	A_{TT}	D
O	6	A_{TO}	E
C	7		

Table 3 Hexadecimal numbers of kinematic orientations of joints

Kinematic orientation	*Corresponding number*	*Kinematic orientation*	*Corresponding number*
v	0	yx	7
x	1	yy	8
y	2	yz	9
z	3	zx	A
xx	4	zy	B
xy	5	zz	C
xz	6		

6. TOPOLOGICAL HOMOMORPHISM

A very familiar problem revealed in the structural analysis of mechanisms is the isomorphism identification between two kinematic chains or mechanisms. Two kinematic chains or mechanisms are said to be isomorphic if they have the same topological structures. Similarly, it is possible for two MVTs with the same topological structures at one or more topology states. In such situation, these two MVTs are said to be *topological homomorphic*. More precisely, if

MVT m_a have one or more identical topological structures to another MVT m_b, we said that m_a is topological homomorphic to m_b. Furthermore, if there is a sequentially one-to-one correspondence of topological structures between m_a and m_b, they are said to be *mutually topological homomorphic*. For example, Figs. 5(a)-(c) show three simplest mechanisms (with two links and one joint only) and (a1)-(c1) are their graph representations. Figure 5(c) has five topology states whereas Figs. 5(a) and (b) both have only four. By inspecting their joint sequences, Figs. 5(a)-(c) possess the joint topologies with $J^{P,R,P,R}_{x,x,x,x}$, $J^{R,P,R,P}_{x,x,x,x}$, and $J^{R,P,R,P,R}_{x,x,x,x,x}$, respectively. It is obvious that there exists a sequentially one-to-one correspondence to the joint codes of Figs. 5(a) and (b), i.e., a cyclic R_x-P_x-R_x-P_x period. So these two mechanisms shown in Figs. 5(a) and (b) are mutually topological homomorphic. Furthermore, Figs. 5(a) and (b) have four identical joint codes (mechanism topologies) which are corresponding to the parts of that in Fig. 5(c). So the mechanisms shown in Figs. 5(a) and (b) are topological homomorphic to that in Fig. 5(c). Note that if two MVTs are mutually topological homomorphic, each of their topological structures at the corresponding topology state is automatically one-to-one isomorphic.

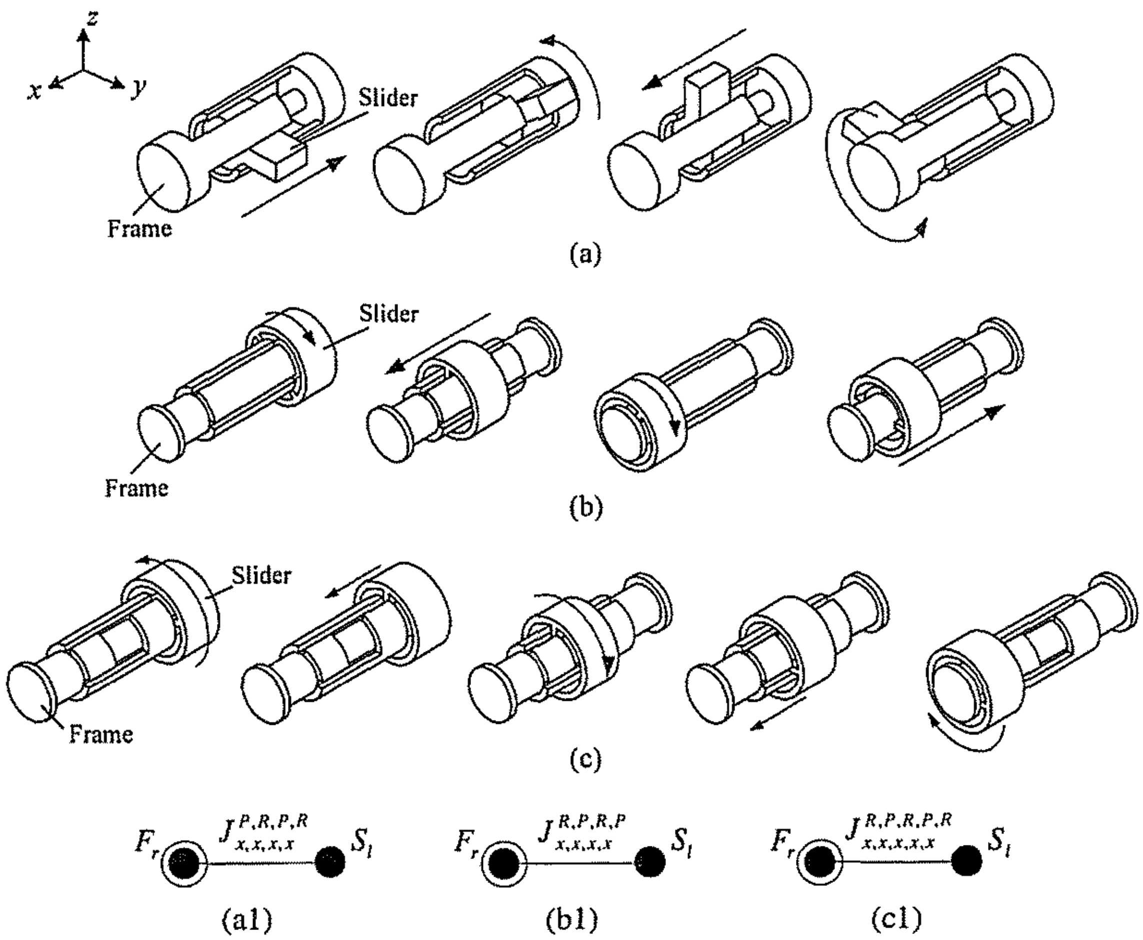


Figure 5 Three topological homomorphic mechanisms

The topological homomorphism between two MVTs, M_1 and M_2, can be identified mathematically by using the hexadecimal topology matrices. The procedure is summarized as follows:

Step 1. Write down their hexadecimal topology matrices, $M_{HT,1}$ and $M_{HT,2}$, respectively. For example, the hexadecimal topology matrices, $M_{HT,a}$, $M_{HT,b}$, and $M_{HT,c}$, of the mechanisms shown in Figs. 5(a)-(c) are:

$$M_{HT,a} = \begin{bmatrix} 0000 & 3232 \\ 1111 & 0000 \end{bmatrix} \tag{4}$$

$$M_{HT,b} = \begin{bmatrix} 0000 & 2323 \\ 1111 & 0000 \end{bmatrix} \tag{5}$$

$$M_{HT,c} = \begin{bmatrix} 00000 & 23232 \\ 11111 & 00000 \end{bmatrix} \tag{6}$$

Step 2. Respectively decompose $M_{HT,1}$ and $M_{HT,2}$ as two matrix sequences with n sub-matrices, $M_{DHT,1}$ and $M_{DHT,2}$, in which n is the number of topology states. For the nth matrix ($M_{DHT,1n}$ and $M_{DHT,2n}$) in $M_{DHT,1}$ and $M_{DHT,2}$, the entries $m_{DHT,1n_{ij}}$ and $m_{DHT,2n_{ij}}$ are identical to the nth numbers of $m_{HT,1_{ij}}$ and $m_{HT,2_{ij}}$, respectively. For example, the matrix sequences, $M_{DHT,a}$, $M_{DHT,b}$, and $M_{DHT,c}$, decomposed from $M_{HT,a}$, $M_{HT,b}$, and $M_{HT,c}$ are:

$$M_{DHT,a} = \begin{bmatrix} 0 & 3 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 3 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} \tag{7}$$

$$M_{DHT,b} = \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 3 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 3 \\ 1 & 0 \end{bmatrix} \tag{8}$$

$$M_{DHT,c} = \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 3 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 3 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} \tag{9}$$

Step 3. Compare with all sub-matrices in $M_{DHT,1}$ and $M_{DHT,2}$ one by one. If there are one or more identical sub-matrices between them, these two mechanisms are topological homomorphic. If there is a sequentially one-to-one correspondence between them, they are mutually topological homomorphic. Otherwise, they are distinct. For example, through Eqs. (7)-(9), it is obvious that there exists a sequentially on-to-one correspondence between $M_{DHT,a}$ and $M_{DHT,b}$, so their corresponding mechanisms, Figs. 5(a) and (b), are mutually topological homomorphic. And, $M_{DHT,a}$ and $M_{DHT,b}$ have four identical sub-matrices to that in $M_{DHT,c}$, so the corresponding mechanisms, Figs. 5(a) and (b), are topological homomorphic to that in Fig. 5(c).

When this procedure is applied into the structural identification of MVTs, one should notice that there may have the permutation problem in the matrix operation. As in the traditional schemes of isomorphism identification, the isomorphic mechanisms and kinematic chains can be detected by some permutations to the columns and rows of the corresponding matrix (such as link adjacency matrix, etc). Similarly, the identification of topological homomorphism among MVTs can be undertaken with all possible link permutations that correspond to permute the columns and rows of the hexadecimal topology matrices. The permutation techniques are similar to the traditional methods provided by many famous approaches. Here we skip it to give more space.

7. MOTION STATE REPRESENTATIONS

In the aforementioned approaches, the representations of topological structures of MVTs are clearly described. However, another important property, the motion state characteristics of MVTs, must be addressed for the completeness of representations. An MVT can change its topological structures under one or more controllable inputs to come up with the predictable motion. At each topology state, activating different inputs or controlling by other ways may lead the mechanism into the next different topological structure. So when all of the topological structures of an MVT are described, the relationships between them are needed to be addressed. To identify these relationships, the concept of *finite-state machines* is adopted [14]. The finite-state machines [16,17] are broadly used in network systems with a set of transformations of a set of states. It not only describes the transformations between these states, but also recognizes the similarity of different systems.

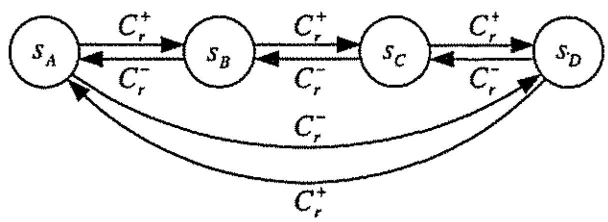
Finite-State Machines

A *finite-state machine* is an algebra structure $\{S, I, Y, M, \delta\}$, where S, I, and Y are finite sets of states, inputs, and outputs, respectively, M is a mapping from $S \times I$ into S, and δ is a mapping from S into Y.

Mapping the algebra structure of finite-state machines into MVTs, S, M, and Y are respectively corresponding to the state set, transformation, and the next state set when the input sequence I acts on the state set. The state set S selects all possible topology states of MVTs. The input sequence I informs the operation strategy of MVTs. The transformation M defines the rules of topology variation of MVTs. And the next state set Y concludes the correspondence between topology states and input variables. Since in MVTs we in general concern only with the relationships between state set and input sequence as well as output, the mapping function δ is neglected in this representation.

State Graphs and State Tables

The representations of finite-state machines include the *state graph* and *state table*. In the state graph, there is one node for each state in M and one edge leaving each node for each input symbol in I. The paths on this graph show state changes by the machine for all input sequences. In the state table, the basic rules of state transformations in M are constructed. The abscissa labels the input variables, the longitudinal axis labels total states, and the intersection denotes the output subject to the present state and input variable. For example, Fig. 6 shows the state graph and state table of the Geneva mechanism depicted in Fig. 1 in which "+" and "–" denote the clockwise and counterclockwise rotations of the crank, respectively. By inspection of the state graph and state table, it reveals that the Geneva mechanism contains four topology states and one manipulating input link with two rotating directions. And, in general, the input sequence I is suitable for $\{C_r^+, C_r^+, C_r^+, C_r^+\}$ or $\{C_r^-, C_r^-, C_r^-, C_r^-\}$ for a continuous rotation.



Present state s	Next state $M(s,i)$	
	$i = C_r^+$	$i = C_r^-$
s_A	s_B	s_D
s_B	s_C	s_A
s_C	s_D	s_B
s_D	s_A	s_C

Figure 6 State graph and state table of the Geneva mechanism

The finite-state machine representations are especially powerful for the MVTs with multiple inputs. For example, Fig. 7 [18] is an aircraft horizontal tail control mechanism with four possible topological structures. It has fourteen links and seventeen joints (including one multiple revolute joint and one slider joint). Member 2 is input-I from the control stick, member 12 is input-II from the control flap, the actuator (members 7 and 14) is input-III for the purpose of stability augmentation, and member 8 is the output link. This mechanism can be manipulated by one of the input combinations {I,II,III}, {I,II}, {I,III}, and {I}. Each input scheme produces distinct topological structures as follows:

(1) Input {I,II,III}:
In this situation, all links and joints are active. So this mechanism is containing fourteen links and seventeen joints (including the multiple revolute joint g). It therefore has three degrees of freedom.

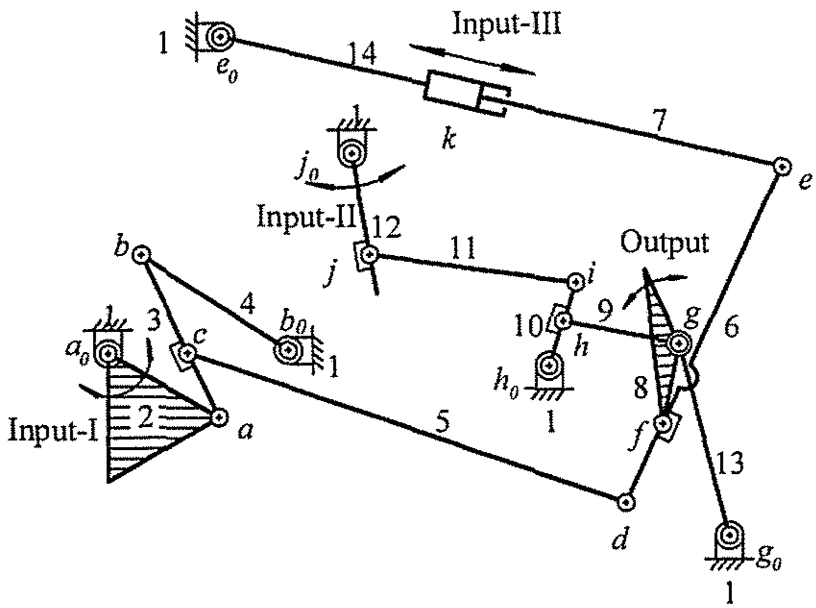


Figure 7 An aircraft horizontal tail control mechanism with three inputs [18]

(2) Input {I,II}:

In this situation, the actuator is inactive. Hence joint k becomes a fixed pair and members 7 and 14 constitute a fixed-length link. So this mechanism can be treated as containing thirteen links and sixteen joints (including the multiple revolute joint g). It therefore has two degrees of freedom.

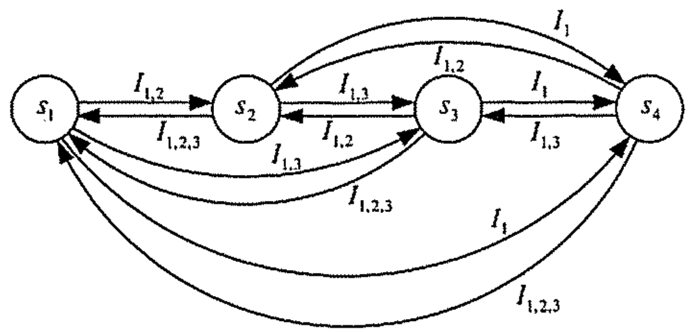
(3) Input {I,III}:

In this situation, the control flap is inactive. Since the control flap does not make any rotation, members 9, 10, 11, 12, and 13 will be stationary and form as a structure. Besides, joint g will become a fixed pivot. So this mechanism can be treated as containing nine links and eleven joints (excluding the multiple revolute joint g). It therefore has two degrees of freedom.

(4) Input {I}:

In the situation, this mechanism can be treated as containing eight links and ten joints (excluding the multiple revolute joint g). It therefore has one degree of freedom.

For identifying its motion state characteristics, the corresponding state graph and state table are established in Fig. 8. By inspecting this figure, the manipulation of this mechanism is readily and clearly obtained.



Present state s	Next state $M(s,i)$			
	$i = I_{1,2,3}$	$i = I_{1,2}$	$i = I_{1,3}$	$i = I_1$
$s_1/I_{1,2,3}$	-	s_2	s_3	s_4
$s_2/I_{1,2}$	s_1	-	s_3	s_4
$s_3/I_{1,2}$	s_1	s_2	-	s_4
s_4/I_1	s_1	s_2	s_3	-

Figure 8 State graph and state table of the aircraft horizontal tail control mechanism

8. MOTION HOMOMORPHISM

In addition to the application of state graphs and state tables, there are several interesting properties of finite-state machines that can be utilized to MVTs. Before this exploration, the following definitions should be addressed:

Definition 1 A machine M_1 is said to be a *homomorphic image* of a machine M_2 if there exist two surjections such that

$$\varphi_1 : J \to I_1 \text{, where } J \subseteq I_2$$

and

$$\varphi_2 : T \to S_1 \text{, where } T \subseteq S_2$$

and such that

$$\varphi_2(M_2(t,j)) = M_1(\varphi_2(t), \varphi_1(j))$$

for all j in J and t in T. We say that the functions φ_1 and φ_2 form a *machine homomorphism*.

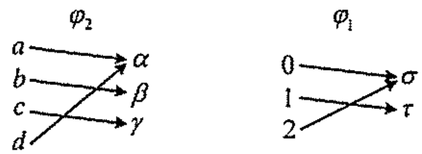
Following example illustrates the machine homomorphism.

Example 1 Suppose M_A and M_B are the machines whose state tables are as follows:

M_A	Next state $M_A(s,i)$		
Present state s	$i=0$	$i=1$	$i=2$
a	c	d	d
b	b	c	c
c	a	a	d
d	c	d	a

M_B	Next state $M_B(s,i)$	
Present state s	$i=\sigma$	$i=\tau$
α	γ	α
β	β	γ
γ	α	α

The surjective mapping functions, φ_1 and φ_2, are shown below, from S_A to S_B and I_A to I_B:



We can verify that for all j in J and t in T, where $J \subseteq I_A$ and $T \subseteq S_A$:

$$\varphi_2(M_A(t,j)) = M_B(\varphi_2(t), \varphi_1(j)) \tag{10}$$

For example,

$$\varphi_2(M_A(a,0)) = \varphi_2(c) = \gamma = M_B(\alpha,\sigma) = M_B(\varphi_2(a), \varphi_1(0)) \tag{11}$$

$$\varphi_2(M_A(a,1)) = \varphi_2(d) = \alpha = M_B(\alpha,\tau) = M_B(\varphi_2(a), \varphi_1(1)) \tag{12}$$

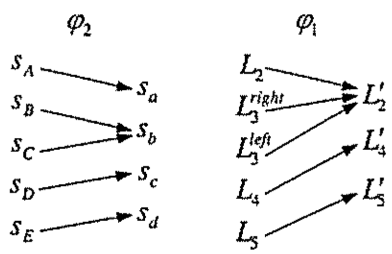
and so on. It follows that $(\varphi_1, \varphi_2): M_A \to M_B$ is a machine homomorphism; that is M_B is a homomorphic image of M_A.

Definition 2 Based upon Def. 1, a machine M_2 can *simulate* M_1 if and only if the surjections φ_1 and φ_2 constitute a machine homomorphism.

Hence in Example 1, we see that M_A simulates M_B. It implies that M_A can do the same function as that in M_B.

The concept of machine homomorphism can be mapped into the MVTs manipulation. If the manipulation of an MVT m_a can imitate the *overall* manipulation of another MVT m_b, we say that m_a can simulate m_b, and m_b is *motion homomorphic* to m_a. Furthermore, if m_a can simulate m_b and vice versa, we say that m_a and m_b are *mutually motion homomorphic.* Two MVTs would have similar operation strategies if they are motion homomorphic; two MVTs will have strictly identical operation strategies if they are mutually motion homomorphic.

The above definitions are illustrated by Figs. 9 and 10. Figure 9 shows a defined mechanism that is manipulated to move block B to the slot on the lower middle. During this manipulation, five topology states based on five input variables are identified as indicated in the state table and state graph. And the process of varying topology forms a machine namely M_A. Let us now define two surjections φ_1 and φ_2 for the input sequence and state set, respectively:



According to these two surjections, we can obtain Table 4 immediately by substituting the mapping elements into M_A. By inspecting Table 4, it reveals that there are several trivial states and input symbols. Thus a more compact form of state table can be rearranged as that in Fig. 10. The resulting state table concludes a machine M_B with four states and three input variables. And its state graph also can be obtained in Fig. 10. Furthermore, after a deliberated conceiving, a possible solution corresponding to the mechanism configuration is received. The resulted mechanism varies its topology with three inputs. And it performs the same function of that in Fig. 9 but with fewer operation steps.

Table 4 State table of surjective mappings of Fig. 9

M_A	Next state $M_A(\varphi_2(s),\varphi_1(i))$				
Present state s	$i=L_2'$	$i=L_2'$	$i=L_2'$	$i=L_4'$	$i=L_5'$
s_a	s_b	-	-	-	-
s_b	-	s_b	-	-	-
s_b	-	-	s_b	s_c	-
s_c	-	-	-	-	s_d
s_d	-	-	-	-	-

State table

M_A	Next state $M_A(s,i)$				
Present state s	$i = L_2$	$i = L_3^{right}$	$i = L_3^{left}$	$i = L_4$	$i = L_5$
s_A	s_B	-	-	-	-
s_B	-	s_C	-	-	-
s_C	-	-	s_B	s_D	-
s_D	-	-	-	-	s_E
s_E	-	-	-	-	-

State graph

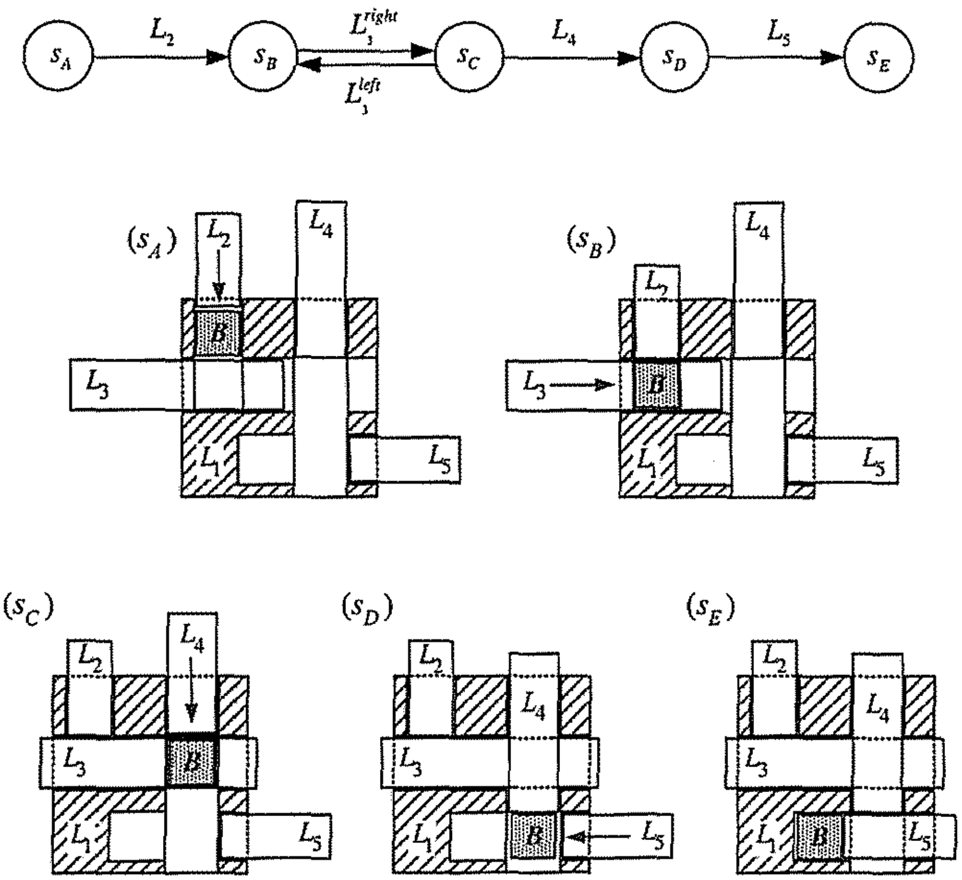
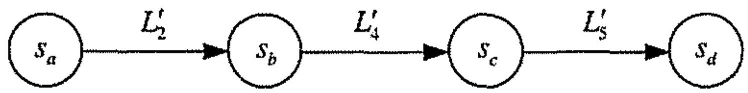


Figure 9 An MVT with five machine states and five input variables

State table

M_B	Next state $M_B(s,i)$		
Present state s	$i = L_2'$	$i = L_4'$	$i = L_5'$
s_a	s_b	-	-
s_b	-	s_c	-
s_c	-	-	s_d
s_d	-	-	-

State graph



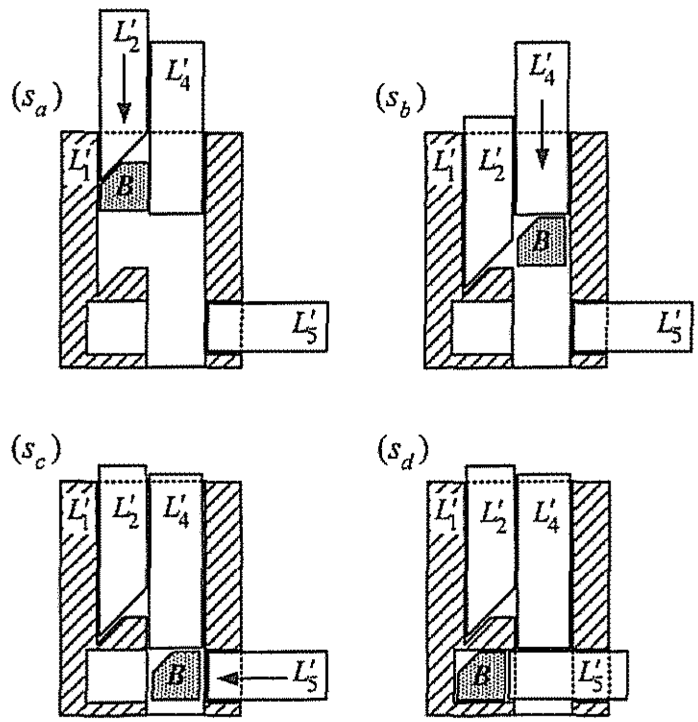


Figure 10 An MVT with four machine states and three input variables

Among the above operation, we produce M_B, that is a homomorphic image of M_A, from M_A such that it can be simulated by M_A. Also, the produced mechanism corresponding to M_B can be simulated by that corresponding to M_A. Briefly speaking, the mechanism shown in Fig. 9 can simulate the other one shown in Fig. 10. So the mechanism in Fig. 10 is motion homomorphic to that in Fig. 9. Since the homomorphic image M_B can be simulated by M_A for all of its own states, an important property for the motion homomorphism and simulation of MVTs is revealed. When an MVT M_2 has a homomorphic image M_1, it major operations can be achieved by M_1 associated with a simpler manipulation; oppositely, all of the operations of M_1 can be simulated by M_2 associated with an elaborate manipulation. The homomorphism and simulation properties of MVTs can reduce and encourage the complexity of their topology variation schemes in an opposite way.

9. CONCLUSIONS

This paper introduces the structural and motion state representations and identifications for the mechanisms with variable topologies. For the structural representations of MVTs, the unified graphs, directionality topology matrices, and hexadecimal topology matrices are suggested. Accordingly, the concept of topological homomorphism is introduced to identify the structural characteristics of MVTs. For the motion state representations of MVTs, the concept of finite-state machines is adopted by the uses of state tables and state graphs. Furthermore, the concept of motion homomorphism is utilized to identify the motion state characteristics of MVTs. The overall approach that organized here provides a logical foundation for the structural and motion state representations and identifications of MVTs. Future works to the structural and motion analysis and synthesis of MVTs can therefore be expected based on the contributions of this work.

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