A THEORETICAL BASIS FOR THE ANALYSIS OF LOGIX GEARS

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ABSTRACT

A novel approach to gear design to produce so-called Logix gears is based on differential geometric methods and results in higher order contact parameters, i.e., reduced stress concentration. Profiles designed in this way admit concave-to-convex mating tooth surfaces and a simple two gear set with such properties is presented as an example. Once a pair of mating gears has been configured it is then shown how to design hobbing racks to cut these gears.

UNE THÉORIE DE BASE POUR L'ANALYSE DES ENGRENAGES LOGIX

RESUME

Une approche neuve à la production des engrenages Logix est basée sur les méthodes de la géométrie différentielle, résultant en un plus haut contact de paramètre et une réduction de la concentration de stress. Ce type de profil permet du contact concave-convex des surfaces des dents conjuguées. On présente un exemple d'une trousse simple avec deux engrenages qui démontre telles propriétés. Après avoir configuré une paire d'engrenages conjugués, on démontre aussi le profil de la crémaillère de base afin de couper ces engrenages.

1. INTRODUCTION

Professor Novikov of former Soviet Union presented the point contact circular gearing. Professor Neumann of Germany presented the Neumann worm transmission[1]. Such researches improved the contact strength of tooth surface in some degree. But circular gearing can only mesh temporally in the same transverse section. The contact of the profile of Neumann worm transmission was still the "second degree "contact. In the early 1990s, the Japanese scholar Komori T *et al.* presented a new gear profile having zero relative curvature at many contact points[2,3] , improved the contact strength of tooth profile. However, after the presentation of this new gear, so called "Logix" gear, there were not many significant researches on this field. In recent years, some institutes in China had been attracted by this gearing approach and started to follow the research of this kind of gearing. But most researches concentrated on the topics such as the foot-cut and interference, which are applications of the ready-made principles[4,5]. After an extensive research on the fundamental theory, the theoretical basis for the Logix gearing will be presented in this paper. In comparison to previous studies, the theory of this paper is more restricted going beyond third order parameters into the fourth order, the teeth profile had one order of contact higher than that of the Logix gearing. So that further improved the contact strength of tooth profile. In the gearing proposed in this work the curvature center of basic rack extends along the pitch line continuously and steadily. The transverse engagement factor is larger than that of Logix gearing. From the viewpoint of derivative geometry, the most simple and strict way to judge the closeness between two bodies is "contact".

2. THE APPROXIMATION OF THE CURVE WITH THE TAYLOR SERIES

The approximation of the curve with the Taylor series

$$\overrightarrow{MM_0} = dr = \sum_{m=1} \frac{1}{m!} \frac{d^m r}{du^m} \Delta u^m \tag{1}$$

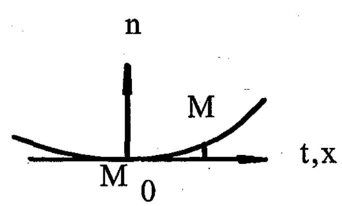

Fig.1 the distance between curve and its tangency

As showed in Fig.1 the distance between curve and its tangency:

$$\delta = \boldsymbol{n} \bullet d\boldsymbol{r} = \sum_{m=1} \frac{1}{m!} \frac{\boldsymbol{n} \bullet d^m \boldsymbol{r}}{du^m} \Delta u^m$$

Since that : $\boldsymbol{n} \cdot \frac{d\boldsymbol{r}}{du} = 0$; the differentia of the curve: $\Delta s = \sqrt{E}\ \Delta u = \Delta x$, so that $\Delta u = \frac{\Delta x}{\sqrt{E}}$

$$\delta = \boldsymbol{n} \bullet d\boldsymbol{r} = \sum_{m=2} \frac{1}{m!} \frac{\boldsymbol{n} \bullet d^m \boldsymbol{r}}{du^m} (\frac{\Delta x}{\sqrt{E}})^m$$

The introduced curve of the two curves can be developed into:

$$\delta_1 - \delta_2 = \sum_{m=2} \frac{1}{m!} \boldsymbol{n} \bullet (\frac{1}{\sqrt{E_1}^m} \frac{d^m \boldsymbol{r}_1}{du_1^m} - \frac{1}{\sqrt{E_2}^m} \frac{d^m \boldsymbol{r}_2}{du_2^m}) \Delta x^m \qquad (2)$$

For most of the gearings, the coefficients of the above equation are not zero. The gap between tooth surfaces at adjacent area of mesh point is an infinite small of the second order. If the first coefficient equals zero , the curvature radius of 2 profiles are the same, the gap between tooth surfaces at adjacent area of mesh point is an infinite small of the third order. If the first and second coefficient equal zero , the gap between tooth surfaces at adjacent area of mesh point is an infinite small of the fourth order. The thought process is: first, suppose that the profile of the basic rack is unknown, Then find out the profile of conjugate surfaces, finally, determine the profile of basic rack in order that the two surfaces will most closely contact with each other.

3. THE CONDITION FOR THE GAP BETWEEN TOOTH SURFACES TO BE AN INFINITE SMALL OF FOURTH ORDER

According to the theories of gear meshing, suppose that the profile of basic rack is: $y = y(x)$, the corresponding profile and normal vector of the gear would be (see Fig.2)

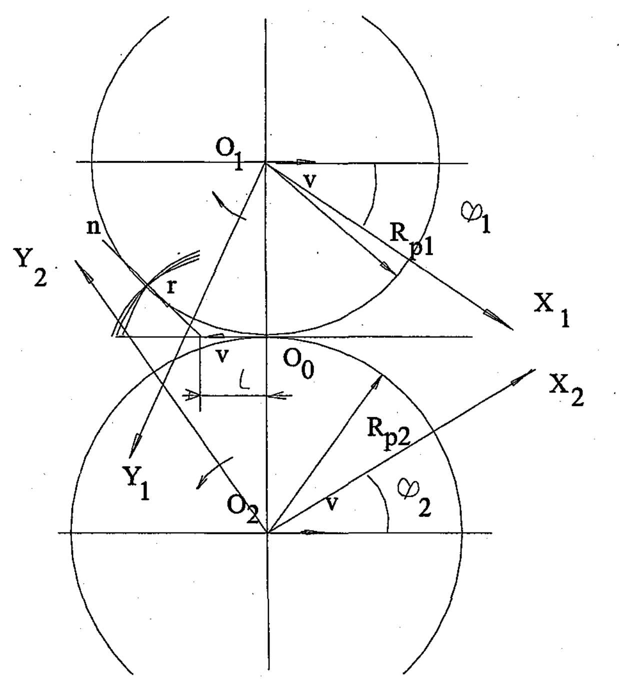

Fig.2 Basic rack and the generation of the conjugate gears

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} -R_p\phi \\ R_p \end{bmatrix} \qquad (3)$$

$$\begin{bmatrix} N_x \\ N_y \end{bmatrix} == \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} n_x \\ n_y \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} -y' \\ 1 \end{bmatrix} / \sqrt{y'^2+1} \qquad (4)$$

According to the theory of gear meshing, the transverse displacement l of rack and the angular displacement of the gear ϕ must satisfy the following equation:

$$\phi = \frac{l}{R_p} = \frac{x+yy'}{R_p} \qquad (5)$$

Equ.3 can be expressed by the form of Matrix:

$$\{R\} = M\{r\} + M\{\Phi\} \qquad (6)$$

Where: $\{R\} = \begin{bmatrix} X \\ Y \end{bmatrix}, M = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}, \{r\} = \begin{bmatrix} x \\ y \end{bmatrix}, \{\Phi\} = \begin{bmatrix} -R_p\phi \\ R_p \end{bmatrix}$

differentiate $\{R\}$ form 1—4th order :

$$\{R\}' = M'\{r\} + M\{r\}' + M'\{\Phi\} + M\{\Phi\}' \qquad (7)$$

$$\{R\}'' = M''\{r\} + 2M'\{r\}' + M\{r\}'' + M''\{\Phi\} + 2M'\{\Phi\}' + M\{\Phi\}'' \qquad (8)$$

$$\{R\}''' = M'''\{r\} + 3M''\{r\}' + 3M'\{r\}'' + M\{r\}''' + M'''\{\Phi\} + 3M''\{\Phi\}' + 3M'\{\Phi\}'' + M\{\Phi\}''' \qquad (9)$$

$$\{R\}^{(4)} = M^{(4)}\{r\} + 4M'''\{r\}' + 6M''\{r\}'' + 4M'\{r\}''' + M\{r\}^{(4)}$$
$$+ M^{(4)}\{\Phi\} + 4M'''\{\Phi\}' + 6M''\{\Phi\}'' + 4M'\{\Phi\}''' + M\{\Phi\}^{(4)} \qquad (10)$$

in the above equation, the differentials of $\{r\}$ and $\{\Phi\}$ are: :

$$\{r\}' = \begin{bmatrix} 1 \\ y' \end{bmatrix}, \quad \{r\}'' = \begin{bmatrix} 0 \\ y'' \end{bmatrix}, \quad \{r\}''' = \begin{bmatrix} 0 \\ y''' \end{bmatrix}, \quad \{r\}^{(4)} = \begin{bmatrix} 0 \\ y^{(4)} \end{bmatrix} \qquad (11)$$

$$\{\Phi\} = R_p \begin{bmatrix} -\phi \\ 1 \end{bmatrix}, \quad \{\Phi\}' = R_p \begin{bmatrix} -\phi' \\ 1 \end{bmatrix}, \quad \{\Phi\}'' = R_p \begin{bmatrix} -\phi'' \\ 1 \end{bmatrix}, \quad \{\Phi\}''' = R_p \begin{bmatrix} -\phi''' \\ 1 \end{bmatrix},$$

$$\{\Phi\}^{(4)} = R_p \begin{bmatrix} -\phi^{(4)} \\ 1 \end{bmatrix} \qquad (12)$$

Since that the original point of coordinate is the intersect point of normal vector and the pitch line of rack. Where: $\phi = \frac{l}{R_p} = \frac{x+yy'}{R_p} = 0$, So that, we have: $M\big|_{\phi=0} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (13)

$$M'\big|_{\phi=0} = \begin{bmatrix} -\phi'\sin\phi & \phi'\cos\phi \\ -\phi'\cos\phi & -\phi'\sin\phi \end{bmatrix}_{\phi=0} = \begin{bmatrix} 0 & \phi' \\ -\phi' & 0 \end{bmatrix} \qquad (14)$$

$$M''\big|_{\phi=0} = \begin{bmatrix} -\phi''\sin\phi - \phi'^2\cos\phi & \phi''\cos\phi - \phi'^2\sin\phi \\ -\phi''\cos\phi + \phi'^2\sin\phi & -\phi''\sin\phi - \phi'^2\cos\phi \end{bmatrix}_{\phi=0} = \begin{bmatrix} 0 & \phi'' \\ -\phi'' & 0 \end{bmatrix} \qquad (15)$$

$$\{R\}'' = \phi'' \begin{bmatrix} y \\ -x \end{bmatrix} + 2\phi' \begin{bmatrix} y' \\ -1 \end{bmatrix} + \begin{bmatrix} 0 \\ y'' \end{bmatrix} + R_p \begin{bmatrix} 0 \\ 2\phi'^2 \end{bmatrix}$$

The normal vector of the gear is determined by the normal vector of the rack, so that we have:

$$\begin{bmatrix} N_x \\ N_y \end{bmatrix}_{\phi=0} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -y' \\ 1 \end{bmatrix} / \sqrt{y'^2+1} = \begin{bmatrix} -y' \\ 1 \end{bmatrix} / \sqrt{y'^2+1} \qquad (16)$$

The second parameter of the Taylor series of the tooth profile $n \cdot \dfrac{d^2\mathbf{r}}{du^2}$ is: :

$$\{N\}^T\{R\}'' = \phi''[-y' \quad 1]\begin{bmatrix} y \\ -x \end{bmatrix} + 2\phi''[-y' \quad 1]\begin{bmatrix} y' \\ -1 \end{bmatrix} + [-y' \quad 1]\begin{bmatrix} 0 \\ y''' \end{bmatrix} + R_p[-y' \quad 1]\begin{bmatrix} 0 \\ 2\phi'^2 \end{bmatrix} =$$

$$\phi''(-y'y - x) + 2\phi'(-y'^2 - 1) + y'' + 2R_p\phi'^2 = -2\phi'(y'^2 + 1 + 2R_p\phi') + y''$$

$$(17) \quad (\because \phi = 0, l = x + yy' = 0)$$

For gear 1 and gear 2 we have the expressions as following respectively:

$$\{N_1\}^T\{R_1\}'' = -2\phi_1'(y'^2 + 1 + 2R_{p1}\phi_1') + y'' \ , \ \phi_1 = \frac{l}{R_{p1}} = \frac{x + yy'}{R_{p1}} \ , \ ,$$

$$\{N_2\}^T\{R_2\}'' = -2\phi_2'(y'^2 + 1 + 2R_{p2}\phi_2') + y'' \ , \ \phi_2 = \frac{x + yy'}{R_{p2}}$$

let the second parameter equal zero: $\{N_1\}^T\{R_1\}'' - \{N_2\}^T\{R_2\}'' = 0$ we have :

$$\phi_1' = \frac{l'}{R_{p1}} = 0, \ \phi_2' = \frac{l'}{R_{p2}} = 0, \ l' = 0 \qquad (x + yy')' = 0, \ 1 + y'^2 + y'' = 0, \qquad (18)$$

The coordinate of the curvature center: $y_c = y + \dfrac{1 + y'^2}{y''} = 0$, the curvature center of basic rack must lay on the pitch line.

4. EXAMPLIE

4.1 First, Choose the Appropriate Value : y_0, x_0, y_0', y_0''

4.2 Create a Shot Curve Passing Through The Point (x_0, y_0) :

$$y = y_0 + y_0'(x - x_0) + \frac{y_0''}{2}(x - x_0)^2 \qquad (19)$$

$$y' = y_0' + y_0''(x - x_0) \qquad (20)$$

$$y'' = y_0'' \qquad (21)$$

4.3 Find The Curvature Center of the Curve :

$$y_c = y + \frac{1 + y'^2}{y''} \qquad (22)$$

$$x_c = x - \frac{y'(1 + y'^2)}{y''} \qquad (23)$$

4.4 Choose a Point at the End of the Curve.

Connect this point and its curvature center. Find out the intersect point of this line and the x

axes :

$$x_c' = x - \frac{y(x - x_c)}{y - y_c} \tag{24}$$

4.5 Let The New Point to be the Center of Curvature, Revise y'' Accordingly.

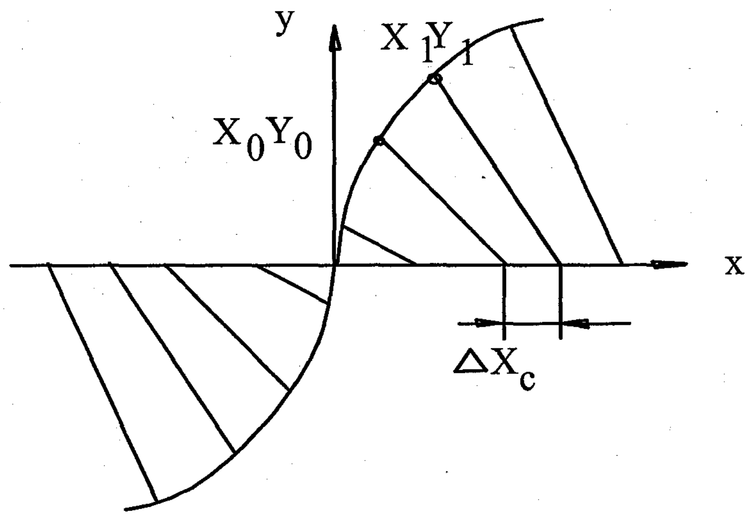

Fig.3. the construction of basic rack of new kind Logix gear

The original data are as following:

$y_0 = 10$ $x_0 = 3.64$ $y_0' = 2.74748$ $y_0'' = -0.8585$

Calculate processes are as table 1

Table 1. the results of calculation

y	x	y'	y''	y_c	x_c	x_c'
6.825	2.64	3.602	-0.8585	-9.45	61.27	27.23
2.2	1.64	5.652	-2.05	-13.87	92.47	14.07
1.56	1.54	7.15	-14.98	-1.92	26.42	14.81
1.25	1.5	8.486	-33.4	-0.936	20.1	12.1
0.7527	1.45	11.406	-58.4	-1.49	27.1	10.06

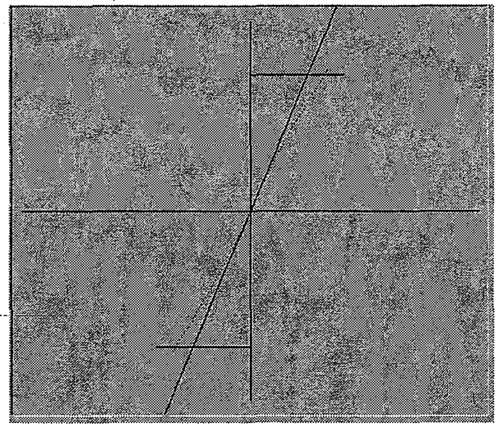

Fig.4. The comparing of standard profile of basic rack to profile of basic rack of Logix gear. The straight line: standard profile of basic rack. The curve like "*ƒ*": profile of basic rack of Logix gear.

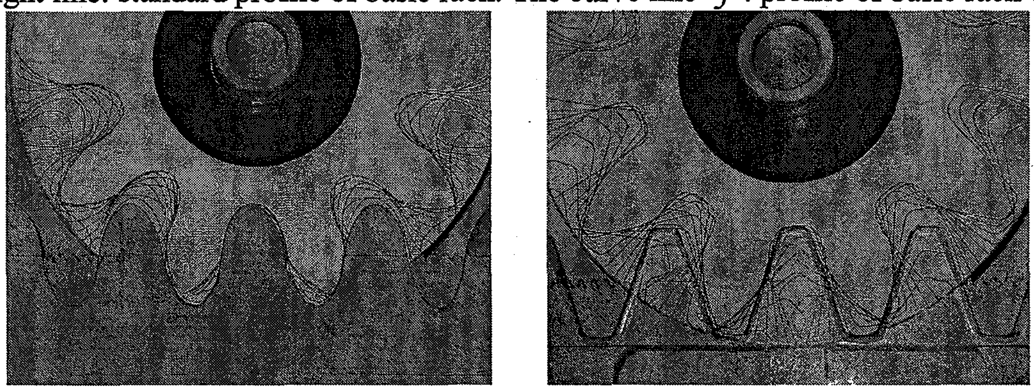

Fig.5. Generation of profile of the Logix gear, which show a profile that highlights the difference between profiles commonly used and profile related to Logix gear. Left: Logix gear, Right: commonly used gear.

5 CONCLUSIONS

Some essential findings are:

(1) If $x+yy'=0$, the two surfaces would conjugate with each other, there are no additional requirements for the profile of basic rack[6,7,8]. The gap between tooth surfaces at adjacent area of mesh point is an infinite small of the second order. The parameter of the second order can be expressed by: $(x+yy')'$.

(2) If $(x+yy')'=0$, the curvature center of basic rack must lay on the pitch line, the gap between tooth surfaces at adjacent area of mesh point is an infinite small of the third order. The parameter of the third order can be expressed by: $(x+yy')''$.

(3) If $(x+yy')''=0$, The gap between tooth surfaces at adjacent area of mesh point is an infinite small of the fourth order. The parameter of the fourth order can be expressed by

$(x + yy')'''$. From the viewpoint of logic it is very regular and reasonable, where $y=y(x)$ is the equation of rack profile.

NOMENCLATURE

$\boldsymbol{r}$	Position vector of surface
u	parameter of the surface
Δu	Differentia of the parameter
$\boldsymbol{n}$	normal vector of surface,
$\boldsymbol{t}$	tangential vector of surface,
M_0	the temporally contact point of the gear.
$\boldsymbol{n}$	normal vector of surface
δ	*Distance* between curve and its tangency:
x, y	the coordinates of the basic rack
n_x, n_y	the normal vector of the basic rack
X, Y	the coordinates of the gear
N_X, N_Y	the normal vector of the gear
ϕ	the angular displacement of the gear
l	the transverse displacement
Rp	the radius of the pitch circle

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