PARAMETER IDENTIFICATION IN A HIGH PERFORMANCE HYDROSTATIC ACTUATION SYSTEM USING THE UNSCENTED KALMAN FILTER

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ABSTRACT

This paper describes an early fault detection strategy for a high performance hydrostatic actuation system, referred to as the ElectroHydraulic actuator (EHA). Safety is crucial for the EHA which is being applied in flight surface actuation systems and in robotics. The proposed fault detection methodology in this manuscript uses a new state/parameter estimation algorithm, referred to as the Unscented Kalman Filter (UKF) to estimate parameters which cannot be measured using sensors. The parameters reflect the health condition of the system and changes in their normal values can be related to the inception and progression of faults in the system. The two parameters of interest in this study are the viscous damping coefficient of a symmetrical actuator and the effective bulk modulus of the hydrostatic system. The feasibility of the approach is demonstrated by a simulation study and using experimental data. Changes in the viscous damping coefficient provide valuable information about the lubricating properties of the oil and the seal conditions of the actuator. Changes in the effective bulk modulus, as a result of air getting trapped in the system, will change the system response, affecting the natural frequency and may cause stability problems. In this paper, the UKF is used for the first time for parameter estimation in a hydraulic system.

L'IDENTIFICATION DE PARAMETRE DANS UN SYSTEME HYDROSTATIQUE A HAUTE PERFORMANCE EN UTILISANT LE UNSCENTED KALMAN FILTER

RESUME

L'article présente une stratégie de détection de défauts dans un système hydraulique hydrostatique à haute performance, appelé l'EHA (Electrohydraulic Actuator). L'application de l'EHA dans l'aéronautique et la robotique requiert un niveau de sécurité élevé. La méthodologie proposée ici pour détecter des défauts est basée sur l'utilisation d'un algorithme nouveau notamment, Unscented Klaman Filter (UKF), pour estimer des états et des paramètres. L'UKF estime des paramètres qui sont non mesurables par des capteurs. Des changements dans les valeurs de ces paramètres révèlent la présence de défauts dans le système hydraulique. Les deux paramètres qui sont estimés sont le coefficient d'amortissement visqueux dans le vérin hydraulique et le module de compressibilité de l'huile hydraulique. La faisabilité de cette méthode est démontrée en simulation et la méthodologie est validée par des résultats expérimentaux. Des changements dans le coefficient d'amortissement indiquent que l'huile hydraulique se dégrade ou que les joints dynamiques, se trouvant à l'intérieur du vérin, s'usent. Le module de compressibilité change par la présence de poches d'air coincées dans le système hydrostatique. La fréquence propre peut changer, pouvant affecter la stabilité du système. L'article présente, pour la première fois, l'utilisation de l'UKF pour estimer des paramètres dans un système hydraulique.

1. INTRODUCTION

Hydraulic systems are found in a wide range of applications and some of their unique features include a broad range of operation for actuators and motors, high torque to mass ratio at the actuation point, and the fluid itself acting as a lubricant [1]. As such, hydraulic systems are widely used in mobile and airborne applications. In this study, fault detection in a high performance hydrostatic system, referred to as the ElectroHydraulic Actuator (EHA) is investigated. The EHA has been introduced in [2] and a brief description will be provided here. A schematic of the EHA prototype is shown in Figure 1 and a photograph in Figure 2.

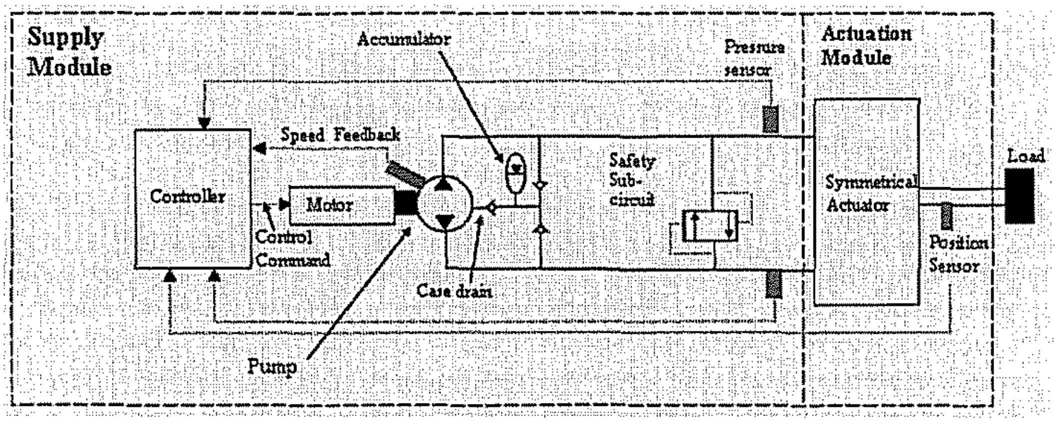

Figure 1: Schematic of the Electrohydraulic Actuator (EHA)

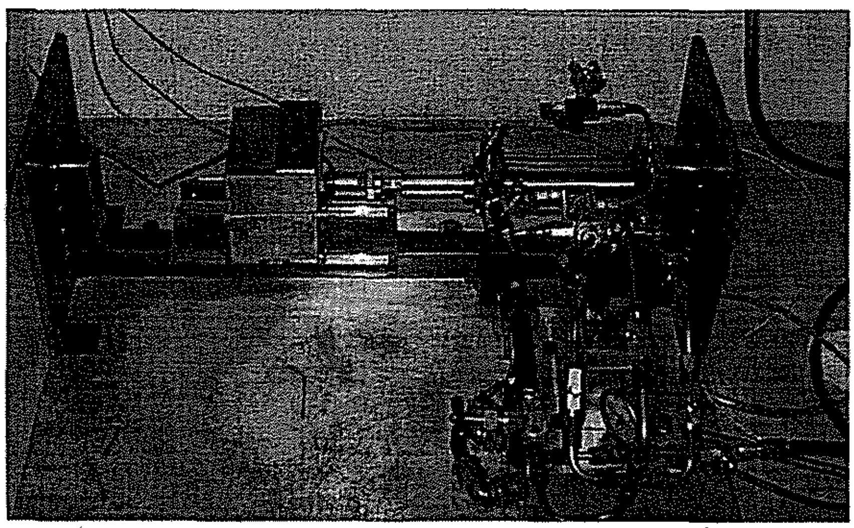

Figure 2: Photograph of the Electrohydraulic Actuator (EHA)

The EHA is a high performance, closed loop hydrostatic system consisting of a variable speed electric motor, a bi-directional fixed displacement gear pump, an accumulator, connecting tubes, a custom made symmetrical actuator and sensors. Potential applications of the EHA are in robotics and the aerospace industry [3]. This actuator offers advantages such as high-energy efficiency, a closed loop position precision of 1 micron with an inertial load of 20 kg, and compactness due to the custom made symmetrical actuator with no dead space, as explained in [2] and [4].

An effective fault detection strategy is important for the EHA because failures can be dangerous and costly. Early fault detection based on detecting degradation in the performance of the system is useful to increase safety and for reducing downtime. The two parameters that will be investigated in this study are the viscous damping coefficient of the symmetrical actuator and the effective bulk modulus of the hydrostatic system. Changes in the viscous damping coefficient, when the oil temperature is kept constant, is a good indication that the actuator seals may be experiencing wear, or that the oil is degrading due to contaminants, resulting in reduced lubricating properties. The bulk modulus is an important fluid property in determining the dynamic performance of hydraulic systems. The bulk modulus of oil decreases as a result of air being trapped in the system causing the fluid to become spongy, lowering the response time, affecting efficiency and causing stability problems [1]. Air can get trapped in the EHA during maintenance and when the system is refilled with oil. In section 2 of this paper, the Unscented Kalman Filter (UKF) is introduced. The algorithm is described in greater detail in the appendix. The UKF is then applied to a state space model of the EHA in section 3 where the viscous damping coefficient is estimated in simulation first, and then using experimental data, followed by section 4 where the effective bulk modulus is estimated. Concluding remarks are given in section 5. The nomenclature is provided in Table 1.

2. INTRODUCTION TO THE UNSCENTED KALMAN FILTER (UKF)

As mentioned in section I, the Unscented Kalman Filter (UKF) is used in this study to estimate two important parameters of the EHA, the viscous damping coefficient and the effective bulk modulus. The Kalman filter, which is a well-known optimal state estimator for dynamic systems [5, 6], is a linear, unbiased and minimum mean error variance algorithm used to optimally estimate the unknown states of a dynamic system from noisy data taken at discrete time intervals [7]. The Kalman gain, which minimizes the error covariance between the estimated state and the actual state, is used to correct the state estimate with the latest sensor measurements. The Kalman filter is a recursive algorithm also consisting of a prediction stage and it uses a mathematical model of the system to predict the states for the next step. A parameter estimation problem is basically a state estimation problem requiring a nonlinear formulation, whereby the unknown parameter is included in the state vector. As such, the formulation of Kalman filter for nonlinear problems referred to as the Extended Kalman Filter (EKF) is required.

The EKF has become a standard estimation technique and has been used in a number of nonlinear estimation problems, including estimating both the states and parameters simultaneously [5]. The EKF has been applied with some success to hydraulic systems as described in [8-11]. An essential operation performed in the EKF algorithm is the propagation of a random variable through the system dynamics, i.e. using the system model. The random variable is actually propagated through the first-order linearization of the nonlinear system (Taylor Series approximation). This can introduce large errors in the mean or estimate and covariance of the transformed Gaussian random variable. Moreover, those errors might be significant enough to lead to sub-optimal performance and sometimes divergence of the filter [6].

In this paper, the Unscented Kalman Filter (UKF), proposed by Julier and Uhlman in 1995 [12], is used instead of the more familiar EKF. The UKF uses a so-called "*deterministic sampling approach*", [13]. The state distribution is represented using a minimal set of carefully chosen sample points, referred to as "*sigma points*"[12-14]. These sample points completely capture the true mean and covariance of the Gaussian random variable, and when propagated through the true non-linear system, capture the posterior mean and covariance accurately to the 3rd order (Taylor series expansion) for any nonlinearity, compared to the EKF which achieves first-order accuracy. Since no linearization is required for the UKF, the computational complexity of the UKF is not much different from that of the EKF. Julier and Uhlman demonstrated the substantial performance gains of the UKF in the context of state-estimation for nonlinear control [12].

The application of the UKF to hydraulic systems is believed to be important because the UKF, a powerful nonlinear estimation technique, can cope with the inherent nonlinearities in hydraulic systems. As such, the UKF is applied for the first time to a hydraulic system in this paper.

3. ESTIMATION OF THE VISCOUS DAMPING COEFFICIENT FOR THE EHA

The UKF procedures and algorithm are summarized in Appendix I and used in this section for the estimation of the states and the viscous damping coefficient for the EHA system. The state and parameter estimation is conducted here by a computer simulation using Matlab/Simulink and then experimentally verified. The mathematical model used in the UKF algorithm to describe the relationship between the pressure difference $(P_1 - P_2)$ in the actuator chambers and piston displacement is as follows:

$$(P_1 - P_2)A = M\ddot{X} + B\dot{X} \qquad (1)$$

The nomenclature is given in Table 1. The input to the UKF is the load pressure and the measurement for the algorithm is the piston displacement, X. The viscous damping coefficient, $X_3(k)$ is included in the discrete state space model for the actuator, as shown in equation (2).

$$\begin{aligned}
X_1(k+1) &= X_1(k) + T_s X_2(k) + T_s w_1(k) \qquad (2)\\
X_2(k+1) &= \frac{U(k)AT_s}{M} - \frac{X_3(k)T_s X_2(k)}{M} + X_2(k) + T_s w_2(k)\\
X_3(k+1) &= X_3(k) + T_s w_3(k)\\
Z(k) &= X_1(k) + v(k)
\end{aligned}$$

where $X_1(k)$, $X_2(k)$, $X_3(k)$ are the displacement, velocity and viscous damping coefficient respectively, $w(k)$ the system noise, $U(k) = (P_1 - P_2)$ the pressure difference between the actuator chambers , $Z(k)$ measurement vector (piston position) and $v(k)$ the measurement or sensor noise. The simulated pressure difference across the chambers of the actuator $(P_1 - P_2)$ was a sine shaped signal and the simulated load pressure was used as the input to the UKF algorithm. The estimation of the viscous damping coefficient in the simulated system using the UKF is illustrated in Figure 3. The other input to the UKF filter was the piston displacement.

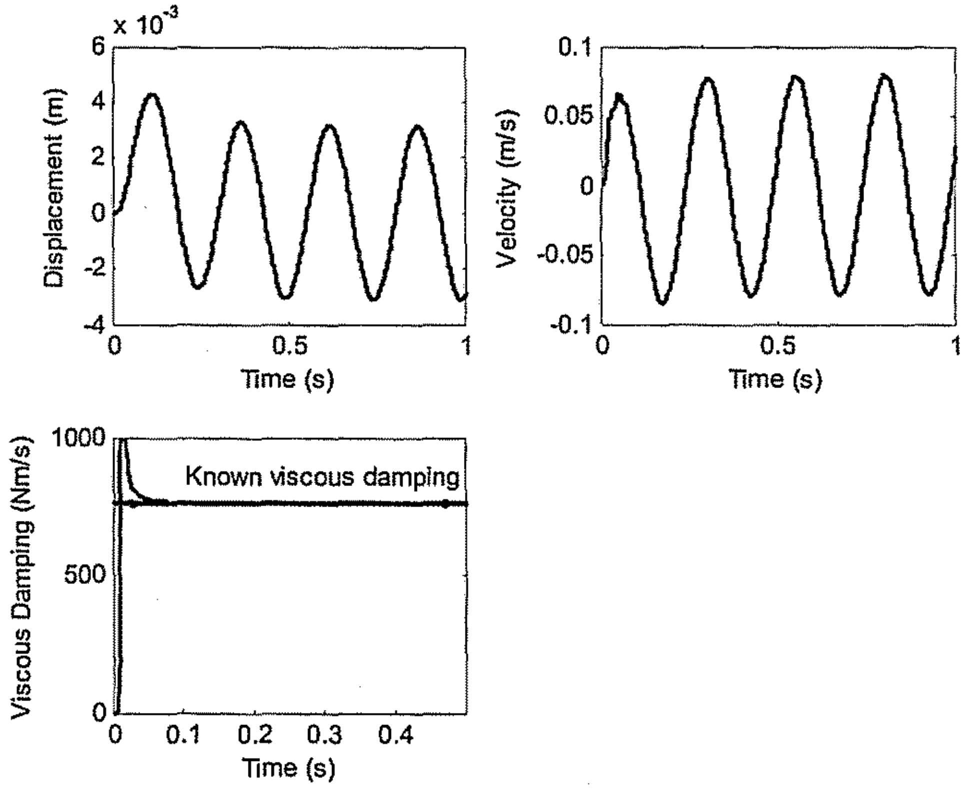

Figure 3a: Estimated states and viscous damping coefficient in simulation

It can be observed that the estimated piston displacement converges to the simulated piston displacement as shown in the first superimposed plot in Figure 3a. The piston velocity is also estimated by the UKF and from the superimposed plot in Figure 3a, it can be observed that the estimated velocity agrees with the simulated velocity. The viscous damping coefficient is estimated successfully by the UKF. The estimated parameter converges to its simulated value with an estimation error of less than 0.5 %. Having verified in simulation that the UKF can be used to estimate the parameter for a healthy EHA system, the feasibility of the fault detection strategy is investigated by changing the simulated viscous damping coefficient and using the UKF to estimate the new value of the parameter. The estimation process is illustrated in Figure 3b. The estimated states match the simulated states closely and the viscous damping coefficient, which was increased by 20% in the simulated model, was estimated with an error of 1.3%. Hence the feasibility of using the UKF for fault detection is demonstrated. The UKF detects a change in the parameter and estimates it accurately.

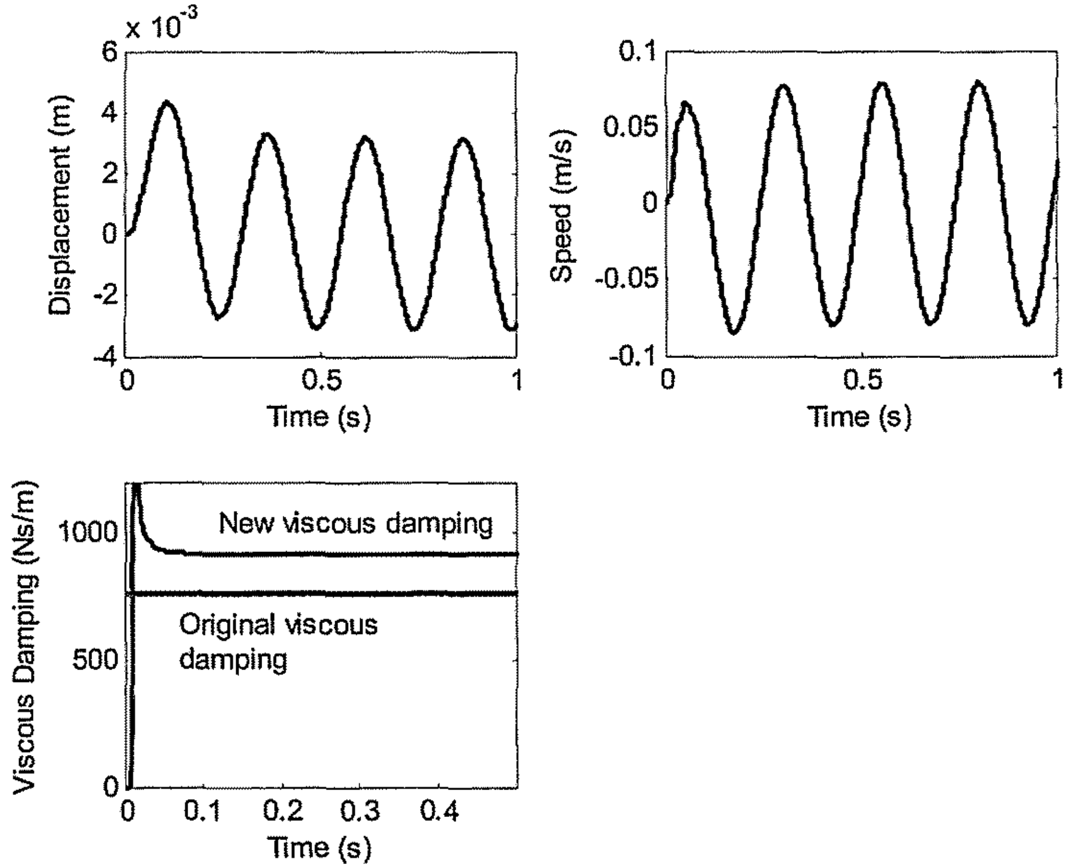

Figure 3b: Estimated states and new viscous damping coefficient by UKF

The methodology which was developed was then applied to the EHA prototype. The UKF was used to estimate the viscous damping coefficient for the EHA prototype and simulated data was replaced by load pressure and piston position measurements from the real system. The estimation process is illustrated in Figure 4. It is observed that the estimated piston position converges to the measured position and the estimated piston velocity shows good agreement with the measured piston velocity, represented by the broken lines. The estimated viscous damping coefficient is 756 Ns/m. The estimations were repeated several times and the results were within a variation of 2 %. The temperature was monitored carefully during the experiments and was kept constant for all the experiments since viscous damping coefficient is known to decrease with an increase in oil temperature. Also, similar to the simulation studies, no a priori information about the states are assumed, i.e. all the states are initialized to zero in the UKF algorithm.

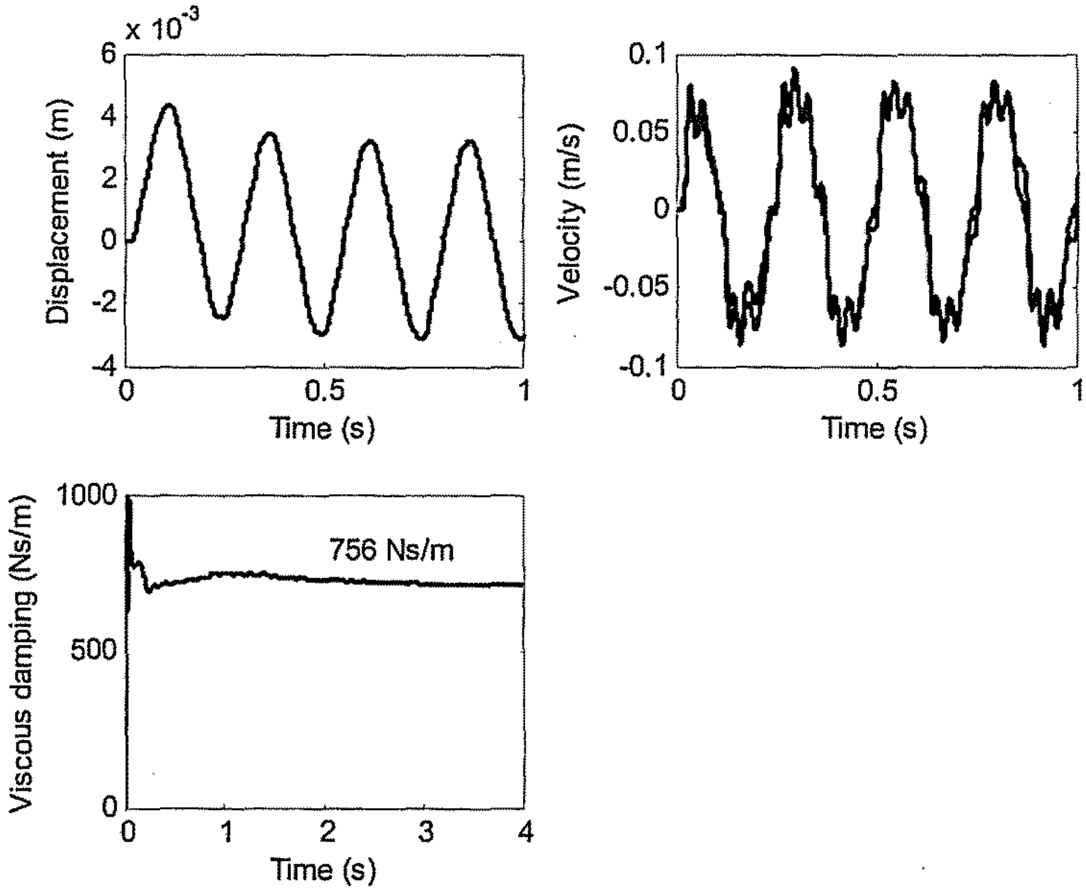

Figure 4: Estimated states and viscous damping coefficient in the EHA prototype

4. ESTIMATION OF THE EFFECTIVE BULK MODULUS FOR THE EHA

The second parameter of interest in this study is the effective bulk modulus of the hydrostatic system. Further to the UKF process specified in the appendix, the first step in applying the UKF to the EHA for the estimation of the effective bulk modulus is to develop a mathematical model for the EHA. A simplified model for the EHA, representing the piston position as output $x(s)$ and the pump angular velocity $\omega_p(s)$ as the input for the hydrostatic system has been introduced in [8] and is given by:

$$\frac{x(s)}{\omega_p(s)} \approx \frac{\dfrac{2D_p\beta_e A}{MV_o}}{s^3 + s^2\left(\dfrac{B}{M} + \dfrac{C_T\beta_e}{V_0}\right) + s\left(\dfrac{2\beta_e A^2}{MV_o} + \dfrac{C_T B\beta_e}{MV_0}\right)} \tag{3}$$

Parameter values are shown in Table 1 which is found in the nomenclature. The mathematical model (the state space model) embedded in the UKF is described below:

$$\dddot{X}+\ddot{X}\left(\frac{B}{M}+\frac{C_T\beta_e}{V_0}\right)+\dot{X}\left(\frac{2\beta_e A^2}{MV_0}+\frac{B\beta_e C_T}{MV_0}\right)=\omega_p\left(\frac{2D_p\beta_e A}{MV_0}\right) \tag{4}$$

A state space formulation of the EHA system can be written as follows:

$$\begin{aligned}
\dot{X}_1 &= X_2+w_1 \\
\dot{X}_2 &= X_3+w_2 \\
\dot{X}_3 &= -X_4X_2\left(\frac{2A^2}{MV_0}+\frac{BC_T}{MV_0}\right)-X_3\left(\frac{B}{M}+\frac{C_TX_4}{V_0}\right)+\omega_pX_4\left(\frac{2D_pA}{MV_0}\right)+w_3 \\
\dot{X}_4 &= w_4
\end{aligned} \tag{5}$$

where X_1 is the state variable X, X_2 the state variable $\dot{X}$, X_3 the state variable $\ddot{X}$, X_4 the effective bulk modulus and w_1, w_2, w_3 , w_4 the system noise. The piston position is used as measurement for the UKF algorithm and the input to the UKF is the pump angular velocity. Using a first order approximation, the discrete state space model of the EHA is determined to be as follows:

$$\begin{aligned}
X_1(k+1) &= X_1(k)+T_sX_2(k)+T_sw_1(k) \\
X_2(k+1) &= X_2(k)+T_sX_3(k)+T_sw_2(k) \\
X_3(k+1) &= -X_2(k)T_s\left(\frac{2X_4(k)A^2}{MV_0}+\frac{BX_4(k)C_T}{MV_0}\right)-X_3T_s\left(\frac{B}{M}+\frac{C_TX_4(k)}{V_0}\right) \\
&\quad +X_3(k)+\frac{2D_pAT_sX_4(k)}{MV_0}\omega_p+T_sw_3(k) \\
X_4(k+1) &= X_4(k)+T_sw_4(k)
\end{aligned} \tag{6}$$

The feasibility of the approach was demonstrated in simulation first and the UKF was used to estimate the effective bulk modulus using a simulated model of the EHA. The input of the simulated closed loop EHA model was a 25 Hz sine wave signal, with 0.005 m amplitude. This input was chosen because the effective bulk modulus was dominant at a frequency close to the natural frequency of the system. The input to the simulated closed loop EHA model was the desired piston position. The simulation study was conducted using Matlab/Simulink. The input to the UKF algorithm was the simulated electric motor angular velocity. The effective bulk modulus was estimated by the UKF by using the simulated electric motor (or pump) angular velocity, simulated piston displacement and simulated piston velocity. The sampling time was set to 0.001s. The estimation process is illustrated by Figure 5a. Note that the initial conditions, namely the initial state vector, error covariance matrix, system noise covariance matrix and the measurement noise covariance matrix were all set by trial and error.

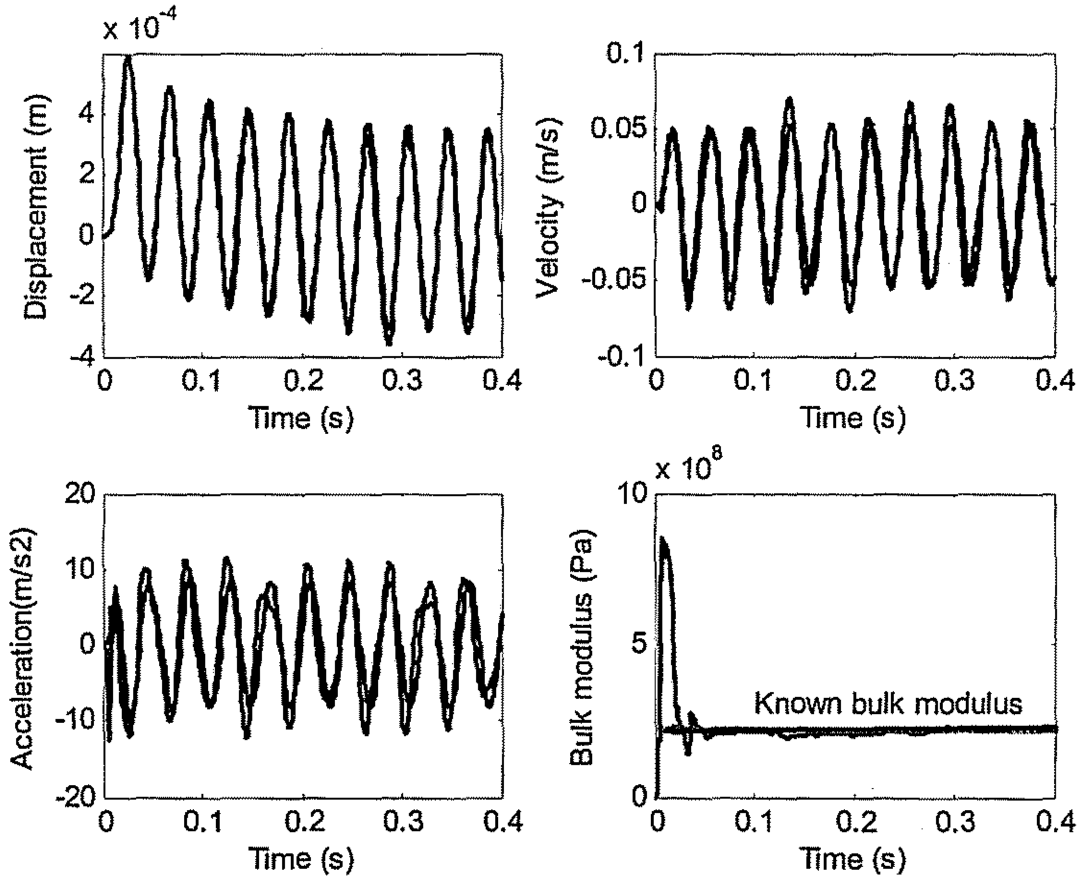

Figure 5a: Estimated states and effective bulk modulus in simulation using the UKF

The estimated states, namely the piston displacement, velocity and acceleration converge to the simulated states (represented by the broken lines) as illustrated by the superimposed plots in Figure 5a. Also, the estimated effective bulk modulus converges to the value used in the simulation as shown, with an estimation error of 0.3 %. Thus, the feasibility of using the UKF to estimate the effective bulk modulus was demonstrated.

Using the same tuned filter, the next step was to verify that the UKF was able to detect a change in the effective bulk modulus of the hydrostatic system. Therefore, the value of the effective bulk modulus was changed in the simulated EHA model. The UKF then used the simulated piston displacement and velocity to estimate the new value for the parameter. The estimated states for a lower effective bulk modulus, a situation which could arise in the prototype when air gets trapped as a result of maintenance activities, are shown in Figure 5b. Using the simulated model, the effective bulk modulus was reduced by 50 % and this change was estimated accurately with an estimation error of 0.22 %. Furthermore, as illustrated by the superimposed plots of Figure 5b, the estimated states converged to the simulated ones.

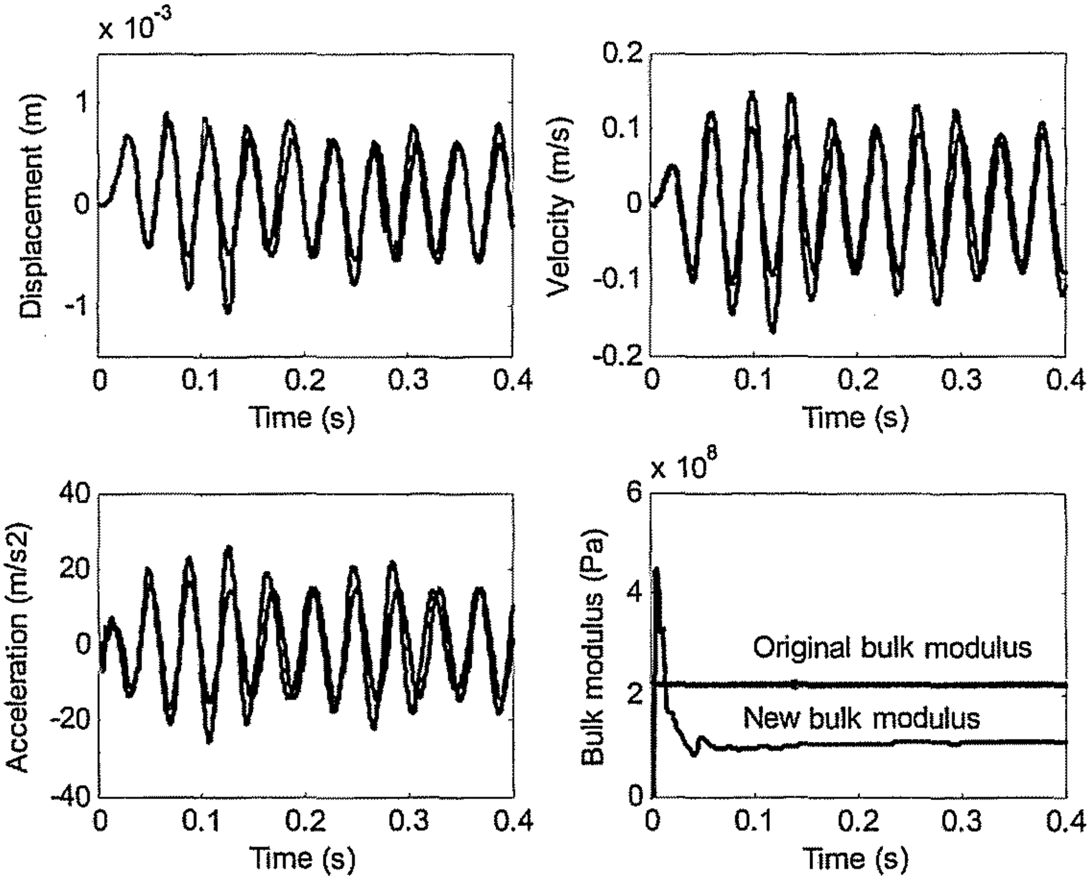

Figure 5b: Estimated states of the simulated EHA model with a reduction in the effective bulk modulus.

The effective bulk modulus was further changed and each time the UKF algorithm estimated the new parameter values successfully, with an estimation error of less than 1%. It is worth mentioning that the viscous damping coefficient was also changed in the simulated model and its effect on the estimation of the effective bulk modulus was investigated. It was found that the effective bulk modulus was not affected by the changes in viscous damping coefficient because of the frequency of the input signal which was used.

Having developed the methodology for the estimation of the effective bulk modulus in simulation, the same approach was used to estimate this important parameter using the EHA prototype. The UKF used the measured pump angular velocity, piston displacement and velocity. The UKF estimated the states and parameter successfully, as illustrated in Figure 6. From Figure 6, it is observed that the estimated piston position, velocity and acceleration by the UKF show good agreement with their measurement and derived values from the prototype. Similar to the simulation study, no priori information is assumed to be known about the state vector, i.e. all states are initialized to zero in the UKF algorithm. The initial matrices for the UKF algorithm are identical to the simulation study. The estimated effective bulk modulus value for the ElectroHydraulic Actuator prototype was 2.25×10^8 Pa and the result was found to be repeatable within 2% variations in the estimations.

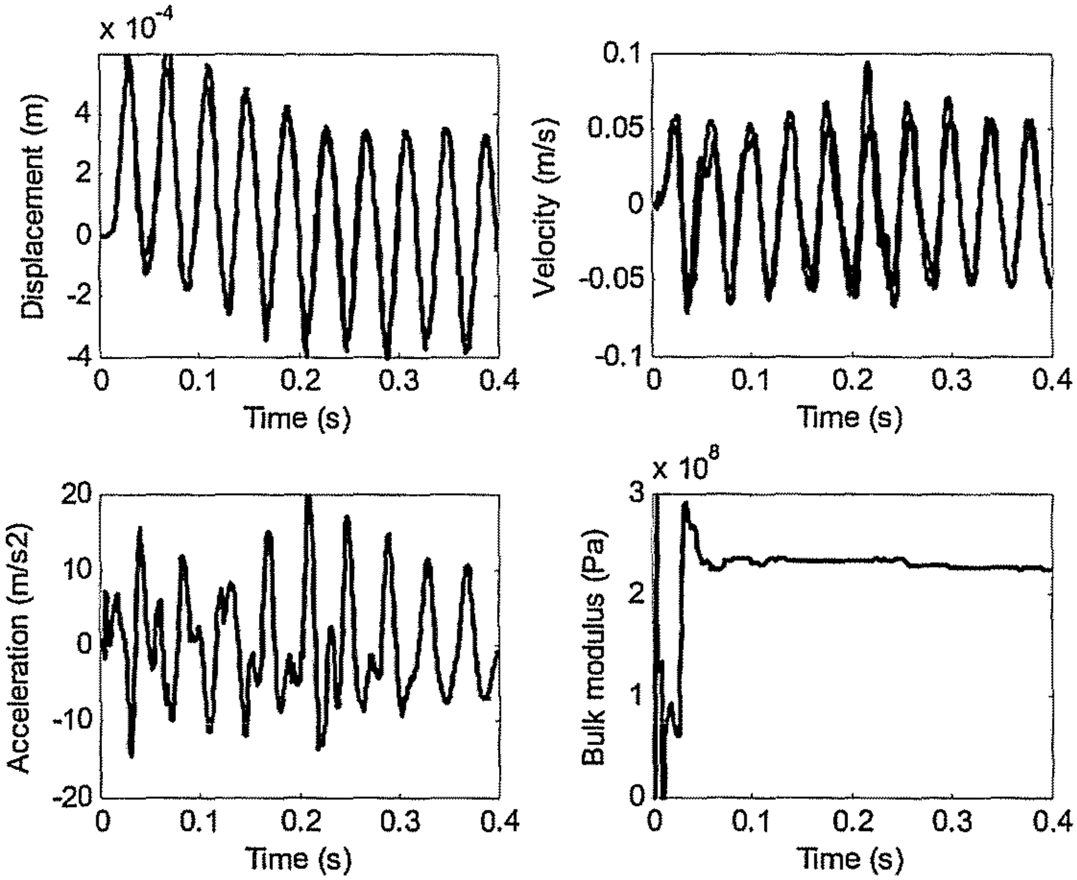

Figure 6: Estimated effective bulk modulus in the EHA prototype using the UKF

5. CONCLUSION

In this paper, a recently proposed state/parameter estimation technique, the Unscented Kalman Filter (UKF) was used to estimate the viscous damping coefficient and the effective bulk modulus in a high performance hydrostatic actuation system referred to as the ElectroHydraulic Actuator (EHA). The UKF algorithm was implemented within the Matlab/Simulink environment using a simulated EHA model to show the feasibility of the approach. This simulation study was then followed by the implementation of the UKF on an EHA prototype to demonstrate its practical application and validity. The UKF demonstrated its ability to successfully estimate parameters and states, without any priori knowledge of the states. An acceptable estimation error of less than 1% was achieved in simulation when the two parameters were estimated. In simulation, changes in the parameters which could be linked to inception of faults in the system were detected and estimated accurately. Experimental results are also presented in this paper and the parameter estimations are within a variation of 2% about the mean value, which is considered acceptable. The Unscented Kalman Filter (UKF) is appealing since, unlike the most commonly used Extended Kalman Filter (EKF), no linearization of the internal model of the filter is required. As such, this study shows the feasibility of using the UKF algorithm for parameter estimation in hydraulic systems and the results are very promising for the application of the UKF for fault detection in nonlinear systems.

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APPENDIX: THE UNSCENTED KALMAN FILTER (UKF) ALGORITHM

The standard UKF algorithm described in this section is for a L-dimensional state vector $X(k)$, with mean $\hat{X}(k)$ and covariance $P(k)$. The a priori state vector $\hat{X}^-(k)$ (one which has not been updated using measurements and the corrective gain) and its error covariance is $P^-(k)$ are initially predicted by using a mathematical model of the system under consideration or the EHA in this case. The system and measurement noise covariance matrices are given by $Q(k)$ and $R(k)$ respectively. Further to the UKF procedure as specified in [12, 13] , the a priori state vector estimate $\hat{X}^-(k)$ is approximated by $2L+1$ weighted samples or sigma points $\chi^-(k)$. Using the a priori state estimate and error covariance, the sigma points (weighted samples) are selected using :

$$\chi^-(k) = \left[\hat{X}^-(k) \quad \hat{X}^-(k) + \gamma\sqrt{P^-(k)} \quad \hat{X}^-(k) - \gamma\sqrt{P^-(k)}\right] \tag{8}$$

where $\gamma = \sqrt{(L+\kappa)}$ and is a scaling parameter.
Equation (8) can be expressed alternatively as equations (9), (10) and (11).

$$\chi_0(k) = \hat{X}_0(k) \tag{9}$$

$$\chi_i^-(k) = \hat{X}^-(k) + [\sqrt{(L+\kappa)P^-(k)}]_i \quad \text{where } i=1,2,\ldots\ldots L \tag{10}$$

$$\chi_{i+L}^-(k) = \hat{X}^-(k) - [\sqrt{(L+\kappa)P^-(k)}]_i \quad \text{where } i=1,2,\ldots\ldots L \tag{11}$$

Associated with each sigma point is a weight such that for the initial weight :

$$W_0 = \frac{\lambda}{(L+\lambda)} \tag{12}$$

The weight associated with the i^{th} point is calculated using:

$$W_i = \frac{1}{2(L+\kappa)}, \ i=1,\ldots\ldots\ldots 2L \tag{13}$$

Once $\chi^-(k)$ is computed, the prediction step is performed.

$$\chi^-(k+1) = f(\chi^-(k), u(k)) \tag{14}$$

The predicted mean and error covariance using the projected sigma points are computed as:

$$\hat{X}^-(k+1) = \sum_{i=0}^{2L} W_i \chi^-(k+1) \tag{15}$$

$$P^-(k+1) = \sum_{i=0}^{2L} W_i \left[\chi^-(k+1) - \hat{X}^-(k+1)\right]\left[\chi^-(k+1) - \hat{X}^-(k+1)\right]^T + Q(k) \tag{16}$$

To compute the correction step, it is necessary to transform the sigma point by using the measurement function. Therefore,

$$\Upsilon^-(k) = H\left[\chi^-(k+1)\right] \tag{17}$$

The estimated output (using a weighted sample or sigma points) is given by:

$$\hat{y}^-(k) = \sum_{i=0}^{2L} W_i \Upsilon^-(k) \tag{18}$$

The error covariance are expressed as a cross correlation $P_{xy}(k)$ of state and measurement, as shown in equation (19) and as an auto-correlation $P_{yy}(k)$ of measurement as shown in equation (20).

$$P_{xy}(k) = \sum_{i=0}^{2L} W_i \left[\chi^-(k+1) - \hat{X}^-(k+1)\right]\left[\Upsilon^-(k) - \hat{y}^-(k)\right]^T \tag{19}$$

$$P_{yy}(k) = \sum_{i=0}^{2L} W_i \left[\Upsilon^-(k) - \hat{y}^-(k)\right]\left[\Upsilon^-(k) - \hat{y}^-(k)\right]^T + R(k) \tag{20}$$

In the UKF formulation, the Kalman gain is calculated using $P_{xy}(k)$ and $P_{yy}(k)$ as:

$$K(k) = P_{xy}(k) P_{yy}^{-1}(k) \tag{21}$$

The a posteriori or refined state estimation obtained using a linear combination of the a priori estimate and the product of the error between the measurement and predicted output and UKF gain is :

$$\hat{X}(k) = \hat{X}^-(k+1) + K(k)\left(z(k) - \hat{y}^-(k)\right) \tag{22}$$

The error covariance for the a posteriori estimate is calculated as:

$$P(k) = P^{-}(k+1) - K(k)P_{yy}(k)K^{T}(k) \tag{23}$$

The algorithm is repeated and the sigma points using the refined state estimate and error covariance are calculated at each sampling interval.

NOMENCLATURE

Table 1 Parameter values for the EHA

Symbol	Definitions	Values (Prototype)
β_e	Effective bulk modulus.	2.2×10^{8} Pa
A	Piston area in symmetrical actuator	$5.051\times10^{-4}\,m^2$
B	Viscous damping coefficient.	760 Ns/m
C_T	Lumped pump and actuator leakage coefficient	$5\times10^{-13}\,m^3/s/Pa$
D_p	Pump volumetric displacement.	$1.6925\times10^{-7}\,m^3/rad$
M	Load mass	20 Kg
V_0	Oil volume	$6.85\times10^{-5}\,m^3$