ELASTIC MODULUS ADJUSTMENT PROCEDURES (EMAP) IN METAL FORMING ANALYSIS

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ABSTRACT

The estimation of exact loads or forces that cause plastic flow of the material in a metal forming process is often difficult. The main objective of this paper is to estimate the limit loads for some well-known metal forming processes, using a new generation of robust simplified methods. These methods, based on iterative linear elastic finite element analyses, are implemented by modifying the local elastic modulus of the material during subsequent iterations. On account of the possibility of local plastic collapse, the reference volume concept is invoked in order to identify the kinematically active and dead zones in the metal component. The reference volume method is shown to give a reasonable prediction of the limit load. The estimates of limit loads are then compared with corresponding results obtained using inelastic finite element analysis, and analytical solutions, with good effect.

PROCÉDURES DE RÉGLAGE ÉLASTIQUES DE MODULE (EMAP) EN MÉTAL FORMANT L'ANALYSE

RESUME

L'évaluation des charges ou des forces exactes qui causent l'écoulement plastique du matériel dans un métal formant le processus est souvent difficile. L'objectif principal de cet article est d'estimer les charges de limite pour un certain métal bien connu formant des processus, en utilisant une nouvelle génération des méthodes simplifiées robustes. Ces méthodes, basées sur l'élément fini élastique linéaire itératif analyse, sont mises en application en modifiant le module élastique local du matériel pendant des itérations suivantes. À cause de la possibilité d'effondrement en plastique local, le concept de volume de référence est appelé afin d'identifier les zones cinématiquement actives et mortes dans le composant en métal. La méthode de volume de référence est montrée pour donner une prévision raisonnable de la charge de limite. Les évaluations des charges de limite sont alors comparées à la correspondance résulte obtenu en utilisant l'analyse finie non élastique d'élément, et les solutions analytiques, avec le bon effet.

INTRODUCTION

The elastic-plastic response of a metal forming process is loading history dependent and is determined using analytical and numerical methods, which can often be complex and time consuming. In the numerical simulation of metal forming problems, the occurrence of large deformations not only add to the complexity from a mathematical standpoint, but can also cause rapid solution degradation due to element distortion; therefore, adaptive re-meshing techniques are often employed [1]. Exact solutions require that both equilibrium of forces and geometrically self-consistent strain patterns are satisfied simultaneously everywhere within the deforming body as well on its surface. To simplify the problem, upper and lower bound limit load evaluation methods have been suggested by several authors [2, 3]. Furthermore, there are some methods based on an iterative linear elastic finite element analyses that make use of modification to the local elastic modulus of material at each subsequent iteration in order to achieve strain and stress distributions that enable good estimation of upper and lower bound limit loads, even during early iterations.

Jones and Dhalla [4] were one of the earliest users of elastic modulus adjustment procedures (EMAP) in their research work. Highly stressed regions of the component or structure were systematically softened by a reduction of their modulus of elasticity in an attempt to simulate local inelastic action. Marriott [5] developed an iterative procedure for estimating lower bound limit loads on the basis of linear elastic finite element analysis (FEA) by generating statically admissible stress fields, and then using them in conjunction with established theorems of limit analysis. Seshadri and Fernando [6] made use of the elastic modulus adjustment procedure to determine lower bound limit loads by adopting "reference stress" concepts in creep design. Mackenzie and Boyle [7] utilized the elastic modulus adjustment procedure suggested by Marriott [5] and Seshadri [8], (generally referred to as the "elastic compensation method") and obtained for every iteration lower- and upper-bound limit loads by invoking established theorems of limit analysis.

In metal forming analysis, there is great interest in predicting loads that will ensure plastic deformation of the body in order to produce the desired shape. An upper bound analysis predicts a load that is at least equal to or greater than the exact load needed to cause plastic flow, while simply focusing on satisfying self-consistent patterns of deformations.

In this paper, EMAP is applied to well-known metal forming processes involving conical and inclined dies. The results of the proposed upper bound methods are then compared with the analytical solution [9] and elastic-plastic finite element analyses.

LIMIT LOAD MULTIPLIERS

Consider a structure made of an elastic perfectly-plastic material that is in equilibrium with the surface traction T_i applied on the surface S_T. On the surface S_u, the constraint $u_i = 0$ is applied as shown in Figure 1. It is assumed that the surface traction is applied as a proportional loading, that is, external tractions are assumed to be γT_i, where γ is a monotonically increasing parameter. For small values of γ, the structure will be in an elastic state. As γ is gradually increased, plastic flow starts to occur at a specific location and then spreads to several parts of the structure. When the load mT_i is applied where $\gamma = m$, the structure will be in a state of impending plastic collapse. Here, m is the limit load multiplier, which is also known as the safety factor. The collapse load of a structure is then evaluated as:

$$T_i\big|_{collapse} = mT_i \tag{1}$$

In this section, different multipliers corresponding to the upper bound formulation are discussed. By using the iterative linear elastic procedure, relevant quantities are defined for the purpose of calculating limit load multipliers.

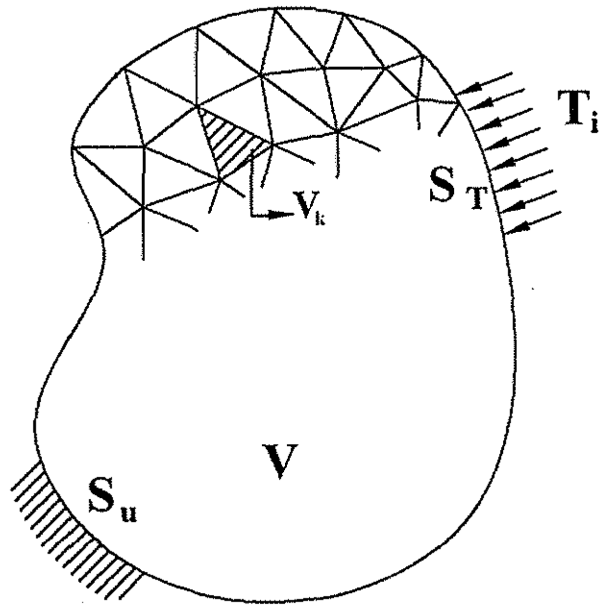

Figure 1: A general component with elastic perfectly-plastic material

Classical Upper Bound Multiplier

According to the upper bound limit load theorem, if for a given load set the rate of internal dissipation of energy in a body is equal to the rate at which the external forces do work for any postulated mechanism of deformation, then the applied load set will be equal to or greater than the plastic collapse load [3]. The classical upper-bound multiplier, m_U, can be obtained from the equation

$$\int_{S_T} T_i u_i \, dS \leq \int_{V_T} D \, dV \tag{2}$$

where T_i is the traction acting on the surface S_T, u_i is the compatible displacement, D is the plastic dissipation energy per unit volume and V_T is the total volume. So long as linear elastic analysis is used, it is possible to substitute strains and displacements with their corresponding rates.

For the von Mises yield criterion, the plastic dissipation energy is given as

$$D = \sigma_y \int_{V_T} \sqrt{\frac{2}{3}\left(\varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2\right)}\, dV \tag{3}$$

where ε_i (i=1, 2, 3) are the three principal strains. In the case of a Tresca perfectly plastic material, the plastic dissipation energy (D) can be expressed as:

$$D = \sigma_y \int_{V_T} \varepsilon_{\max}\, dV \tag{4}$$

where $\varepsilon_{\max}$ is the maximum principal strain.
Using Eq. (2) in conjunction with the divergence theorem, we have:

$$\int_{S_T} T_i\, u_i\, dS = \int_{V_T} \sigma_{ij}\, \varepsilon_{ij}\, dV \tag{5}$$

Substituting Eqs. (2) and (4) into Eq. (1), and making use of the equality sign, the upper-bound multiplier m_U for the von Mises yield criterion can be obtained as

$$m_U = \frac{\int_{V_T} \sigma_y\, \varepsilon_{eq}\, dV}{\int_{V_T} \sigma_{ij}\, \varepsilon_{ij}\, dV} \tag{6}$$

where ε_{eq} is the equivalent strain can be directly evaluated for any linear elastic FEA results.

Multiplier m_1^0

In classical limit analysis, the statically admissible stress field (equilibrium set) cannot lie outside the yield surface, and the stress associated with a kinematically admissible strain rate field in calculating the plastic dissipation should lie on the yield surface. Mura et al. [2] proposed an approach to eliminate such a requirement, and replaced it by the concept of "integral mean of yield" in the context of a variational formulation. The integral mean of yield criterion can be expressed as

$$\int_{V_T} \mu^0 \left[f(\bar{s}_{ij}^0) + (\phi^0)^2 \right] dV = 0 \tag{7}$$

The superscript "0" corresponds to a statically admissible condition, and $\bar{s}_{ij}^0$ is the

statically admissible deviatoric stress ($\bar{s}_{ij}^0 = \bar{\sigma}_{ij}^0 - \frac{1}{3}\bar{\sigma}_{kk}^0\delta_{ij}$) where δ_{ij} is the Kronecker delta, and $\mu \geq 0$. Since $\bar{s}_{ij}^0$ corresponds to the stress state for impending plastic flow, $\bar{s}_{ij}^0 = m^0 s_{ij}^0$, where s_{ij}^0 represents the stress state for applied traction, T_i. The quantity φ^0 is a point function that takes on a value of zero if $\bar{s}_{ij}^0$ is at yield, and remains positive below yield.
The von Mises yield criterion is given by

$$f(s_{ij}) = \frac{3}{2}\bar{s}_{ij}\bar{s}_{ij} - \sigma_y^2 \tag{8}$$

The associated flow rule can be expressed as

$$\dot{\varepsilon}_{ij} = \mu\left(\frac{\partial f}{\partial s_{ij}}\right) \tag{9}$$

Mura and co-workers [2] have shown that m^0, μ^0 and φ^0 (parameters associated with a statically admissible stress state) can be determined by rendering the functional F stationary in

$$F = m^0 - \int_{V_T} \mu^0\left[f(\bar{s}_{ij}^0) + (\varphi^0)^2\right]dV \tag{10}$$

leading to the set of equations

$$\frac{\partial F}{\partial m^0} = 0; \quad \frac{\partial F}{\partial \mu^0} = 0; \quad \frac{\partial F}{\partial \varphi^0} = 0; \tag{11}$$

For the von Mises yield criterion the Eq. (10) becomes

$$F = m^0 - \int_{V_T} \mu^0\left[\frac{3}{2}(m^0)^2\, s_{ij}^0 s_{ij}^0 - \sigma_y^2 + (\varphi^0)^2\right]dV \tag{12}$$

Assuming an unspecified, but constant flow parameter μ^0 in Eq. (12), the solution of the foregoing functional for a uniaxial state of stress becomes

$$\varphi^0 = 0 \tag{13}$$

$$m_1^0 = \frac{\sigma_y\sqrt{V_T}}{\sqrt{\int_{V_T}(\sigma_{eq})^2\, dV}} \tag{14}$$

Multiplier m_2^0

Equation (14) implies that the calculation of m_1^0 is based on the total volume V_T. If plastic collapse occurs over a localized region of the structure, m_1^0 will be significantly overestimated. To overcome this problem, Pan and Seshadri [10] have proposed a new formulation for evaluating m^0, namely m_2^0. It is based on the notion that m^0 is a distributed parameter that characterizes the degree of plastic flow at a given location, and can be readily evaluated as

$$m_2^0 = \sigma_y \left[\frac{\int_{V_T} \left(\frac{\sigma_{eq}}{\varepsilon_{eq}} \right) dV}{\int_{V_T} \sigma_{eq} \varepsilon_{eq} \, dV} \right]^{1/2} \tag{15}$$

ELASTIC MODULUS ADJUSTMENT PROCEDURES

In this paper, we consider two elastic modulus adjustment procedures that are used to analyze metal forming problems: (1) Direct EMAP and (2) Reference Volume Method.
In the section to follow, details of the above methods are outlined.

(1) The aim of the Direct EMAP is to establish a suitable stress field by modifying the local elastic modulus in an effort to obtain inelastic-like stress redistribution. Numerous sets of statically admissible and kinematically admissible distributions are generated in this manner that makes it possible to calculate both lower and upper bound limit loads.

An arbitrary load set along with the original elastic modulus (E_0) is applied in the first iteration of the linear elastic FEA. Subsequently, the elastic modulus of each element is modified in each successive iteration by following equation:

$$E^{i+1} = \left(\frac{\sigma_{ref}}{\sigma_{eq}^{i}} \right)^{q} E^{i} \tag{16}$$

q is the elastic modulus adjustment parameter, σ_{ref} is a reference stress, σ_{eq} is the equivalent stress, and the subscript "i" is the iteration counter (i=1 for the initial elastic analysis). Depending on the geometry and loading conditions, the value of q could be specified as an initial guess. For example, q equal to 1 and 2 are suitable for plane strain and plane stress configurations, respectively. To avoid this guessing however, a variable q scheme has been suggested by Adibi-Asl et al.[11]. For each element during different iterations q is calculated from the respective linear elastic FEA solutions by making the use of the following expression:

$$q = \frac{\ln\left(\frac{2\sigma_{ref}^2}{\sigma_{eq}^2 + \sigma_{ref}^2}\right)}{\ln\left(\frac{\sigma_{ref}}{\sigma_{eq}}\right)} \tag{17}$$

The reference stress σ_{ref}^i is given by the expression [12]:

$$\sigma_{ref}^i = \left[\frac{\int_{V_T} \sigma_{eq}^2 \, dV}{V_T}\right]_i^{1/2} \tag{18}$$

The above equation describes the manner in which the elastic modulus at a given location with an equivalent stress (σ_{eq}) is updated from the i^{th} to the $(i+1)^{th}$ linear elastic FEA iteration. This procedure is repeated until suitable convergence during a subsequent iteration is achieved (A flow chart of this procedure is presented in the Appendix).

To provide a measure for the degree of convergence of a given linear elastic FEA iteration, Seshadri and Indermohan [13] introduced parameter $G(\zeta)$ that can be expressed as

$$G(\zeta) = \sqrt{\frac{\sum_{k=1}^{N}\left[\left(m^0 \frac{\sigma_{eq}}{\sigma_y}\right)^2 - 1\right]^2 \Delta V}{4V_T}} \tag{19}$$

A value of $G = 0$ indicates that a converged distribution has been reached, whereas non-zero G points to a non-converged distribution.

(2) In the Reference Volume Approach, it is assumed that the plastic collapse occurs over a localized region of the mechanical component or structure. Clearly, m_1^0 will be significantly overestimated since it is based on total volume, V_T. The concept of reference volume [12] is introduced to identify the "kinematically active" region of the component or structure that effectively participates in plastic action.

Consider next a component that is subjected to an arbitrary loading condition as shown in Figure 2. The component is divided into two regions: (1) reference volume (V_R), which is the kinematically active volume; and (2) the dead zone (V_D). This means that some portions of the component take part in plastic action while the remaining parts do not contribute to plastic flow. If V_T is the total volume then $V_T = V_R + V_D$, where V_R is the reference volume and V_D is the volume of the dead zone in the component.

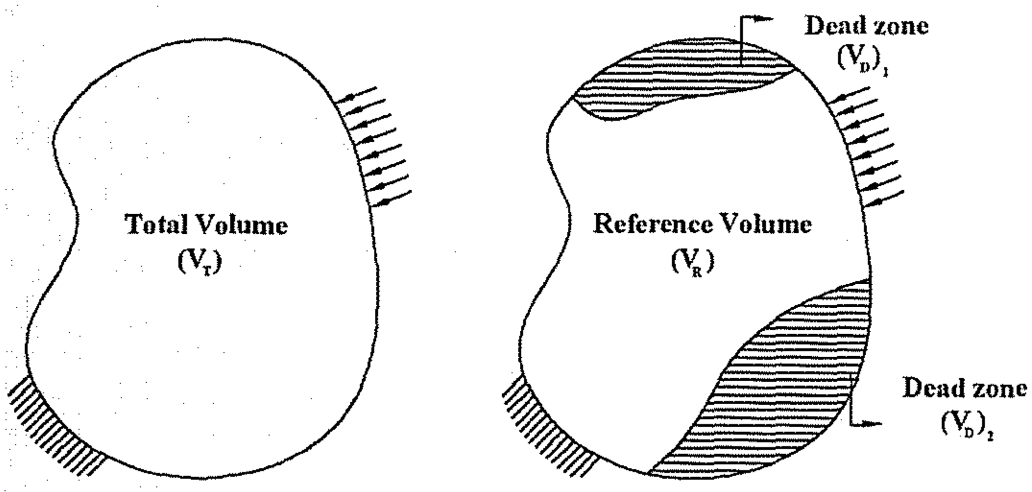

Figure 2: Total and Reference volumes

Therefore, the multiplier m_2^0 can be written in terms of the reference volume and the dead zone volume as

$$m_2^0 = \sigma_y \frac{\sqrt{\int_{V_R} \frac{\varepsilon_{eq}}{\sigma_{eq}} dV + \int_{V_D} \frac{\varepsilon_{eq}}{\sigma_{eq}} dV}}{\sqrt{\int_{V_R} \sigma_{eq}\varepsilon_{eq}\, dV + \int_{V_D} \sigma_{eq}\varepsilon_{eq}\, dV}} \tag{20}$$

If we assume that the dead zone has no deformation occurring, then the equation (15) can be simplified as

$$m_2^0 = \frac{\sigma_y \sqrt{\int_{V_R} \frac{\varepsilon_{eq}}{\sigma_{eq}} dV}}{\sqrt{\int_{V_R} \sigma_{eq}\varepsilon_{eq}\, dV}} \tag{21}$$

The magnitude of the multiplier, m_2^0, would therefore depend on the sub-volume, V_η, where

$$V_\eta = \sum_{k=1}^{\eta} \Delta V_k \tag{22}$$

In order to identify the reference volume V_R and the corresponding multipliers $m_2^0(V_R)$, two methods are suggested as follows:

Method 1: The stress distribution within the elements obtained from linear elastic finite element analysis is sorted in descending order, i. e., $\sigma_{eq}^{(1)} > \sigma_{eq}^{(2)} > \sigma_{eq}^{(3)} > \ldots > \sigma_{eq}^{(N)}$. Corresponding to the stresses $\sigma_{eq}^{(1)}$, $\sigma_{eq}^{(2)}$, $\sigma_{eq}^{(3)}, \ldots,$ $\sigma_{eq}^{(N)}$ are the volumes $\Delta V^{(1)}$, $\Delta V^{(2)}$, $\Delta V^{(3)}, \ldots, \Delta V^{(N)}$, respectively. Here, V_η is the sub-volume that represents the volume summations started from the element with the highest equivalent stress to the η^{th} element.

For the maximum stress element of volume $\Delta V^{(1)}$, m_2^0 increases with increasing i. On the other hand m_2^0, evaluated on the basis of the total volume, would decrease with increasing i. Therefore, for some intermediate volume V_R (where $\Delta V_1 < V_R < V_T$ corresponding to $\eta = \eta^*$), the multiplier m_2^0 would be invariant, i. e., $(m_2^0)_I = (m_2^0)_{II}$. The schematic of variation of $m_2^0(\overline{V}_\eta)$ with the iteration variable is shown in Figure 3 , where $\overline{V}_\eta = V_\eta / V_T$.

Method 2: Similar to the Method 1, the stress sequence of the elements obtained from finite element analysis is sorted in descending order. Next, the variation of upper bound multiplier in each subsequent iteration, $(m_2^0)_i$, is plotted against the volume ratio, $\overline{V}_\eta$, as illustrated in Figure 4. All such distributions will intersect at a specific location, $\overline{V}_\eta = \overline{V}_{\eta^*}$, showing that the multiplier m_2^0 would be invariant, i. e., $(m_2^0)_I = (m_2^0)_{II}$.

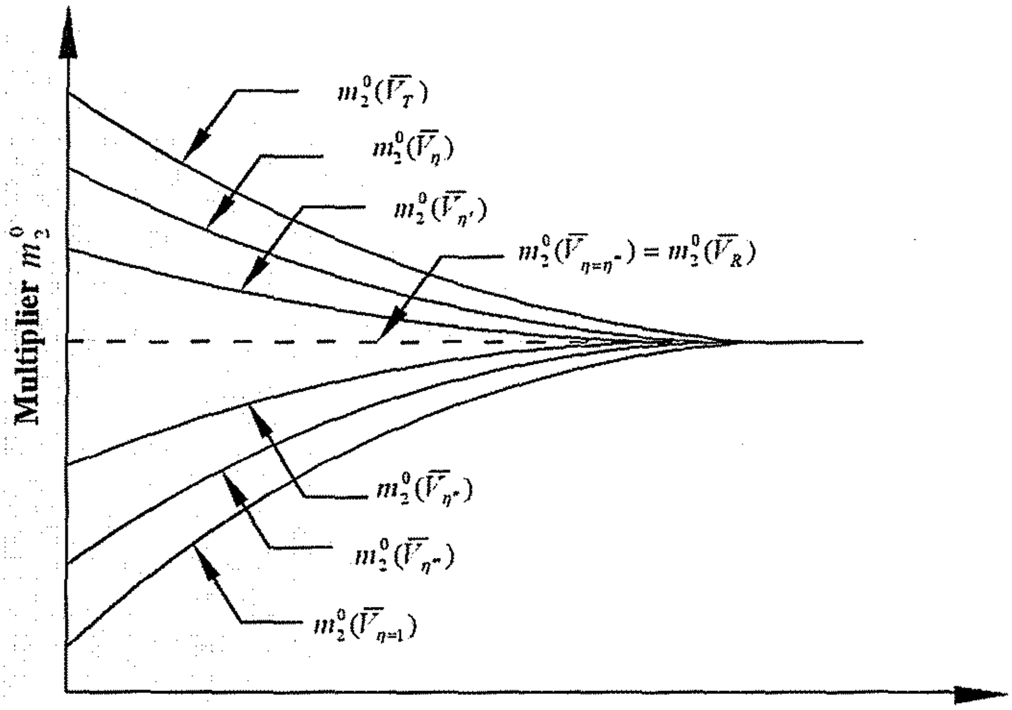

Figure 3: Variation of m_2^0 with Elastic Iterations

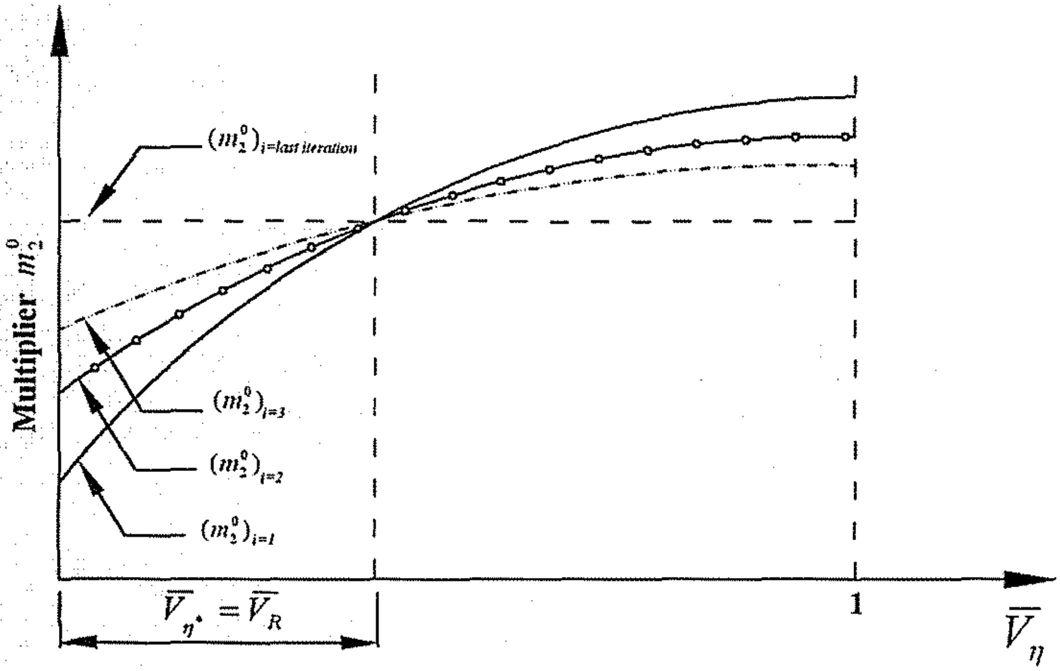

Figure 4: Determination of Reference Volume

NUMERICAL EXAMPLES

In this section numerical simulation of some practical geometric configurations of metal forming processes using the Direct EMAP and the Reference Volume Approach is carried out. Other than the aim of demonstrating the advantages of the methods concerning the simulation, specific features regarding the numerical parameters are also addressed. On the basis of a comparison with other established methods, more can be learnt about the way these factors affect the solution process. The various problems are modeled using the ANSYS finite element software program (university research version [14]).

Flow through conical converging dies

Axisymmetric extrusion and rod-and-wire drawing are common metal forming processes that occur through conical converging dies. This example is particularly of interest because of the practical significance. The problem can also be analyzed by other methods, such as the slip line field and the upper bound analysis. The underlying process is schematically shown in Figure 5. The billet is a cylindrical rod of diameter D_0; the rod is reduced to diameter D_f by forcing it to pass through the conical converging die. The reduction, r, is defined as the ratio of the cross-section area of the billet at the entrance and the die (A_f) to that at the exit (A_0), leading to following equation:

$$r = \left[1 - \left(\frac{D_f}{D_0}\right)^2\right] \tag{23}$$

Due to the variation of velocity fields, the billet is divided into three zones in which the velocity fields are assumed to be continuous. In zones I and III the velocity fields are uniform, and only have components in x direction; however in zone II, the velocity field is directed towards the apex , O, of the cone.

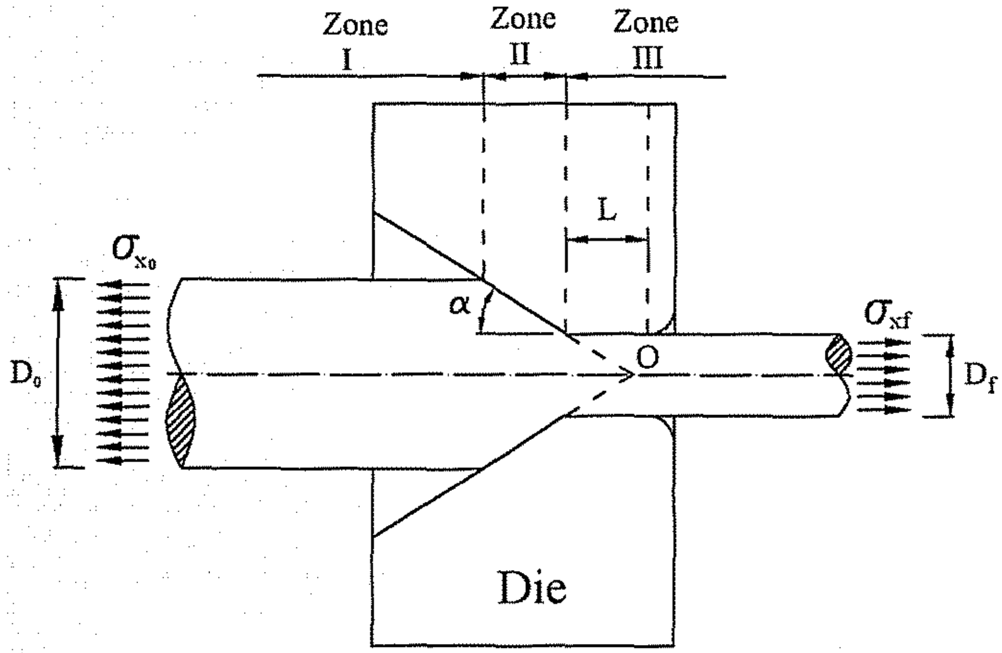

Figure 5: General geometry of axisymmetric extrusion/drawing process

Using the upper bound formulation and assuming a constant shear stress, irrespective of pressure between die and material, the upper bound stresses required for extrusion and drawing have been obtained as follows [9]:

For extrusion (Fig. 5):

$$\frac{\sigma_{x0}}{\sigma_y} = \frac{\sigma_{xf}}{\sigma_y} - 2f(\alpha)\ln\left(\frac{D_0}{D_f}\right) - \frac{2}{\sqrt{3}}\left[\frac{\alpha}{\sin^2\alpha} - \cot\alpha + \overline{m}(\cot\alpha)\ln\left(\frac{D_0}{D_f}\right) + 2\overline{m}\frac{L}{D_f}\right] \tag{24}$$

For drawing:

$$\frac{\sigma_{xf}}{\sigma_y} = \frac{\sigma_{x0}}{\sigma_y} + 2f(\alpha)\ln\left(\frac{D_0}{D_f}\right) + \frac{2}{\sqrt{3}}\left[\frac{\alpha}{\sin^2\alpha} - \cot\alpha + \overline{m}(\cot\alpha)\ln\left(\frac{D_0}{D_f}\right) + 2\overline{m}\frac{L}{D_f}\right] \tag{25}$$

where the function $f(\alpha)$ can be calculated by the following equation

$$f(\alpha)=\frac{1}{\sin^2\alpha}\left[1-(\cos\alpha)\sqrt{1-\frac{11}{12}\sin^2\alpha}+\frac{1}{\sqrt{11\times 12}}\ln\left(\frac{1+\sqrt{\frac{11}{12}}}{\sqrt{\frac{11}{12}}\cos\alpha+\sqrt{1-\frac{11}{12}\sin^2\alpha}}\right)\right] \tag{26}$$

For this particular example, contact elements were used on the tool work piece interface with a shear factor of $\overline{m}$ =0.02 (only at zone II); eight noded axisymmetric rectangular elements were used for other regions; and a ram pressure of 20 MPa was applied. The billet material is copper with an elastic modulus of E=100 MPa, yield stress σ_y=40 MPa, and the die semi-angle is 16 degrees.

The variation of limit load multipliers with successive iterations for the Direct EMAP is plotted in Figure 6. Likewise, the variation in the value of G is shown in Figure 7. The analytical solution, equation (25), predicts a higher value of the limit load than m_2^0 and m_U. On the other hand, inelastic FEA predicts a lower value of limit load than the present method. As can be inferred from Figure 7, the G value tends to decrease during successive iterations, eventually reaching a converged state. The difference between m_1^0 and m_2^0 which has been described earlier is due to local plastic regions in the body.

Using the Reference Volume Approach, it is possible to determine the active volume, and the related multipliers, as described before in Method 1 and Method 2. The variation of $m_2^0(\eta)$ with elastic iterations for axisymmetric drawing is presented in Figure 8. As it can be seen from the figure, at $\overline{V}_\eta$=0.19, the value of m_2^0 is almost constant during successive iterations and represents a good estimate of the exact solution. Figure 9 represents Method 2, in which variation of m_2^0 with respect to $\overline{V}_\eta$ for different iterations is plotted. It can be seen from the figure that all the curves intersect at a specific point ($\overline{V}_\eta$=0.19) which represents the reference volume.

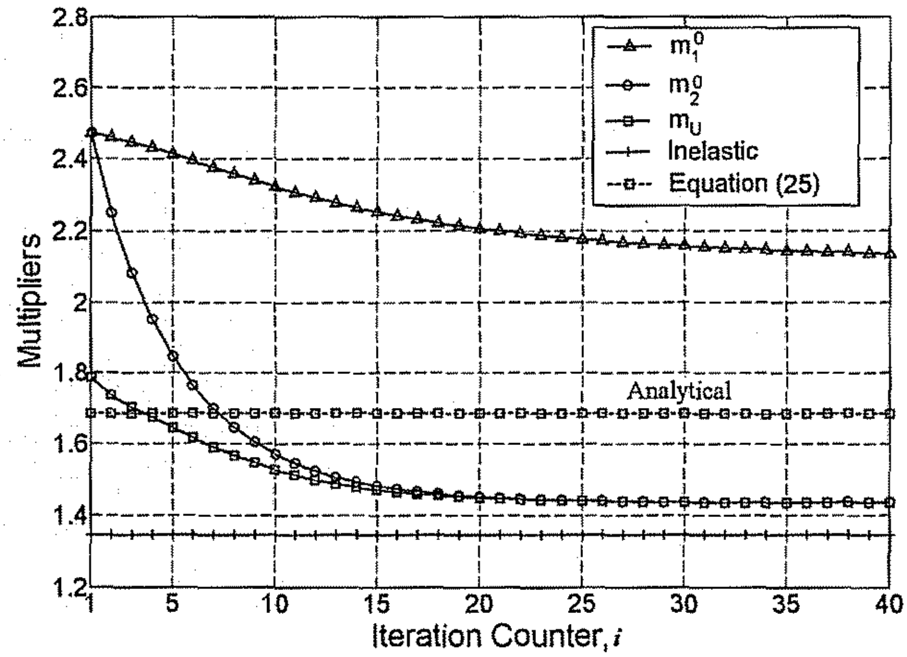

Figure 6: Variation of limit load multipliers for axisymmetric drawing- Direct EMAP

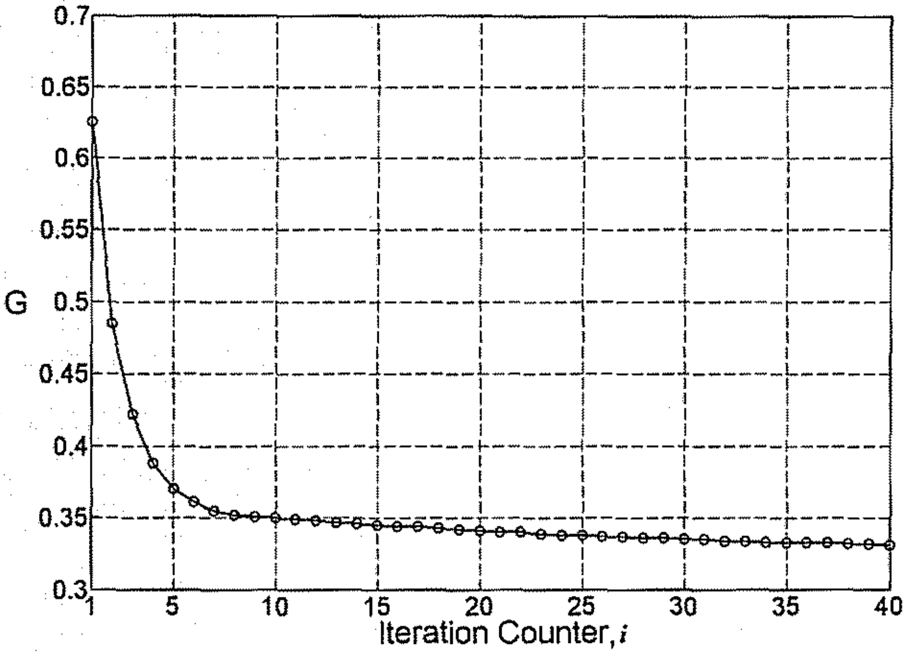

Figure 7: Variation of G for axisymmetric drawing- Direct EMAP

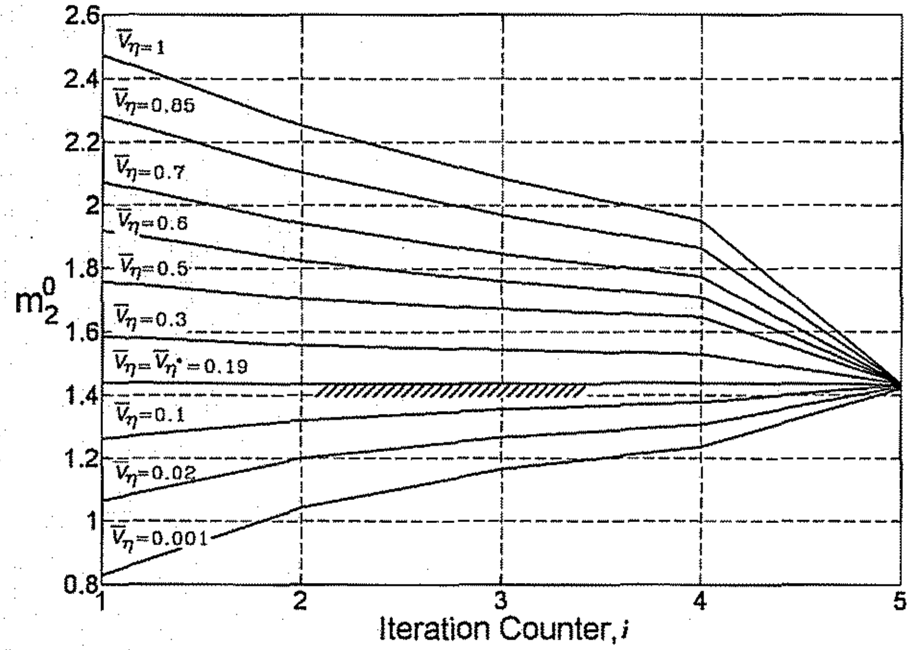

Figure 8: Variation of m_2^0 with elastic iterations for axisymmetric drawing - Reference Volume Approach (Method 1)

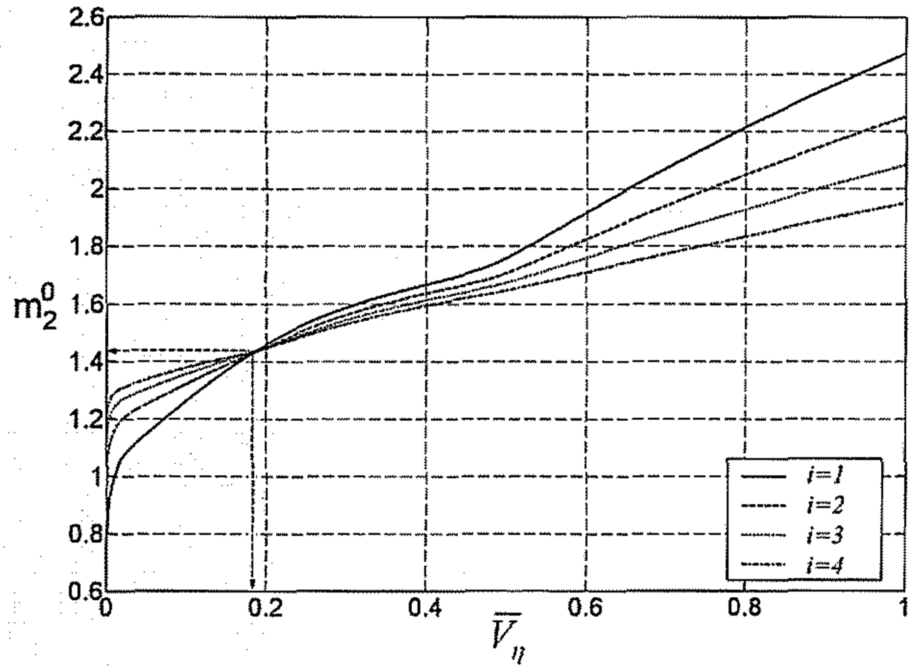

Figure 9: Variation of m_2^0 versus $\overline{V}_\eta$ for axisymmetric drawing -Reference Volume Approach (Method 2)

Flow through inclined plates

Extrusion/drawing through inclined plates is a plastic deformation process in which a block of metal (billet) is pushed to flow by compression through the die opening of a smaller cross-sectional area than that of the original billet as shown in Figure 10. Similar to the conical die, the billet is divided into three zones. The thickness of billet is much smaller than its length and width. The original thickness of the billet is t_0 and the final thickness is t_f. The reduction in area is given by expression:

$$r = \left[1 - \left(\frac{t_f}{t_0}\right)\right] \tag{27}$$

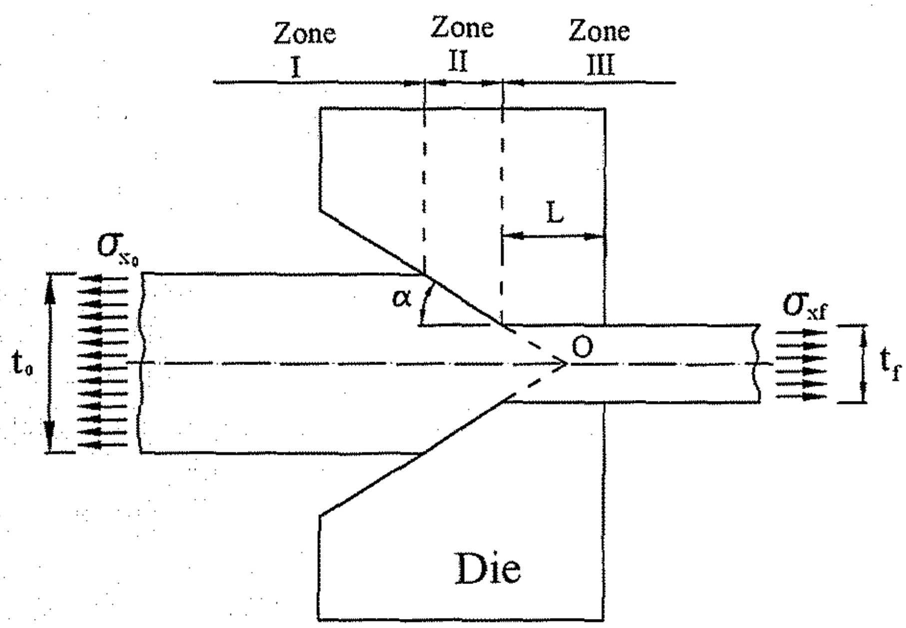

Figure 10: General geometry of plane strain extrusion/drawing process

Using the upper bound theory, appropriate stresses for extrusion and drawing can be obtained as [9]:

For extrusion:

$$\frac{\sigma_{x0}}{\sigma_y} = \frac{\sigma_{xf}}{\sigma_y} - \frac{2}{\sqrt{3}}\left[\frac{Ei\left(\frac{\sqrt{3}}{2}, \alpha\right)}{\sin\alpha}\ln\left(\frac{t_0}{t_f}\right) + \frac{1-\cos\alpha}{\sin\alpha} + \frac{\overline{m}}{2}(\cot\alpha)\ln\left(\frac{t_0}{t_f}\right) + \overline{m}\frac{L}{t_f}\right] \tag{28}$$

For drawing:

$$\frac{\sigma_{xf}}{\sigma_y}=\frac{\sigma_{x0}}{\sigma_y}+\frac{2}{\sqrt{3}}\left[\frac{Ei\left(\frac{\sqrt{3}}{2},\alpha\right)}{\sin\alpha}\ln\left(\frac{t_0}{t_f}\right)+\frac{1-\cos\alpha}{\sin\alpha}+\frac{\overline{m}}{2}(\cot\alpha)\ln\left(\frac{t_0}{t_f}\right)+\overline{m}\frac{L}{t_f}\right] \tag{29}$$

where $Ei\left(\frac{\sqrt{3}}{2},\alpha\right)$ is an elliptic integral of the second kind, i. e.,

$$Ei\left(\frac{\sqrt{3}}{2},\alpha\right)=\int_0^{\alpha}\sqrt{1-\frac{3}{4}\sin^2\theta}\,d\theta \tag{30}$$

Except for a semi die angle of 12 degrees, a similar material model, loading condition and geometry are used as in the axisymmetric drawing (plane-strain problem). The variation of multipliers (Direct EMAP) is presented in Figure 11. As well, the corresponding variation in the value of G is presented in Figure 12. Comparing the results with equation (29), the Direct EMAP calculation predicts lesser value of the limit load.

The variation of $m_2^0(\eta)$ with elastic iterations for plane strain drawing (Method 1) is presented in Figure 13 . As it can be seen from the figure, for $\overline{V}_\eta$=0.31 the value of m_2^0 is almost constant during the successive iterations leading to a good estimate of the exact solution. Using Reference Volume Approach (Method 2), the variation of m_2^0 with respect to $\overline{V}_\eta$ for different iteration number is plotted in Figure 14. All curves intersect at a specific point ($\overline{V}_\eta$=0.31), representative of the kinematically active volume of the component. Again, by calculating upper bound multiplier (m_2^0), a good approximation of the limit load multiplier can be obtained.

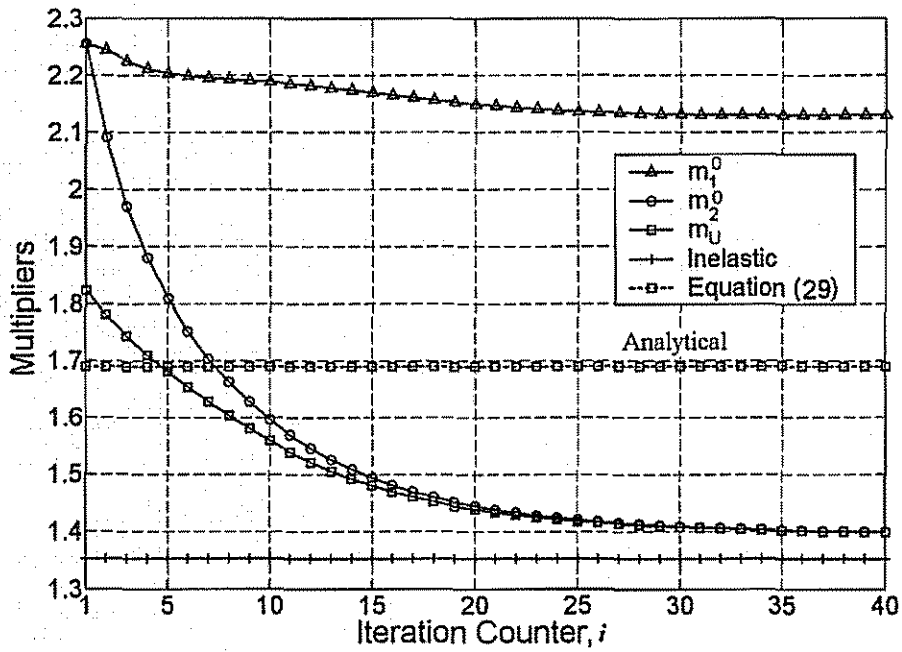

Figure 11: Variation of limit load multipliers for plane-strain drawing-Direct EMAP

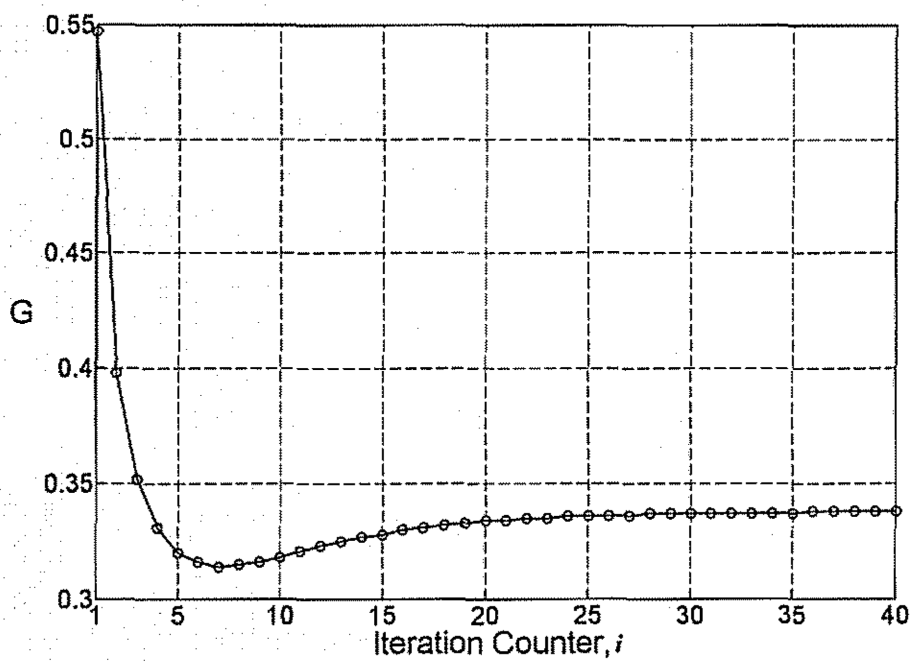

Figure 12: Variation of *G* for plane strain drawing-Direct EMAP

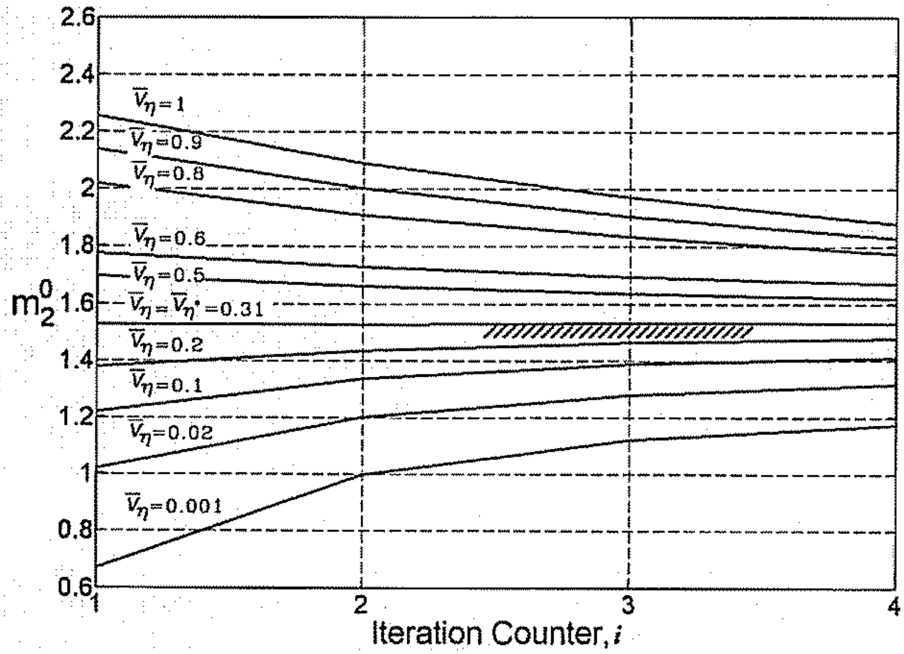

Figure 13: Variation of m_2^0 with elastic iterations for plane strain drawing - Reference Volume Approach (Method 1)

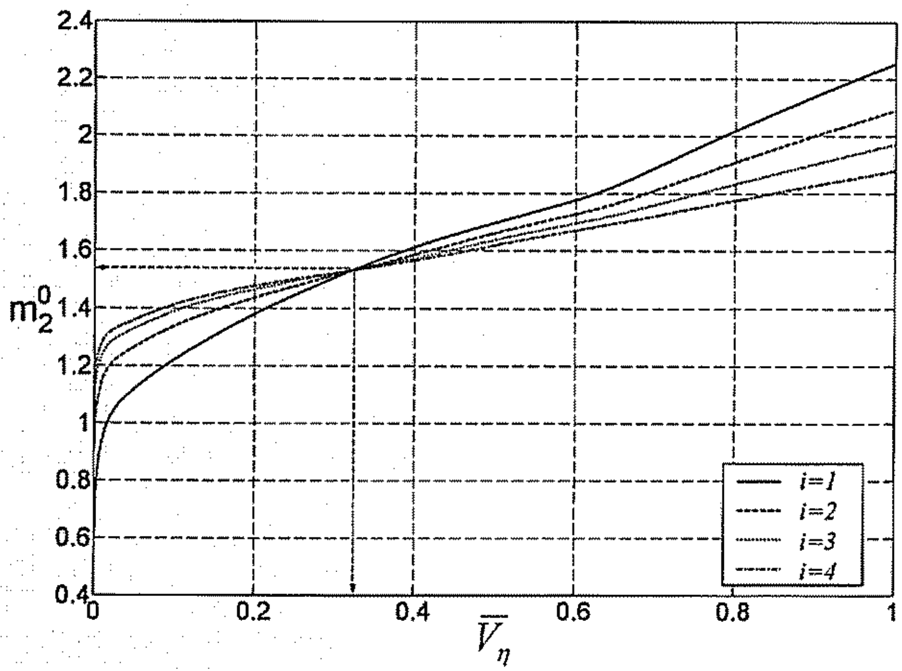

Figure 14: Variation of m_2^0 versus $\overline{V}_\eta$ - Plane strain drawing - Reference Volume Approach (Method 2)

CONCLUSION

The application of EMAP for the estimation of load requirements in metal forming processes is simple and can be easily implemented. In addition, the limit load predicted in this manner represents conservative cost effective results that can be obtained rapidly, making it an attractive alternate method for the analysis of metal forming processes. The results from Direct EMAP indicates a significant improvement in the limit load estimates ensuring adequate deformation of raw material as required to form the desired components. Improved convergence criterion incorporated in this method yields a stable iterative solution process and eliminates the convergence difficulties usually encountered with full inelastic finite element analysis.

The parameter G evaluated in this paper acts as a convergence measure, and is indicative of any deviation of statically admissible stress distributions from the exact limit state. Lower the asymptotic value of G, the better is the estimate of the limit load, i.e., $G \to 0$ would correspond to the exact solution. The reference volume methods, which enable the identification of "kinematically active" regions in a component, are powerful EMAP-based method for estimating the limit load.

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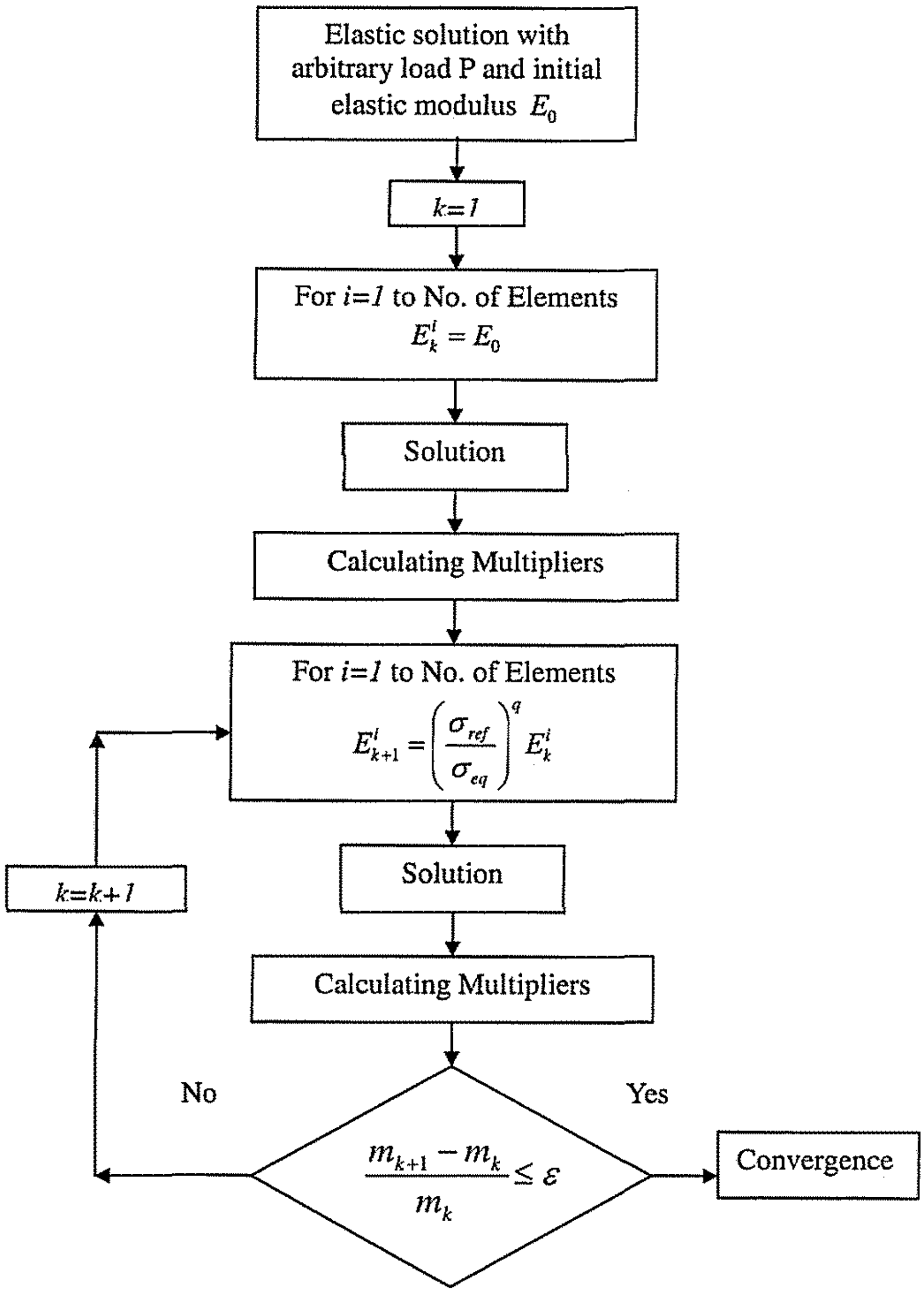
APPENDIX:

Flowchart of Elastic Modulus Adjustment Procedure (EMAP)



NOMENCLATURE

A	area
A_0	original area
A_f	final area
D	energy dissipation
D_0	original diameter
D_f	final diameter
E	modulus of elasticity
G	convergence-measure parameter
i	iteration variable
L	contact length of die in zone III
m	exact limit load multiplier
m_U	classical upper bound limit load multiplier
m^0	upper bound limit load multiplier
m_1^0	upper bound limit load multiplier based on total volume
m_2^0	upper bound limit load multiplier based on kinematically active volume
$\overline{m}$	shear factor
q	elastic modulus index
r	reduction ratio
S	surface
$\overline{s}_{ij}^0$	statically admissible deviatoric stress field corresponding to the traction $m^0 T_i$
s_{ij}^0	statically admissible deviatoric stress field corresponding to the traction T_i
t_0	original thickness
t_f	final thickness
T_i	traction
u	displacement
V_D	dead zone volume
V_T	total volume
V_R	kinematically active or reference volume
α	semi-die angle
δ_{ij}	Kronecker delta
ε	strain
ε_1, ε_2, ε_3	principal strain
ε_{eq}	equivalent strain
ε_{max}	maximum strain
ϕ^0	point function defined in conjunction with yield criterion

μ^0	plastic flow parameter
σ	stress
σ_{eq}	equivalent stress
σ_{ref}	reference stress
σ_y	yield stress

Superscript

0	statically admissible

Abbreviation

EMAP	Elastic Modulus Adjustment Procedures
FEA	Finite Element Analysis