MULTI-PHASE MOTION GENERATION OF FIVE-BAR MECHANISMS WITH PRESCRIBED RIGID-BODY TOLERANCES

MAJDI H. MUSA[a], KEVIN RUSSELL[b], RAJ S. SODHI[a]

[a]Department of Mechanical Engineering, New Jersey Institute of Technology, Newark, NJ 07102-1982, U.S.A

[b]Armaments Engineering and Technology Center, US Army Research, Development and Engineering Center, Picatinny Arsenal, NJ 07806-5000, U.S.A
kevin.russell1@us.army.mil

Received March 2005, Accepted October 2006
No. 05-CSME-11, E.I.C. Accession 2863

ABSTRACT

This work presents a technique for synthesizing adjustable planar, five-bar motion generators to approximate prescribed rigid-body positions and satisfy rigid-body positions with prescribed tolerances. This method is an extension of the adjustable planar five-bar motion generation method introduced by the authors (1). By incorporating rigid-body point tolerances in the rigid-body displacement matrices, the calculated circle and center point curves of the five-bar mechanism become circle and/or center point regions. From these regions, fixed and moving pivots for the five-bar mechanism can be selected that approximate the prescribed rigid-body positions and satisfy rigid-body positions with prescribed tolerances. The example in this work considers two-phase moving pivot adjustments in the planar five-bar motion generator.

GÉNÉRATION DE MOUVEMENTS POLYPHASIQUE D'UN MÉCANISME À CINQ BARRES AVEC DES TOLÉRANCES DE CORPS RIGIDE PRESCRITES

RÉSUMÉ

Cet article présente une technique synthétisant des générateurs de mouvements planaires ajustables à cinq barres pour se rapprocher des positions de corps rigide prescrites et d'y satisfaire avec les tolérances prescrites. Cette méthode est un prolongement de la méthode de génération de mouvements planaires ajustables à cinq barres présentée par les auteurs (1). En incorporant les tolérances de points de corps rigide dans les matrices de déplacement de corps rigide, les courbes du cercle et des points de centre caclulées du mécanisme à cinq barres deviennent les régions du cercle et des points de centre. Au moyen de ces régions, il est possible de sélectionner les pivots fixes ou mobiles du mécanisme à cinq barres afin de se rapprocher des positions de corps rigide prescrites et d'y satisfaire avec les tolérances prescrites. L'exemple de cet article illustre des ajustements à un pivot mobile diphasique dans le générateur de mouvements planaire à cinq barres.

1. INTRODUCTION

1.1 The Planar Five-Bar Mechanism

The planar four-bar mechanism and planar five-bar mechanism (Figure 1) share several distinct characteristics that make the two practical solutions (in design and implementation) for mechanical applications. Both mechanisms typically share the same joint type (revolute joint). The two mechanisms are also bounded by planar workspaces, and subsequently, share the same parallel joint axis orientations. The additional link in the five-bar mechanism however gives it an advantage over its four-bar cousin. Due to the additional link in the planar five-bar mechanism (and possibly, the additional degree of freedom), it can achieve more intricate combinations of rigid-body orientations than the planar four-bar mechanism.

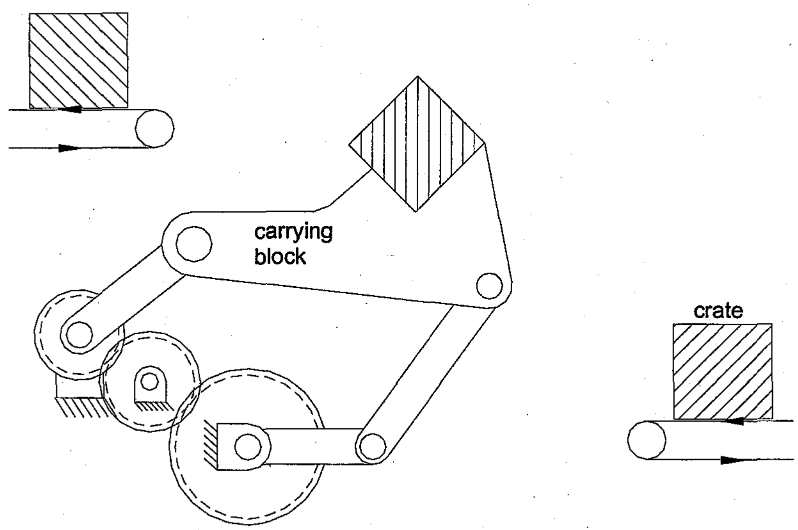

Figure 1. Five-bar mechanism used in package handling

1.2 Motion Generation And Multi-Phase Motion Generation

In **motion generation**, it is required that a rigid-body affixed to a mechanism approximate a single series of prescribed positions. Using the loading mechanism illustrated in Figure 1 as an example, the rigid-body could be represented by the "carrying block" and the prescribed rigid-body positions could be those needed in the carrying block to deliver a crate from one conveyor belt to the next.

Unlike the single series of prescribed rigid-body positions required in motion generation, in **multi-phase motion generation** a mechanism must approximate multiple phases of prescribed rigid-body positions. Each phase of prescribed rigid-body positions can be approximated given a particular mechanism configuration. In order for a single mechanism to achieve a multi-phase motion generation application, the mechanism must be designed with adjustable features (adjustable fixed and moving pivots for example). The user need only adjust the adjustable features of the mechanism to obtain the necessary configurations required to achieve the particular phase of rigid-body positions.

A five-bar mechanism with adjustable moving pivots **a** and **c** is illustrated in Figure 2. When moving pivots $\mathbf{a}_1$ and $\mathbf{c}_1$ are incorporated, one series (or phase) of prescribed rigid-body positions is achieved and a different series of positions is achieved when $\mathbf{a}_{1n}$ and $\mathbf{c}_{1n}$ are incorporated. The appeal of adjustable motion generators is that multiple phases of prescribed rigid-body positions can be achieved using essentially the same mechanism hardware.

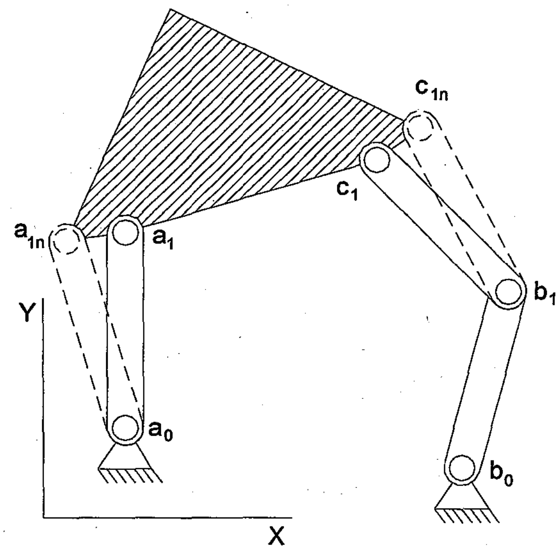

Figure 2. Five-bar motion generator with adjustable moving pivots

1.3 Rigid-Body Tolerance Consideration

Two phases of prescribed rigid-body positions are illustrated in Figure 3. The x and y-coordinates of each corner point denote the position and orientation of the rigid-body in 2-D space. Although position 1, 2, 4 and 6 denote precise rigid-body positions, position 3 and 5 do not. The latter "positions" are in fact planar regions that the respective rigid-body points must line within. The regions in positions 3 and 5 represent tolerances placed on these rigid-body positions. The challenge in Figure 3 is to design an adjustable five-bar mechanism to approximate the precise prescribed rigid-body positions and satisfy the prescribed tolerance constraints of the rigid-body position with tolerances.

In motion generation applications, it may not be necessary for all of the prescribed rigid-body positions to have precise orientation in space. A particular rigid-body position may have several possible orientations that are acceptable for the application. The "possible orientations" of the rigid-body position can be expressed as tolerances placed on a position.

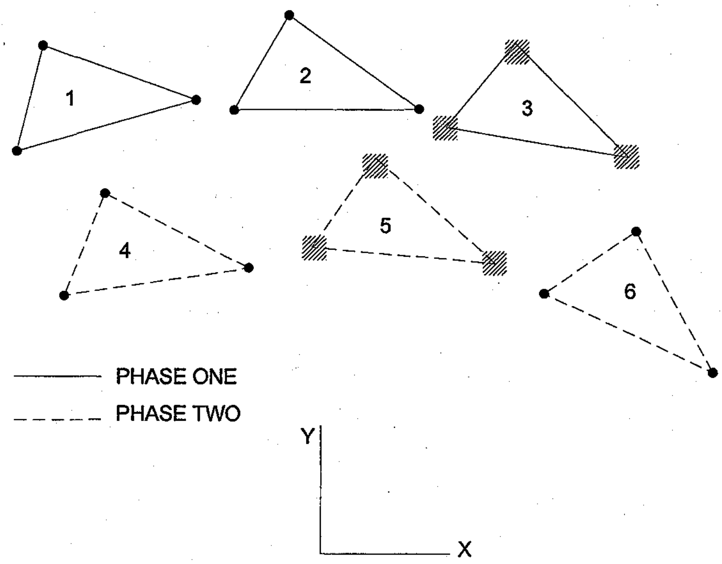

Figure 3. Rigid-body positions and positions with tolerances

1.4 Literature Review and Scope of Work

In the area five-bar mechanism design, Balli and Chand (2) introduced a complex number method for the synthesis of a planar five-bar motion generator with prescribed timing. The authors also introduced a method to synthesize a planar five-bar mechanism of variable topology type with transmission angle contol (3). Nokleby and Podhorodeski (4) introduced an optimization method to synthesize Grashof five-bar mechanisms. Wang and Yan (5) introduced an approach for synthesizing planar five-bar linkages with five prescribed precision positions. Basu and Farhang (6) introduced a mathematical formulation for the approximate analysis and design of two-input, small-crank five-bar mechanisms for function generation. Dou and Ting (7) introduced a method to identify to rotatability and branch condition in linkages containing simple geared five-bar chains. Lin and Chaing (8) extended pole method for use in the synthesis planar, geared five-bar function generators. Ge and Chen (9) introduced a software-based approach for the atlas method on path synthesis of geared five-bar mechanisms. The two authors also studied the effect of ling length, crank angles and gear tooth ratio on the motion of the geared five-bar linkage (10). Li and Dao (11) introduced a complex number method for the synthesis for geared, five-bar guidance mechanisms.

This work presents a technique for synthesizing adjustable planar, five-bar motion generators to approximate prescribed precise rigid-body positions and satisfy rigid-body positions with prescribed tolerances. By incorporating rigid-body point tolerances in the rigid-body displacement matrices, the calculated circle and center point curves of the five-bar mechanism become circle and/or center point regions. From these regions, fixed and moving pivots for the five-bar mechanism can be selected that approximate the prescribed precise rigid-body positions and satisfy rigid-body positions with prescribed tolerances. The example in this work considers two-phase moving pivot adjustments in the planar five-bar motion generator.

2. PLANAR RIGID-BODY GUIDANCE

2.1 Precise Position Rigid-Body Guidance

In this work, links $\mathbf{a}_0$-$\mathbf{a}_1$ and $\mathbf{b}_0$-$\mathbf{b}_1$ in Figure 4 are the driving links (denoted by driving link angles θ and ϕ). Links $\mathbf{a}_0$-$\mathbf{a}_1$ and $\mathbf{b}_1$-$\mathbf{c}_1$ of the planar five-bar mechanism are synthesized using the constant length condition. The constant length constraints given in Equation 1 and Equation 2 (12,13) are satisfied when synthesizing links $\mathbf{a}_0$-$\mathbf{a}_1$ and $\mathbf{b}_1$-$\mathbf{c}_1$ of the planar five-bar mechanism. The angle ϕ_1 in variable $\mathbf{b}_1$ is the initial orientation angle for link $\mathbf{b}_0$-$\mathbf{b}_1$ and is prescribed.

$$(\mathbf{a}_j - \mathbf{a}_0)^T(\mathbf{a}_j - \mathbf{a}_0) = (\mathbf{a}_1 - \mathbf{a}_0)^T(\mathbf{a}_1 - \mathbf{a}_0) \quad j = 2, 3, \ldots . n \tag{1}$$

$$(\mathbf{c}_j - \mathbf{b}_j)^T(\mathbf{c}_j - \mathbf{b}_j) = (\mathbf{c}_1 - \mathbf{b}_1)^T(\mathbf{c}_1 - \mathbf{b}_1) \quad j = 2, 3, \ldots . n \tag{2}$$

where

$$\mathbf{a}_0 = (a_{0x}, a_{0y}, 1) \qquad \mathbf{a}_1 = (a_{1x}, a_{1y}, 1) \qquad \mathbf{a}_j = [D_{ij}]\mathbf{a}_1$$

$$\mathbf{b}_1 = [b_{0x} + R_2\cos(\phi_1), b_{0y} + R_2\sin(\phi_1), 1] \qquad \mathbf{b}_j = [T_j]\mathbf{b}_1$$

$$\mathbf{c}_1 = (c_{1x}, c_{1y}, 1) \qquad \mathbf{c}_j = [D_{ij}]\mathbf{c}_1$$

and

$$[D_{ij}] = \begin{bmatrix} p_{jx} & q_{jx} & r_{jx} \\ p_{jy} & q_{jy} & r_{jy} \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} p_{ix} & q_{ix} & r_{ix} \\ p_{iy} & q_{iy} & r_{iy} \\ 1 & 1 & 1 \end{bmatrix}^{-1} \tag{3}$$

$$[T_j] = \begin{bmatrix} \cos(\delta\phi_j) & -\sin(\delta\phi_j) & -b_{0x}\cos(\delta\phi_j) + b_{0y}\sin(\delta\phi_j) + b_{0x} \\ \sin(\delta\phi_j) & \cos(\delta\phi_j) & -b_{0x}\sin(\delta\phi_j) - b_{0y}\cos(\delta\phi_j) + b_{0y} \\ 0 & 0 & 1 \end{bmatrix} \tag{4}$$

Equation 1 and Equation 2 can be rewritten as Equation 5 and Equation 6 respectively. In Equation 5, the variable R_1 represents the length of link $\mathbf{a}_0$-$\mathbf{a}_1$ and the variable R_3 represents the length of link $\mathbf{b}_1$-$\mathbf{c}_1$ in Equation 6. The variable R_2 in Equation 2 represents the length of link $\mathbf{b}_0$-$\mathbf{b}_1$.

$$(\mathbf{a}_j - \mathbf{a}_0)^T(\mathbf{a}_j - \mathbf{a}_0) = R_1^2 \quad j = 2, 3, \ldots . n \tag{5}$$

$$(\mathbf{c}_j - \mathbf{b}_j)^T(\mathbf{c}_j - \mathbf{b}_j) = R_3^2 \quad j = 2, 3, \ldots . n \tag{6}$$

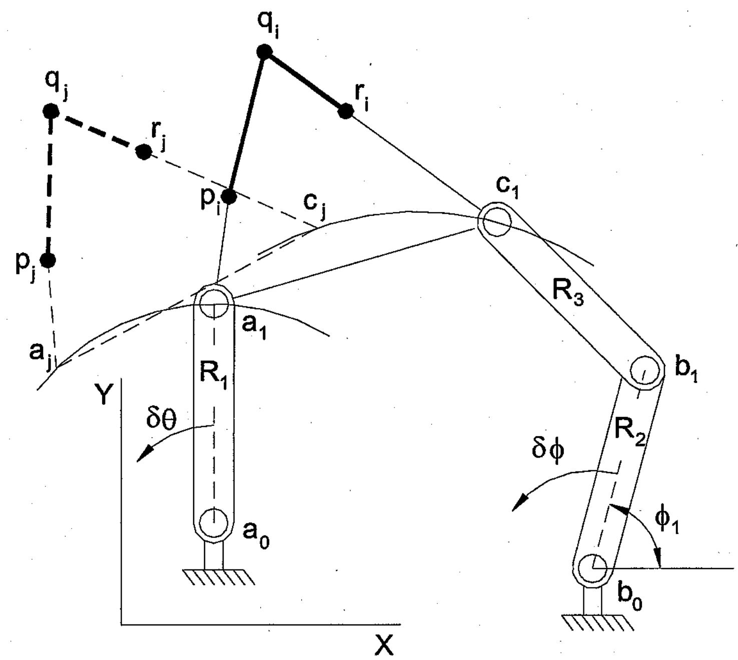

Figure 4. Planar five-bar mechanism with driving link angles θ and ϕ and rigid-body points **p**, **q** and **r**

Equation 3 is a rigid body displacement matrix. It is a derivative of the spatial rigid-body displacement matrix (12, 13). Given the coordinates for a rigid-body in position "i" and the subsequent "j," matrix $[D_{ij}]$ is the transformation matrix required to transform coordinates from position "i" to position "j."Variables **p**, **q** and **r** in Equation 3 represent the position of the rigid-body in two-dimensional space. Although the position of a rigid-body in two-dimensional space is commonly described by a single point and a displacement angle (**p** and θ for example), the authors chose to describe the rigid-body using three points for computational purposes. Since there are four variables in each equation (a_{0x}, a_{0y}, a_{1x}, a_{0y} and b_{1x}, b_{1y}, c_{1x} and c_{1y}), a maximum of five rigid-body positions can be prescribed, with no arbitrary choice of parameter for a single phase (see Table 1).

Points **p, q** and **r** should not all lie on the same line in each rigid-body position. Taking this precaution prevents the rows in the rigid-body displacement matrix (Equation 3) from becoming proportional. With proportional rows, this matrix cannot be inverted.

In Table 1, the **maximum** numbers of prescribed rigid-body positions for the adjustable planar five-bar motion generator for several phases are given. The number of fixed and moving pivot coordinates for the links to be synthesized (links $\mathbf{a}_0$-$\mathbf{a}_1$ and $\mathbf{b}_1$-$\mathbf{c}_1$) determine the maximum number of rigid-body positions. In the example problem in this work, an adjustable planar five-bar mechanism is designed to achieve a two-phase moving pivot adjustment application.

Table 1. Prescribed rigid-body position and phase variations for the adjustable planar five-bar mechanism

		Link a_0-a		**Link b_1-c**	
number of phases	max. number of rigid-body positions	number of unknowns	number of free choices	number of unknowns	number of free choices
1	5	4	0	6	2
2	8	6	0	8	2
3	11	8	0	10	2
m	5 + 3(m - 1)	2 + 2m	0	6+2(m – 1)	2

2.2 Rigid-Body Guidance With Tolerances

To incorporate rigid-body point tolerances, the rigid-body displacement matrix $[D_{ij}]$ in Equation 7 must be used in the constant length constraints in Equation 5 and Equation 6. In Equation 7, $\delta\mathbf{p}$, $\delta\mathbf{q}$ and $\delta\mathbf{r}$ represent the specified upper and lower rigid-body point tolerances.

$$[D_{ij}]=\begin{bmatrix} p_{jx}\pm\delta p_{jx} & q_{jx}\pm\delta q_{jx} & r_{jx}\pm\delta r_{jx} \\ p_{jy}\pm\delta p_{jy} & q_{jy}\pm\delta q_{jy} & r_{jy}\pm\delta r_{jy} \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} p_{ix}\pm\delta p_{ix} & q_{ix}\pm\delta q_{ix} & r_{ix}\pm\delta r_{ix} \\ p_{iy}\pm\delta p_{iy} & q_{iy}\pm\delta q_{iy} & r_{iy}\pm\delta r_{iy} \\ 1 & 1 & 1 \end{bmatrix}^{-1} \quad (7)$$

Since points **p**, **q** and **r** lie on a rigid-body, these points must satisfy the constant length constraints of a rigid-body with and without tolerances. In other words, the distances between **p** and **q**, **q** and **r**, and **p** and **r** must equal the distances between $\mathbf{p}\pm\delta\mathbf{p}$ and $\mathbf{q}\pm\delta\mathbf{q}$, $\mathbf{q}\pm\delta\mathbf{q}$ and $\mathbf{r}\pm\delta\mathbf{r}$, and $\mathbf{p}\pm\delta\mathbf{p}$ and $\mathbf{r}\pm\delta\mathbf{r}$ respectively. In this work, the coordinates of the rigid-body positions and the rigid-body position tolerances were prescribed using Computer-Aided-Design (CAD) software. Using such software, points **p**, **q** and **r** can be dynamically oriented and measured in two-dimensional space (thus judiciously prescribing the rigid-body positions and prescribing the position tolerances without violating the constant length constraints of the rigid-body).

3. ADJUSTABLE FIVE-BAR MECHANISM AND MOTION GENERATION EQUATIONS

This work considers two-phase moving pivot adjustments of the five-bar mechanism (with constant link lengths). In such a problem, the required unknowns are $\mathbf{a}_0$, $\mathbf{a}_1$, $\mathbf{a}_{1n}$, $\mathbf{b}_1$, $\mathbf{c}_1$ and $\mathbf{c}_{1n}$ (see Figure 2). Variables $\mathbf{a}_1$ and $\mathbf{c}_1$ represent the moving pivots required to achieve the prescribed phase 1 rigid-body positions. Variables $\mathbf{a}_{1n}$ and $\mathbf{c}_{1n}$ represent the moving pivots required to achieve the prescribed phase 2 rigid-body positions. Since variables $\mathbf{a}_0$, $\mathbf{a}_1$, $\mathbf{a}_{1n}$, $\mathbf{c}_1$ and $\mathbf{c}_{1n}$ have two unknown components each (the x and y-component) and variable $\mathbf{b}_1$ has four unknown components (b_{0x}, b_{0y}, R_2 and ϕ_1), there are a total of 14 variables to determine.

$$\mathbf{a}_0 = (a_{0x}, a_{0y}) \qquad \mathbf{a}_1 = (a_{1x}, a_{1y}) \qquad \mathbf{a}_{1n} = (a_{1nx}, a_{1ny})$$

$$\mathbf{b}_1 = [b_{0x} + R_2\cos(\phi_1), b_{0y} + R_2\sin(\phi_1)] \qquad \mathbf{c}_1 = (c_{1x}, c_{1y}) \qquad \mathbf{c}_{1n} = (c_{1nx}, c_{1ny})$$

Equations 8 through 10 were used to calculate three of the four unknowns in $\mathbf{a}_0$ and $\mathbf{a}_1$. The variable a_{0x} and the link length R_1 were specified. Equations 8 through 10 represent the phase 1 adjustment of link $\mathbf{a}_0$-$\mathbf{a}$.

$$([D_{12}]\mathbf{a}_1 - \mathbf{a}_0)^T([D_{12}]\mathbf{a}_1 - \mathbf{a}_0) - R_1^2 = 0 \tag{8}$$

$$([D_{13}]\mathbf{a}_1 - \mathbf{a}_0)^T([D_{13}]\mathbf{a}_1 - \mathbf{a}_0) - R_1^2 = 0 \tag{9}$$

$$([D_{14}]\mathbf{a}_1 - \mathbf{a}_0)^T([D_{14}]\mathbf{a}_1 - \mathbf{a}_0) - R_1^2 = 0 \tag{10}$$

Equations 11 through 13 were used to calculate three of the four unknowns in $\mathbf{a}_0$ and $\mathbf{a}_{1n}$. The variable a_{0x} and the link length R_1 were specified. Equations 11 through 13 represent the phase 2 adjustment of link $\mathbf{a}_0$-$\mathbf{a}$.

$$([D_{56}]\mathbf{a}_{1n} - \mathbf{a}_0)^T([D_{56}]\mathbf{a}_{1n} - \mathbf{a}_0) - R_1^2 = 0 \tag{11}$$

$$([D_{57}]\mathbf{a}_{1n} - \mathbf{a}_0)^T([D_{57}]\mathbf{a}_{1n} - \mathbf{a}_0) - R_1^2 = 0 \tag{12}$$

$$(\mathbf{a}_{1n} - \mathbf{a}_0)^T(\mathbf{a}_{1n} - \mathbf{a}_0) - R_1^2 = 0 \tag{13}$$

In gear-driven (Figure 1), chain-driven and belt-driven five-bar mechanisms, the driving link angles have a functional relationship (e.g. $\delta\phi = f(\delta\theta) = k^*\delta\theta$). This relationship depends on the ratios of the gears, sprockets or pulleys connecting both driving links in the mechanism. To accommodate such drive types for the planar five-bar mechanism, the driving link angles $\delta\theta$ must be determined first. These angles are the angles between $[D_{1j}]\mathbf{a}_1$-$\mathbf{a}_0$-$\mathbf{a}_1$ in phase 1 and $[D_{5j}]\mathbf{a}_{1n}$-$\mathbf{a}_0$-$\mathbf{a}_{1n}$ in phase 2. After calculating the $\delta\theta$ angles, the user can then establish a $\delta\phi = f(\delta\theta) = k^*\delta\theta$ relationship and calculate the driving link angles $\delta\phi$.

Equations 14 through 16 were used to calculate $\mathbf{c}_1$ and R_3. The components of variable $\mathbf{b}_1$ (b_{0x}, b_{0y}, ϕ_1 and the link length R_2) were specified. Equations 14 through 16 represent the phase 1 adjustment of link $\mathbf{b}_1$-$\mathbf{c}$.

$$([D_{12}]\mathbf{c}_1 - [T_1]\mathbf{b}_1)^T([D_{12}]\mathbf{c}_1 - [T_1]\mathbf{b}_1) - R_3^2 = 0 \tag{14}$$

$$([D_{13}]\mathbf{c}_1 - [T_2]\mathbf{b}_1)^T([D_{13}]\mathbf{c}_1 - [T_2]\mathbf{b}_1) - R_3^2 = 0 \tag{15}$$

$$([D_{14}]\mathbf{c}_1 - [T_3]\mathbf{b}_1)^T([D_{14}]\mathbf{c}_1 - [T_3]\mathbf{b}_1) - R_3^2 = 0 \tag{16}$$

Equations 17 through 19 were used to calculate $\mathbf{c}_{1n}$ and R_3. The components of variable $\mathbf{b}_1$ (b_{0x}, b_{0y}, ϕ_1 and the link length R_2) were specified. Equations 17 through 19 represent the phase 2 adjustment of link $\mathbf{b}_1$-$\mathbf{c}$.

$$([D_{56}]\mathbf{c}_{1n} - [T_6]\mathbf{b}_1)^T([D_{56}]\mathbf{c}_{1n} - [T_6]\mathbf{b}_1) - R_3^2 = 0 \tag{17}$$

$$([D_{57}]\mathbf{c}_{1n} - [T_7]\mathbf{b}_1)^T([D_{57}]\mathbf{c}_{1n} - [T_7]\mathbf{b}_1) - R_3^2 = 0 \tag{18}$$

$$(\mathbf{c}_{1n} - \mathbf{b}_1)^T(\mathbf{c}_{1n} - \mathbf{b}_1) - R_3^2 = 0 \tag{19}$$

4. MOVING PIVOT ADJUSTMENT EXAMPLE

The synthesis of a two-phase moving pivot adjustable motion generator is exemplified in this section. Listed in Table 2 are the dimensionless X and Y-coordinates of rigid-body points **p**, **q** and **r** for seven prescribed rigid-body positions (where $\delta = \pm 0.01$).

To construct the boundaries for the regions representing variables $\mathbf{a}_0$, $\mathbf{a}_1$ and $\mathbf{a}_{1n}$ (the solid bold lines in Figure 5), a locus of solutions were calculated using Equations 8 through 13. The two boundaries for each region for variables $\mathbf{a}_0$, $\mathbf{a}_1$ and $\mathbf{a}_{1n}$ in Figure 5 were produced by calculating two solution loci from Equations 8 through 13 (one using δ = +0.01 and the other using δ = -0.01). Using Equations 8 through 10, variables a_{1x}, a_{1y} and a_{0y} were calculated for a specified range of values for a_{0x} (where a_{0x} =-0.1, -0.08....0.1). Using Equations 11 through 13, variables a_{1nx}, a_{1ny} and a_{0y} were calculated for a specified range of values for a_{0x} (where a_{0x} =-0.1, -0.08...0.1). The initial guesses for Equations 8 though 13 are the following (with R_1 =1 specified):

$$\mathbf{a}_{0y} = 0.02 \qquad \mathbf{a}_1 = (0.1, 0.9) \qquad \mathbf{a}_{1n} = (0.1, 0.9)$$

With the solution regions for variables $\mathbf{a}_0$, $\mathbf{a}_1$ and $\mathbf{a}_{1n}$ constructed, solutions were then selected from them as illustrated in Figure 5. The solutions selected are listed immediately below. The constant length condition R_1 =1 from Equations 8 through 13 was maintained during the selection process. It is important to note that like the $\mathbf{a}_0$ regions in Figure 5, there are overlapping portions of regions $\mathbf{a}_1$ and $\mathbf{a}_{1n}$. A selection from the overlap represents a single moving pivot that would satisfy phase 1 and phase 2 rigid-body positions and tolerances. Rather that employing such an option, the authors selected one solution from each region since the goal is to produce a five-bar mechanism with adjustable moving pivots.

$$\mathbf{a}_0 = (0, 0) \qquad \mathbf{a}_1 = (0.1736, 0.9848) \qquad \mathbf{a}_{1n} = (0.1101, 0.9939)$$

The displacement angles $\delta\theta$ for link $\mathbf{a}_0$-$\mathbf{a}_1$ and $\mathbf{a}_0$-$\mathbf{a}_{1n}$ were then calculated for each rigid-body displacement ($[D_{1j}]\mathbf{a}_1$ and $[D_{5j}]\mathbf{a}_{1n}$). These displacement angles are 10°, 15° and 20° for phase 1 (link $\mathbf{a}_0$-$\mathbf{a}_1$) and -15° and -20° for phase 2 (link $\mathbf{a}_0$-$\mathbf{a}_{1n}$). The displacement for angles $\delta\phi$ for the prescribed link $\mathbf{b}_0$-$\mathbf{b}_1$ were calculated using the relationship $\delta\phi = f(\delta\theta) = 0.5^*\delta\theta$. This relationship produced the displacement angles of 5°, 7.5° and 10° for phase 1 and -7.5° and -10° for phase 2.

To construct the boundaries for the regions representing variables $\mathbf{c}_1$ and $\mathbf{c}_{1n}$ (the solid bold lines in Figure 5) in Figure 5, a locus of solutions were calculated using Equations 14 through 19. The two boundaries for each region for variables $\mathbf{c}_1$ and $\mathbf{c}_{1n}$ in Figure 5 were produced by calculating two solution loci from Equations 14 through 19 (one using δ = +0.01 and the other using δ = -0.01). Using Equations 14 through 16, variables c_{1x}, c_{1y} and R_3 were calculated for a specified range of values for R_2 (where R_2 =0.5, 0.55....1). Using Equations 17 through 19, variables c_{1nx}, c_{1ny} and R_3 were calculated for a specified range of values for R_2 (where R_2 =0.5, 0.55...1). The bold dashed line in Figure 5 represents variable $\mathbf{b}_1$. This variable is based on the specified values of $\mathbf{b}_0$, ϕ_1 and R_2. The initial guesses for Equations 8 though 13 are the following (with b_{0x} =0.7842, b_{0y} = -0.0018, ϕ_1 = 30° specified):

$$\mathbf{R}_3 = 1 \qquad \mathbf{c}_1 = (0.7, 1.1) \qquad \mathbf{c}_{1n} = (0.8, 1.1)$$

With the solution regions for variables $\mathbf{c}_1$ and $\mathbf{c}_{1n}$ constructed, solutions were then selected from them as illustrated in Figure 5. The solutions selected are listed immediately below. It is important to note that there are overlapping portions of regions $\mathbf{c}_1$ and $\mathbf{c}_{1n}$. A selection from the overlap represents a single moving pivot that would satisfy phase 1 and phase 2 rigid-body positions and tolerances. Rather that employing such an option, the authors selected one solution from each region since the goal is to produce a five-bar mechanism with adjustable moving pivots.

$\mathbf{b}_1$ = (1.3977, 0.3732) $\mathbf{c}_1$ = (0.7550, 1.1393) $\mathbf{c}_{1n}$ = (0.8204, 1.1897)

Table 2. Prescribed rigid-body positions for the adjustable planar five-bar motion generator

	p	**q**	**r**
		Phase 1	
Pos. 1	0.0536, 1.0541	0.2896,1.3620	0.8139,1.2572
Pos. 2	-0.1092, 1.0853	0.1674, 1.3573	0.6720, 1.1804
Pos. 3	-0.1882 $\pm \delta$, 1.0911	0.1119 $\pm \delta$, 1.3369	0.5984 $\pm \delta$, 1.1151
Pos. 4	-0.2606, 1.0927	0.0705, 1.2948	0.5218, 1.0079
		Phase 2	
Pos. 5	0.0536, 1.0541	0.2896,1.3620	0.8139,1.2572
Pos. 6	0.3008 $\pm \delta$, 0.9851	0.5013 $\pm \delta$, 1.3172	1.0340 $\pm \delta$, 1.2710
Pos. 7	0.3785, 0.9474	0.5657, 1.2871	1.0998, 1.2621

Note: Rigid-body positions 1 and 5 are shared

The synthesized adjustable five-bar motion generator is illustrated in Figure 6. To maintain the $\delta\phi = f(\delta\theta) = 0.5^*\delta\theta$ relationship, a gear train with an input-output ratio between the driving links of 2:1 was incorporated. The same input-output rations is required if pulleys, sprockets or motors are incorporated between the driving links of the mechanism. The rigid-body positions achieved by the adjustable planar five-bar motion generator are included in Table 3. The positions approximate those in Table 2 and satisfy the prescribed rigid-body point tolerances. The displacement angles are 10°, 15° and 20° for link $\mathbf{a}_0$-$\mathbf{a}_1$ and -15° and -20° for link $\mathbf{a}_0$-$\mathbf{a}_{1n}$.

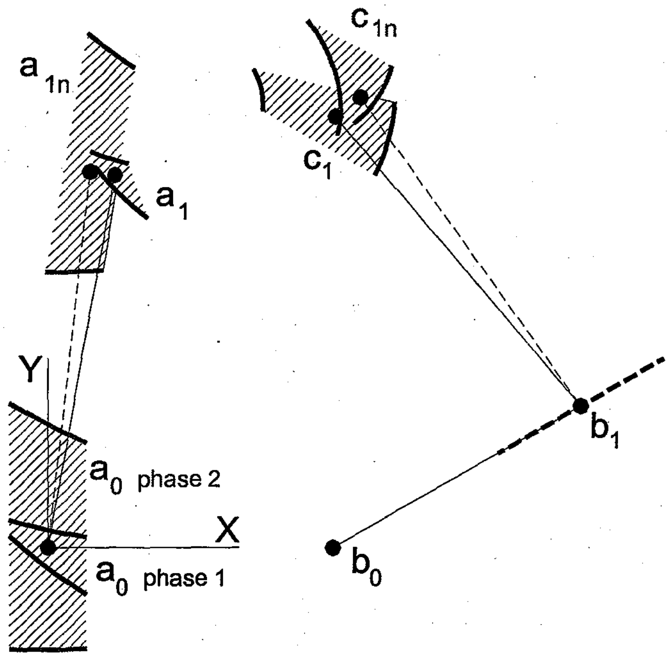

Figure 5. Tolerance regions and pivot selections for 5-bar motion generator

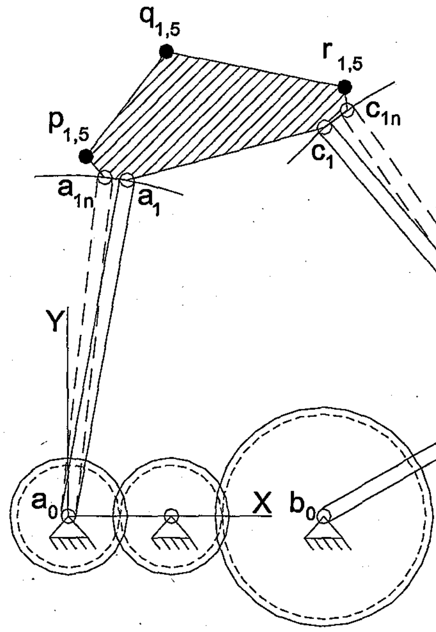

Figure 6. Synthesized adjustable 5-bar mechanism

Table 3. Rigid body positions achieved by the adjustable planar five-bar motion generator

	p	q	r
		Phase 1	
Pos. 1	0.0536, 1.0541	0.2896,1.3620	0.8139,1.2572
Pos. 2	-0.1030, 1.0852	0.1680, 1.3627	0.6761, 1.1961
Pos. 3	-0.1797, 1.0898	0.1096, 1.3482	0.6052, 1.1473
Pos. 4	-0.2530, 1.0878	0.0565, 1.3215	0.5339, 1.0807
		Phase 2	
Pos. 5	0.0536, 1.0541	0.2896,1.3620	0.8139,1.2572
Pos. 6	0.2991, 1.0049	0.5094, 1.3308	1.0405, 1.2686
Pos. 7	0.3768, 0.9737	0.5772, 1.3058	1.1100, 1.2597

Note: Rigid-body positions 1 and 5 are shared

5. DISCUSSION

The region boundaries produced and illustrated in Figure 5 are comparable to Burmester Curves (12, 13). Burmester Curves represent a locus of fixed/moving pivot solutions and are often used in motion generation applications. When adjusting the moving pivots of the synthesized five-bar mechanism in Figure 6, the gear train (or pulley, motor or sprocket system) remains fixed. To accommodate this adjustment, detachable connections should exist between both driving links and their respective gears. Although the position of a rigid-body in two-dimensional space is commonly described by a single point and a displacement angle (**p** and θ for example), the authors chose to describe the rigid-body using three points for computational purposes. Computer-Aided-Design (CAD) software was to prescribe the mechanism parameters in this work and mathematics software was used to compute the mechanism solutions. This software enabled the tabulated prescribed and calculated mechanism parameters to be expressed with four decimal places.

6. CONCLUSION

A new technique for synthesizing adjustable planar, five-bar motion generators to approximate prescribed rigid-body positions and satisfy rigid-body positions with prescribed tolerances is presented in this work. By incorporating rigid-body point tolerances in the rigid-body displacement matrices, the calculated circle and center point curves of the five-bar mechanism become circle and/or center point regions. From these regions, fixed and moving pivots for the five-bar mechanism can be selected that approximate the prescribed rigid-body positions and satisfy rigid-body positions with prescribed tolerances. This work considers two-phase moving pivot adjustments in the planar five-bar motion generator.

REFERENCES

1 K. Russell and R. S. Sodhi, "Kinematic Synthesis of Adjustable Planar Five-Bar Mechanisms for Multi-Phase Motion Generation," *JSME International Journal*, Series C, Vol. 47, No. 1, pp. 345-349 (2004).

2 S. S. Balli and S. Chand, "Five-Bar Motion and Path Generators with Variable Topology for Motion Between Extreme Positions," *Mechanism and Machine Theory*, Vol. 37, No. 11, pp. 1435-1445 (2002).

3 S. S. Balli and S. Chand, "Synthesis of a Five-Bar Mechanism for Variable Topology Type with Transmission Angle Control," *Journal of Mechanical Design*, Vol. 126, No. 1, pp. 128-134 (2004).

4 S. B. Nokleby and R. P. Podhorodeski, "Optimization-Based Synthesis of Grashof Geared Five-Bar Mechanisms," *Journal of Mechanical Design*, Vol. 123, No. 4, pp. 529-534 (2001).

5 A. Wang and H. Yan, "Rigid-Body Guidance of Planar Five-Bar Linkages for Five Precision Positions," *Journal of the Chinese Society of Mechanical Engineers*, Vol. 12, No. 2, pp. 159-165 (1991).

6 P. S. Basu and K. Farhang, "Kinematic Analysis and Design of Two-Input, Five-Bar Mechanisms Driven by Relatively Small Cranks," *22nd Biennial Mechanisms Conference, Scottsdale, Arizona* (1992).

7 X. Dou and K. Ting, "Branch Identification in Geared Five-Bar Chains," *Journal of Mechanical Design,* Vol. 118, No. 3, pp. 384-389 (1996).

8 C. Lin and C. H. Chaing, "Synthesis of Planar and Spherical Geared Five-Bar Function Generators by the Pole Method," *Mechanism and Machine Theory*, Vol. 27, No. 2, pp. 131-141 (1992).

9 W. Ge and Z. Chen, "Study of Geared Five-Bar Curves Based on Computer Mechanism Simulation with Variable Parameters," *Mechanical Science and Technology,* Vol. 18, No. 3, pp. 435-437 (1999).

10 W. Ge and Z. Chen, "Application Oriented Study of Characteristics of Jointn Loci of Geared Five-Bar Linkage," *Journal of Northwestern Polytechnical University,* Vol. 15, No. 4, pp. 542-546 (1997).

11 T. Li and W. Gao, "Kinematic Synthesis of Geared Linkage Mechanism for Body Guidance with Input Timing," *Mechanical Science and Technology,* Vol. 17, No. 6, pp. 869-870 (1999).

12 C. H. Suh and C. W. Radcliffe, "Kinematics and Mechanism Design," John Wiley and Sons, New York (1978).

13 G. N. Sandor and A. G. Erdman, “Advanced Mechanism Design: Analysis and Synthesis,” Prentice-Hall, Englewood Cliffs (1984).