ADAPTIVE BACKSTEPPING CONTROL OF A PIEZO-POSITIONING MECHANISM WITH HYSTERESIS

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ABSTRACT

Piezo-positioning mechanisms are often used in high-precision positioning applications. Due to their materials, nonlinear hysteretic behavior is commonly observed in such mechanisms and can be described by a LuGre model. In this paper, we develop two robust adaptive backstepping control algorithms for piezo-positioning mechanisms. In the first scheme, we take the structure of the LuGre model into account in the controller design, if the parameters of the model are known. A nonlinear observer is designed to estimate the hysteresis force. In the second scheme, there is no apriori information required from these parameters and thus they can be allowed totally uncertain. In this case, the LuGre model is divided into two parts. While the unknown parameters of one part are incorporated with unknown system parameters for estimation, the effect of the other part is treated as a bounded disturbance. An update law is used to estimate the bound involving this partial hysteresis effect and the external load. For both schemes, it is shown that not only global stability is guaranteed by the proposed controller, but also both transient and asymptotic performances are quantified as explicit functions of the design parameters so that designers can tune the design parameters in an explicit way to obtain the required closed loop behavior.

COMMANDE ADAPTATIVE PAR BACKSTEPPING POUR UN MÉCANISME DE POSITIONNEMENT PIÈZE AVEC HYSTÉRÉSIS

RESUME

Les mécanismes de positionnement pièze sont souvent utilisés pour des applications de haute précision. Le matériel fait souvent l'objet d'un comportement hystérétique qui peut être décrit par le modèle de LuGre. Dans cet article, nous développons deux algorithmes de commande adaptative par backstepping pour des mécanismes de positionnement pièze. La première technique prend en compte le modèle de LuGre, si les paramètres du modèle sont connus. Un observateur non linéaire est conçu pour estimer la force de l'hystérésis. Dans la seconde technique, il n'y a pas d'information requise a priori des paramètres. Donc, ces derniers peuvent être supposés incertains. Dans ce cas, le modèle de LuGre est divisé en deux parties. Les paramètres inconnus d'une part sont incorporés dans la conception de l'estimateur, et d'autre part on considère la perturbation comme étant bornée. Une loi mise à jour est utilisée pour estimer la borne. L'effet partiel de l'hystérésis et la force externe sont alors considérés. Dans les deux cas, il est démontré que la stabilité globale est garantie. Finalement, les performances sont quantifiées en tant que fonctions explicites des paramètres de conception, et donc le designer a la possibilité d'ajuster ces derniers pour obtenir le comportement désiré en boucle fermée.

1 INTRODUCTION

Piezo-positioning mechanisms are often used in high-precision positioning applications, such as nanometer. Since the materials of the piezo-positioning mechanisms are ferroelectric, nonlinear hysteretic behavior is commonly observed in such mechanisms in response to an applied electric field. This leads to problems of severe inaccuracy, instability, and restricted system performance due to the hysteresis nonlinearity. Moreover, the hysteresis characteristics are usually unknown and the states to represent hysteresis dynamics are often unavailable. These usually cause the increasing difficulties in servo control design with high performance requirement for piezo-positioning mechanisms. Studies of this problem have been reported in the works of [1; 2]. In [2], a feedforward model-reference control was designed to improve scanning accuracy of PZT piezoelectric actuator. Studies of controlling hysteresis nonlinearity have also been reported in the works of [3; 4; 5; 6; 7; 8]. In [4; 5; 6], an adaptive hysteresis inverse cascaded with the plant was employed to cancel the effects of hysteresis. In [7; 8] a dynamic hysteresis model was defined to pattern a hysteresis rather than constructing an inverse model to mitigate the effects of the hysteresis. In [9], a reinforcement discrete neuro-adaptive control was proposed for unknown piezoelectric actuator systems with dominant hysteresis. In [10], an adaptive wavelet neural network control was proposed to control of a piezo-positioning mechanism with hysteresis estimation. However, it is assumed that system uncertain parameters must be within some known intervals. It is also assumed that system states must be inside a compact set in order to ensure the error due to neural network approximation bounded. This implies that the closed-loop system is bounded-input bounded-output stable even before the controller is designed.

In this paper, in order to consider the hysteresis, the Lugre model presented in [11] is used and the proposed hysteresis model with parametrization is integrated into a mechanical motion dynamics with lumped external load to completely represent the overall dynamics of a piezo-positioning mechanism. We develop two simple adaptive backstepping control schemes for the piezo-positioning mechanism. In the first scheme, we take the structure of the Lugre model into account in the controller design, if parameters of the model are known. A nonlinear observer is designed by using Lyapunov technique to estimate the unavailable state. In the second scheme, there is no apriori information required from these parameters and thus they can be allowed totally uncertain. In this case, the LuGre model is divided into two parts. While the unknown parameters of one part is incorporated with unknown system parameters for estimation, the effect of the other part is treated as a bounded disturbance. An update law is used to estimate the bound involving this partial hysteresis effect and the external load. Besides showing global stability of the system for both schemes, the transient performance in terms of L_2 norm of the tracking error is derived to be an explicit function of design parameters and thus our scheme allows designers to obtain the closed loop behavior by tuning design parameters in an explicit way.

This paper is organized as follows: The problem considered is formulated in Section 2. In Section 3, we present two adaptive control design approaches based on backstepping technique, analyze the stability and establish the performance of the closed loop system. Simulation results are presented to illustrate the effectiveness of our proposed scheme in Section 4. Finally, the paper is concluded in Section 5.

2 PROBLEM FORMULATION

The dynamics of a piezo-positioning system can be represented by the following second-order uncertain nonlinear system [10]

$$M\ddot{x} + D\dot{x} + F_H + F_L = u \tag{1}$$

where M denotes the equivalent mass of the controlled piezo-positioning mechanism, which is positive; x is the displacement of the mechanism; $\dot{x}$ denotes the relative velocity; $\ddot{x}$ denotes the acceleration; D is the linear friction coefficient of the piezo-positioning mechanism; F_L is the external load; F_H denotes the hysteresis friction force function; u denotes the applied voltage to piezo-positioning mechanism. A block diagram representing (1) is shown in Figure 1.

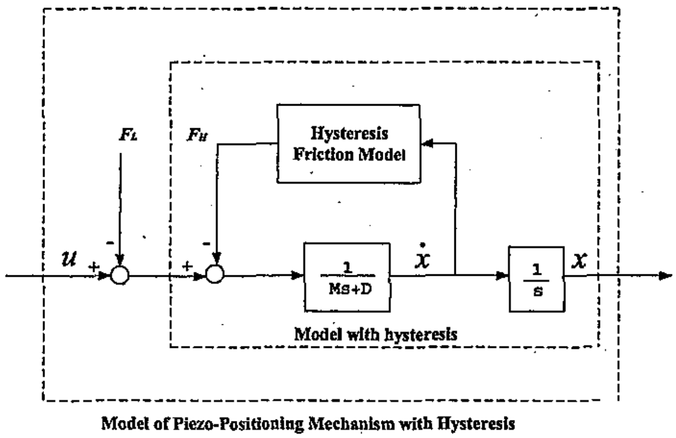

Figure 1: Block diagram of modelling piezo-positioning mechanism

The hysteresis friction force F_H is described by the so-called LuGre model [11] in the following form

$$F_H = \sigma_0 z - \sigma_1 \frac{1}{g(\dot{x})} z|\dot{x}| + (\sigma_1 + \sigma_2)\dot{x} \tag{2}$$

$$\dot{z} = \dot{x} - \frac{|\dot{x}|}{g(\dot{x})} z \tag{3}$$

where z is an unavailable state and is interpreted as the contact force, σ_0, σ_1 and σ_2 are positive constants and can be equivalently interpreted as bristle stiffness, bristle damping, and viscous damping-coefficient. Moreover, the function $g(\dot{x})$ denotes the Stribeck effect curve given by

$$\sigma_0 g(\dot{x}) = f_C + (f_S - f_C)e^{-(\dot{x}/\dot{x}_S)^2} \tag{4}$$

where f_C is the Coulomb friction level, f_S is the level of the stiction force, and $\dot{x}_S$ is the Stribeck velocity, see [12]. The function $g(\dot{x})$ is positive and bounded. As shown in [11], the following lemma holds.

Lemma 1. *Consider nonlinear dynamic system (3). For any piecewise continuous signal x and $\dot{x}$, the output $z(t)$ is bounded.*

The control objective is to design a backstepping adaptive control law for system (1) so that the displacement x of the piezo-positioning mechanism can track any desired bounded reference trajectory x_m.

3 CCONTROLLER DESIGN AND MAIN RESULTS

In this section, we develop two adaptive backstepping design schemes. In Scheme I, we assume that the hysteresis parameters $\sigma_0, \sigma_1, \sigma_2$ and the function g are known. The state $z(t)$ is not measurable and hence has to be observed to estimate the hysteresis force F_H. For this we design an observer $\hat{z}(t)$ to estimate $z(t)$. In Scheme II, we assume that the parameters $\sigma_0, \sigma_1, \sigma_2, f_S, f_C, \dot{x}_S$ in the hysteresis model (3) are all uncertain. The residual effect of the hysteresis is treated as a bounded disturbance with unknown bound. An update law is used to estimate the bound involving the effect of the hysteresis and the external load. To illustrate the backstepping procedures, only the first scheme is elaborated in details.

3.1 Control Scheme I

When the hysteresis parameters $\sigma_0, \sigma_1, \sigma_2$ and the function g are known, we can exploit the structure of the model in our controller design to improve system performance.
We rewrite equations (1) in the following form

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{1}{M}u - \theta x_2 - \frac{1}{M}F_H - \frac{1}{M}F_L \end{aligned} \tag{5}$$

where $x_1 = x, x_2 = \dot{x}$ and $\theta = \frac{D}{M}$, an unknown constant containing uncertain parameters. Note that F_L is bounded with unknown bound F_o. Before presenting the adaptive control design using the backstepping technique to achieve the desired control objectives, the following change of coordinates is made.

$$z_1 = x_1 - x_m \tag{6}$$

$$z_2 = x_2 - \dot{x}_m - \alpha_1 \tag{7}$$

where α_1 is a virtual control and will be determined in later discussion.
The variable $z(t)$ is not measurable and hence has to be observed to estimate the hysteresis force F_H. For this, we design a nonlinear observer to estimate the variable z as follows:

$$\dot{\hat{z}} = \dot{x} - \frac{|x_2|}{g(x_2)}\hat{z} + \phi(t) \tag{8}$$

where $\phi(t)$ is a nonlinear function derived later. Note that $\frac{|x_2|}{g(x_2)}$ is positive and bounded.
• *Step* 1: From (5) to (7), we obtain that

$$\dot{z}_1 = z_2 + \alpha_1 \tag{9}$$

We design the virtual control law α_1 as

$$\alpha_1 = -c_1 z_1 \tag{10}$$

where c_1 is a positive design parameter. From (9) and (10) we have

$$z_1 \dot{z}_1 = -c_1 z_1^2 + z_1 z_2 \tag{11}$$

• *Step* 2: From (5) and (7), we have

$$\dot{z}_2 = \frac{1}{M}u - \theta x_2 - \frac{1}{M}F_H - \frac{1}{M}F_L - \ddot{x}_m - \dot{\alpha}_1 \tag{12}$$

Then function $\phi(t)$ in (8), the control law and parameter update laws are obtained as follows by considering a Lyapunov function in (20) with details given later.

$$\begin{aligned}
\phi(t) &= -\sigma_0 z_2 + \sigma_1 \frac{|x_2|}{g(x_2)} z_2 && (13)\\
u(t) &= \hat{M}\bar{u} + \hat{F}_H - \hat{F}_o sign(z_2) && (14)\\
\bar{u}(t) &= -c_2 z_2 - z_1 + \hat{\theta} x_2 + \ddot{x}_m + \dot{\alpha}_1 && (15)\\
\dot{\hat{\theta}} &= -\gamma_\theta x_2 z_2 && (16)\\
\dot{\hat{M}} &= -\gamma_M \bar{u} z_2 && (17)\\
\dot{\hat{F}}_o &= \gamma_F |z_2| && (18)\\
\hat{F}_H &= \sigma_0 \hat{z} - \sigma_1 \frac{|x_2|}{g(x_2)} \hat{z} + (\sigma_1 + \sigma_2) x_2 && (19)
\end{aligned}$$

where c_2, γ_θ, γ_M and γ_F are positive design parameters, $\hat{\theta}$, $\hat{M}$ and $\hat{F}_o$ are estimates of θ, M and the unknown bound F_o, respectively.

Remark 1. A parameter update law is used to estimate the bound F_o of the external load F_L, so there is no need to know this bound.

We now show how (13)-(19) are derived by the following function.

$$V = \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2 + \frac{1}{2M\gamma_M}\tilde{M}^2 + \frac{1}{2\gamma_\theta}\tilde{\theta}^2 + \frac{1}{2M\gamma_F}\tilde{F}_o^2 + \frac{1}{2M}\tilde{z}^2 \tag{20}$$

where $\tilde{M} = M - \hat{M}$, $\tilde{\theta} = \theta - \hat{\theta}$, $\tilde{F}_o = F_o - \hat{F}_o$ and $\tilde{z} = z - \hat{z}$.
Note that $\frac{1}{M}u$ in (12) can be expressed as

$$\begin{aligned}
\frac{1}{M}u &= \frac{1}{M}\hat{M}\bar{u} + \frac{1}{M}\hat{F}_H - \frac{1}{M}\hat{F}_o sign(z_2)\\
&= \bar{u} - \frac{1}{M}\tilde{M}\bar{u} + \frac{1}{M}\hat{F}_H - \frac{1}{M}\hat{F}_o sign(z_2) && (21)
\end{aligned}$$

Then the derivative of V along with (8), (12) and (21) is given by

$$
\begin{aligned}
\dot{V} &= z_1\dot{z}_1 + z_2\dot{z}_2 + \frac{1}{\gamma_\theta}\tilde{\theta}\dot{\tilde{\theta}} + \frac{1}{M\gamma_M}\tilde{M}\dot{\tilde{M}} + \frac{1}{M\gamma_F}\tilde{F}_o\dot{\tilde{F}}_o + \frac{1}{M}\tilde{z}\dot{\tilde{z}} \\
&= -c_1z_1^2 + z_1z_2 + z_2(\frac{1}{M}u - \theta x_2 - \frac{1}{M}F_H - \frac{1}{M}F_L - \ddot{x}_m - \dot{\alpha}_1) \\
&\quad + \frac{1}{\gamma_\theta}\tilde{\theta}\dot{\tilde{\theta}} + \frac{1}{M\gamma_M}\tilde{M}\dot{\tilde{M}} + \frac{1}{M\gamma_F}\tilde{F}_o\dot{\tilde{F}}_o + \frac{1}{M}\tilde{z}(\dot{z} - \dot{\hat{z}}) \\
&\le -c_1z_1^2 + z_1z_2 + z_2(\bar{u} - \theta x_2 - \ddot{x}_m - \dot{\alpha}_1) - \frac{1}{M}\tilde{M}\bar{u}z_2 + \frac{1}{M}\hat{F}_Hz_2 - \frac{1}{M}F_Hz_2 \\
&\quad - \frac{1}{M}\hat{F}_o|z_2| + \frac{1}{M}F_o|z_2| + \frac{1}{\gamma_\theta}\tilde{\theta}\dot{\tilde{\theta}} + \frac{1}{M\gamma_M}\tilde{M}\dot{\tilde{M}} + \frac{1}{M\gamma_F}\tilde{F}_o\dot{\tilde{F}}_o + \frac{1}{M}\tilde{z}(\dot{z} - \dot{\hat{z}}) \\
&= -c_1z_1^2 + z_2(\bar{u} + z_1 - \theta x_2 - \ddot{x}_m - \dot{\alpha}_1) - \frac{1}{M}z_2(F_H - \hat{F}_H) + \frac{1}{M}\tilde{z}(\dot{z} - \dot{\hat{z}}) \\
&\quad - \frac{1}{\gamma_\theta}\tilde{\theta}^T\dot{\hat{\theta}} - \frac{1}{M\gamma_M}\tilde{M}\big(\gamma_M\bar{u}z_2 + \dot{\hat{M}}\big) + \frac{1}{M\gamma_F}\tilde{F}_o\big(\gamma_F|z_2| - \dot{\hat{F}}_o\big)
\end{aligned} \tag{22}
$$

So $\bar{u}$ given in (15) is designed based on the second term of (22).

Remark 2. Note that the sign function is used to compensate the effects of external load F_L. This will generate a function $-\frac{1}{M}\hat{F}_o|z_2|$ in the derivative of Lyapunov function. It follows that $\frac{1}{M}\tilde{F}_o|z_2| = \frac{1}{M}F_o|z_2| - \frac{1}{M}\hat{F}_o|z_2|$. Then a parameter update law for $\hat{F}_o$ is derived based on the last term of (22).

Then the derivative of V is given by

$$
\begin{aligned}
\dot{V} &\le -c_1z_1^2 - c_2z_2^2 - \frac{1}{M}\frac{|x_2|}{g(x_2)}\tilde{z}^2 + \frac{1}{M}\tilde{z}\big(- \sigma_0z_2 + \frac{\sigma_1|x_2|}{g(x_2)}z_2 - \phi(t)\big) \\
&\quad - \frac{1}{\gamma_\theta}\tilde{\theta}^T\big(\gamma_\theta x_2z_2 + \dot{\hat{\theta}}\big) - \frac{1}{M\gamma_M}\tilde{M}\big(\gamma_M\bar{u}z_2 + \dot{\hat{M}}\big) + \frac{1}{M\gamma_F}\tilde{F}_o\big(\gamma_F|z_2| - \dot{\hat{F}}_o\big)
\end{aligned} \tag{23}
$$

Then $\phi(t), \dot{\hat{\theta}}, \dot{\hat{M}}$ and $\dot{\hat{F}}_o$ in (13), (16), (17) and (18) are obtained by making the last four terms of (23) zero, respectively. Thus,

$$
\dot{V} \le -c_1z_1^2 - c_2z_2^2 - \frac{1}{M}\frac{|x_2|}{g(x_2)}\tilde{z}^2 \tag{24}
$$

Based on (24), we can obtain the result on system stability and performance as stated below.

Theorem 1. *Consider the uncertain nonlinear system (1). With the application of the controller (14), the observer (8) and the parameter update laws (16), (17) and (18), the following statements hold:*

- *The resulting closed loop system is globally bounded input bounded output(BIBO) stable.*
- *The asymptotic tracking is achieved, i.e.,*

$$
\lim_{t\to\infty}[x(t) - x_m(t)] = 0 \tag{25}
$$

- *The transient displacement tracking error performance is given by*

$$\| x(t) - x_m(t) \|_2 \leq \frac{1}{\sqrt{c_1}} \times \left(\frac{1}{2\gamma_\theta}\tilde{\theta}(0)^2 + \frac{1}{2M\gamma_M}\tilde{M}(0)^2 + \frac{1}{2M\gamma_F}\tilde{F}_o(0)^2 + \frac{1}{2M}\tilde{z}(0)^2\right)^{1/2} \tag{26}$$

- *The transient velocity tracking error performance is given by*

$$\| \dot{x} - \dot{x}_m \|_2 \leq \left(\frac{1}{\sqrt{c_2}} + \sqrt{c_1}\right) \times \left(\frac{1}{2\gamma_\theta}\tilde{\theta}(0)^2 + \frac{1}{2M\gamma_M}\tilde{M}(0)^2 + \frac{1}{2M\gamma_F}\tilde{F}_o(0)^2 + \frac{1}{2M}\tilde{z}(0)^2\right)^{1/2} \tag{27}$$

Proof: Equation (24) shows that $V(t)$ is globally uniformly bounded. This implies that z_1, z_2, $\tilde{\theta}$, $\tilde{M}$, $\tilde{F}_o$ and $\tilde{z}$ are bounded. The state variables x_1, x_2 and the parameter estimates $\hat{\theta}, \hat{M}, \hat{F}_o, \hat{z}$ are also bounded. Thus u is bounded from (14) because of the boundedness of $z_1, z_2, \hat{\theta}, \hat{M}, \hat{F}_o, \hat{z}$. By applying the LaSalle-Yoshizawa theorem in [13] to (24), it further follows that $z_i(t) \to 0, i = 1, 2$ as $t \to \infty$, which implies that $\lim_{t\to\infty}[x(t) - x_m(t)] = 0$.

Since V is non increasing from (24), we have

$$\| z_1 \|_2^2 = \int_0^\infty |z_1(\tau)|^2 d\tau \leq \frac{1}{c_1}(V(0) - V(\infty)) \leq \frac{1}{c_1}V(0) \tag{28}$$

The initial value of the Lyapunov function is

$$V(0) = \frac{1}{2}z_1^2(0) + \frac{1}{2}z_2^2(0) + \frac{1}{2\gamma_\theta}\tilde{\theta}(0)^2 + \frac{1}{2M\gamma_M}\tilde{M}(0)^2 + \frac{1}{2M\gamma_F}\tilde{F}_o(0)^2 + \frac{1}{2M}\tilde{z}(0)^2 \tag{29}$$

Note that $\tilde{\theta}(0), \tilde{M}(0), \tilde{F}_o(0)$ and $z_1(0) = x_1(0) - x_m(0)$ are clearly independent of $c_1, \gamma_M, \gamma_\theta$ and γ_F. We can set $z_1(0)$ and $z_2(0)$ to zero by appropriately initializing the reference trajectory $x_m(0)$ and $\dot{x}_m(0)$ as follows

$$x_m(0) = x_1(0) \tag{30}$$

$$\dot{x}_m(0) = x_2(0) \tag{31}$$

Thus, by setting $z_1(0) = z_2(0) = 0$, we obtain

$$V(0) = \frac{1}{2\gamma_\theta}\tilde{\theta}(0)^2 + \frac{1}{2M\gamma_M}\tilde{M}(0)^2 + \frac{1}{2M\gamma_F}\tilde{F}_o(0)^2 + \frac{1}{2M}\tilde{z}(0)^2 \tag{32}$$

a decreasing function of γ_θ, γ_F and γ_M, independent of c_1. This means that the bound resulting from (28) and (32) satisfies

$$\| z_1 \|_2 \leq \frac{1}{\sqrt{c_1}}\left(\frac{1}{2\gamma_\theta}\tilde{\theta}(0)^2 + \frac{1}{2M\gamma_M}\tilde{M}(0)^2 + \frac{1}{2M\gamma_F}\tilde{F}_o(0)^2 + \frac{1}{2M}\tilde{z}(0)^2\right)^{1/2} \tag{33}$$

and can be asymptotically reduced either by increasing c_1 or by simultaneously increasing γ_θ, γ_M and γ_F. Thus the bound for $\| z_1 \|_2$ is an explicit function of design parameters.
From equations (9) to (10), we get

$$\| \dot{x} - \dot{y}_r \|_2 = \| z_2 - c_1 z_1 \|_2 \leq \| z_2 \|_2 + c_1 \| z_1 \|_2 \tag{34}$$

Similarly, we can get $\| z_2 \|_2 \leq \frac{1}{\sqrt{c_2}} \sqrt{V(0)}$. Along with (33) we get

$$\| \dot{x} - \dot{y}_r \|_2 \leq \left(\frac{1}{\sqrt{c_2}} + \sqrt{c_1}\right)\left(\frac{1}{2\gamma_\theta}\tilde{\theta}(0)^2 + \frac{1}{2M\gamma_M}\tilde{M}(0)^2 + \frac{1}{2M\gamma_F}\tilde{F}_o(0)^2 + \frac{1}{2M}\tilde{z}(0)^2\right)^{1/2} \tag{35}$$

Remark 3. From Theorem 1 the following conclusions can be obtained:
• Boundedness of signals in the adaptive system is guaranteed to be global, uniform and ultimate for any positive values of the design parameters $c_1, c_2, \gamma_\theta, \gamma_M$ and γ_F.
• The transient performance depends on the initial estimate errors $\tilde{\theta}(0)$, $\tilde{M}(0)$, $\tilde{F}_o(0)$ and $\tilde{z}(0)$. The closer the initial estimates to the true values, the better the transient performance. The asymptotic behavior is not affected by the initial estimate errors. We can decrease the effects of the initial error estimates on the transient performance by increasing the adaptation gains γ_θ, γ_M and γ_F.
• To improve the displacement tracking error performance we can also increase the gain c_1. However, increasing the gain c_1 will also increase the velocity tracking error as shown above. Improving the closed loop displacement behavior may be done at the expense of the increase in the control signal amplitude. This suggests to fix the gain c_1 to some acceptable value and adjust the other gains. By fixing the gain c_1, increasing the gain c_2 or by simultaneously increasing γ_θ, γ_M and γ_F, we can achieve a velocity tracking error as small as desired.

3.2 Control Scheme II:

In this section, there is no apriori information required from parameters $\sigma_0, \sigma_1, \sigma_2, f_C, f_S, \dot{x}_S$ and thus they can be allowed totally uncertain. The LuGre hysteresis friction force F_H in (2) can be divided into two parts as follows.

$$F_H = (\sigma_1 + \sigma_2)\dot{x} + R(t) \tag{36}$$

$$R(t) = \sigma_0 z - \sigma_1 \frac{|\dot{x}|}{g(\dot{x})} z \tag{37}$$

From Lemma 1, we have that $R(t)$ is bounded. Then we combine $(\sigma_1 + \sigma_2)\dot{x}$ with $D\dot{x}$ in (1) and rewrite equations (1) and (36) in the following form

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{1}{M}u - \theta x_2 - \frac{1}{M}d(t) \end{aligned} \tag{38}$$

where $x_1 = x, x_2 = \dot{x}, \theta = \frac{1}{M}(D+\sigma_1+\sigma_2)$, and $d(t) = R + F_L$. So $d(t)$ is bounded with unknown bound F_o.

Now $d(t)$ can be handled in the same way as F_L in Scheme I. Thus the controller design in this case is similar to the Scheme I and we only give the resulting control laws.

$$
\begin{aligned}
u &= \hat{M}\bar{u} - \hat{F}_o sign(z_2) && (39)\\
\bar{u} &= -c_2 z_2 - z_1 + \hat{\theta} x_2 + \ddot{x}_m + \dot{\alpha}_1 && (40)\\
\alpha_1 &= -c_1 z_1 && (41)\\
\dot{\hat{\theta}} &= -\gamma_\theta x_2 z_2 && (42)\\
\dot{\hat{M}} &= -\gamma_M \bar{u} z_2 && (43)\\
\dot{\hat{F}}_o &= \gamma_F |z_2| && (44)\\
z_1 &= x_1 - x_m && (45)\\
z_2 &= x_2 - \dot{x}_m - \alpha_1 && (46)
\end{aligned}
$$

where c_1, c_2, γ_θ, γ_M and γ_F are designed positive parameters, $\hat{\theta}$, $\hat{M}$ and $\hat{F}_o$ are estimates of θ, M and F_o, respectively.

Following the similar analysis to Scheme I, we can establish that $z_1, z_2, \hat{\theta}, \hat{M}, \hat{F}_o, u$ are all bounded. Thus similar to Theorem 1, the result on system stability and performance can be established and now stated in the following theorem.

Theorem 2. *Consider the uncertain nonlinear system (1). With the application of the controller (39) and the parameter update laws (42), (43) and (44), the following statements hold:*

- *The resulting closed loop system is globally bounded input bounded output(BIBO) stable.*
- *The asymptotic tracking is achieved, i.e.,*

$$\lim_{t\to\infty}[x(t) - x_m(t)] = 0 \tag{47}$$

- *The transient displacement tracking error performance is given by*

$$\| x(t) - x_m(t) \|_2 \le \frac{1}{\sqrt{c_1}} \times \Big(\frac{1}{2\gamma_\theta}\tilde{\theta}(0)^2 + \frac{1}{2M\gamma_M}\tilde{M}(0)^2 + \frac{1}{2M\gamma_F}\tilde{F}_o(0)^2\Big)^{1/2} \tag{48}$$

- *The transient velocity tracking error performance is given by*

$$\| \dot{x} - \dot{x}_m \|_2 \le \Big(\frac{1}{\sqrt{c_2}} + \sqrt{c_1}\Big) \times \Big(\frac{1}{2\gamma_\theta}\tilde{\theta}(0)^2 + \frac{1}{2M\gamma_M}\tilde{M}(0)^2 + \frac{1}{2M\gamma_F}\tilde{F}_o(0)^2\Big)^{1/2} \tag{49}$$

Similar remarks to Remark 3 are also obtained for Scheme II.

4 SIMULATION RESULTS

In this section we test our proposed backstepping controllers on model (1). For simulation studies, the following values are selected as "true" parameters for the system and the hysteresis model:

$\sigma_0 = 5N/m, \sigma_1 = 0.9Ns/m, \sigma_2 = 0.4Ns/m$, $f_C = 1N, f_S = 1.5N, \dot{x}_S = 0.01m/s$, $M = 1kg, D = 0.15Ns/m$ and $F_L = 0.3\sin(t)$.

The design objective is to drive the displacement x of the piezo-positioning mechanism to track the reference trajectory $x_m(t) = 2\sin(2t)$.

When Scheme I is used, we take the following set of design parameters: $\gamma_\theta = 0.4, \gamma_M = 0.2, \gamma_F = 0.4, c_1 = c_2 = 4$. The initials are set as $x(0) = 0.6, \dot{x}(0) = 2, z(0) = 0, \hat{z}(0) = 0$, $\hat{M}(0) = 1.2$, $\hat{F}_O(0) = 0.8$, and $\hat{\theta}(0) = 0.2$, respectively.

When Scheme II is used, we take the following set of design parameters: $\gamma_\theta = 0.4, \gamma_M = 0.2, \gamma_F = 0.4, c_1 = c_2 = 4$. The initials are same with Scheme I.

The simulation results with the proposed two schemes are presented in Figures 2 to 7 using the proposed two schemes, respectively. Figures 2 and 3 show the time history of the displacement x and the trajectory x_m, while figures 4 and 5 show the time history of the control input u. Figure 6 displays the results of the hysteresis behavior. Figure 7 displays the time history of hysteresis behavior F_h and the estimated hysteresis behavior $\hat{F}_H$ with Scheme I.

It is observed that Scheme I is better than Scheme II in improving system performance. This is expected as more apriori knowledge is used. Overall, the simulation results verify our theoretical findings and show the effectiveness of our control schemes.

5 CONCLUSION

In this paper, two backstepping adaptive controllers have been presented for a piezo-positioning mechanism involving hysteretic phenomena. The hysteretic nonlinear behavior is described by the LuGre model. In the first scheme, we take the structure of the hysteresis into account in our controller design, if the parameters of the Lugre model are all known. A nonlinear observer is designed by considering a Lyapunov function. In the second scheme, there is no apriori information required from these parameters and part of the hysteresis effect is treated as a bounded disturbance. It is shown that the proposed controllers can guarantee global uniform ultimate boundedness of signals and achieve tracking to a desired precision. Numerical results show that the designed adaptive control laws work satisfactorily.

REFERENCES

[1] P. Ge and M. Jouaneh, "Tracking control of a piezoceramic actuator," *IEEE Transactions on Control System Technology*, vol. 4, pp. 209–216, 1996.

[2] S. B. Jung and S. W. Kim, "Improvement of scanning accuracy of pzt piezoelectric actuator by feed-forward model-reference control," *Precision Engineering*, vol. 16, pp. 49–55, 1994.

[3] T. E. Pare and J. P. How, "Robust stability and performance analysis of systems with hysteresis nonlinearities," in *Proceedings of the American Control Conference*, 1998, pp. 1904–1908.

[4] G. Tao and P. V. Kokotovic, "Adaptive control of plants with unknown hysteresis," *IEEE Transactions on Automatic Control*, vol. 40, pp. 200–212, 1995.

[5] X. Sun, W. Zhang, and Y. Jin, "Stable adaptive control of backlash nonlinear systems with

bounded disturbance," in *Proceedings of the IEEE Conference on Decision and Control*, 1992, pp. 274–275.

[6] A. Taware, G. Tao, and C. Teolis, "Design and analysis of a hybrid control scheme for sandwich nonsmooth nonlinear systems," *Automatica*, vol. 40, pp. 145–150, 2002.

[7] C. Y. Su, Y. Stepanenko, J. Svoboda, and T. P. Leung, "Robust adaptive control of a class of nonlinear systems with unknown backlash-like hysteresis," *IEEE Transactions on Automatic Control*, vol. 45, no. 12, pp. 2427–2432, 2000.

[8] J. Zhou, C. Wen, and Y. Zhang, "Adaptive backstepping control of a class of uncertain nonlinear systems with unknown backlash-like hysteresis," *IEEE Transactions on Automatic Control*, vol. 49, pp. 1751–1757, 2004.

[9] C. L. Hwang and C. Jan, "A reinforcement discrete neuro-adaptive control for unknown piezoelectric actuator systems with dominant hysteresis," *IEEE Transactions on Neural Networks*, vol. 14, pp. 66–78, 2003.

[10] F.-J. Lin, H.-J. Shieh, and P.-K. Huang, "Adaptive wavelet neural network control with hysteresis estimation for piezo-positioning mechanism," *IEEE Transactions on Neural Networks*, vol. 17, pp. 432–444, 2006.

[11] C. Canudas-de-Wit, H. Olsson, K. J. Astrom, and P. Lischinsky, "A new model for control of systems with friction," *IEEE Transactions on Automatic Control*, vol. 40, pp. 419–425, 1995.

[12] B. Armstrong-Htlouvry, *Control of Machines with Friction*. Boston: MA: Kluwer, 1991.

[13] M. Krstic, I. Kanellakopoulos, and P. V. Kokotovic, *Nonlinear and Adaptive Control Design*. New York: Wiley, 1995.

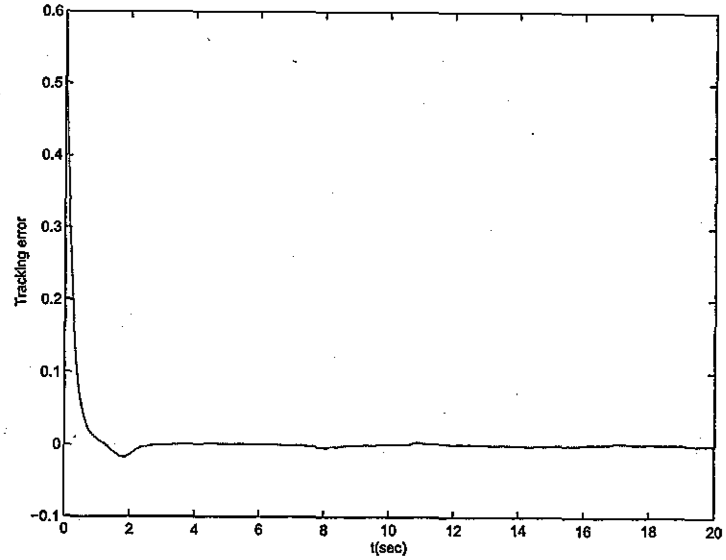

Figure 2: Time history of tracking error $x - x_m$ with Scheme I

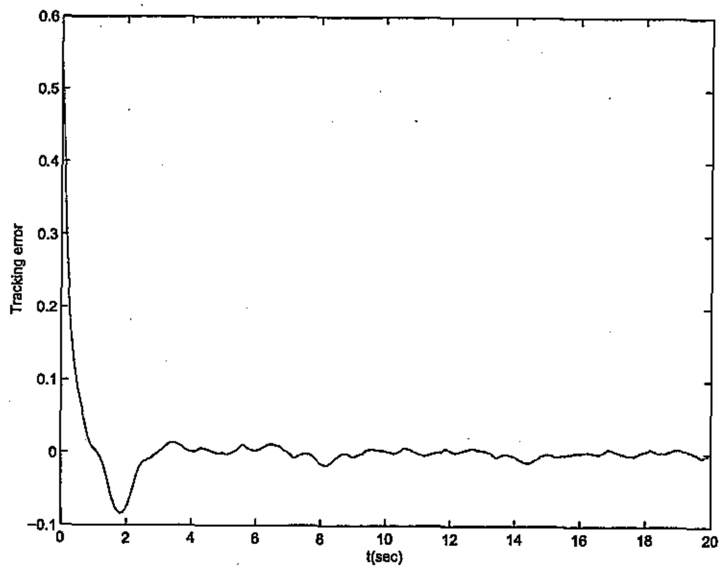

Figure 3: Time history of tracking error $x - x_m$ with Scheme II

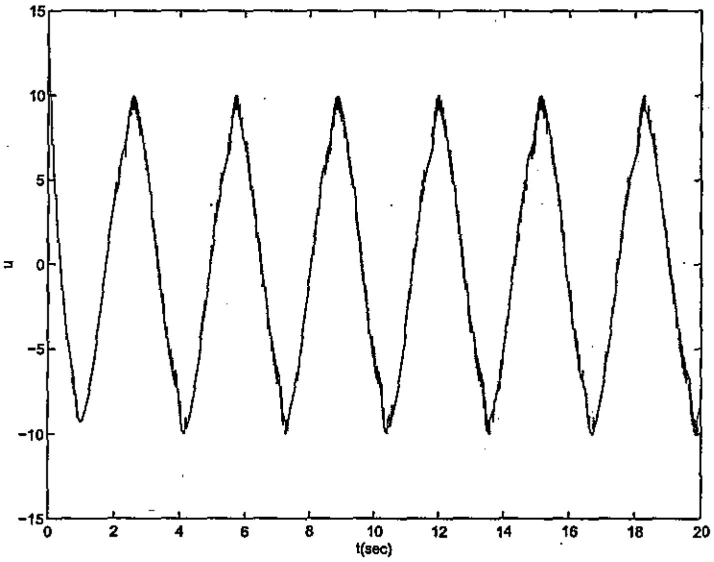

Figure 4: Time history of control input $u(t)$ with Scheme I

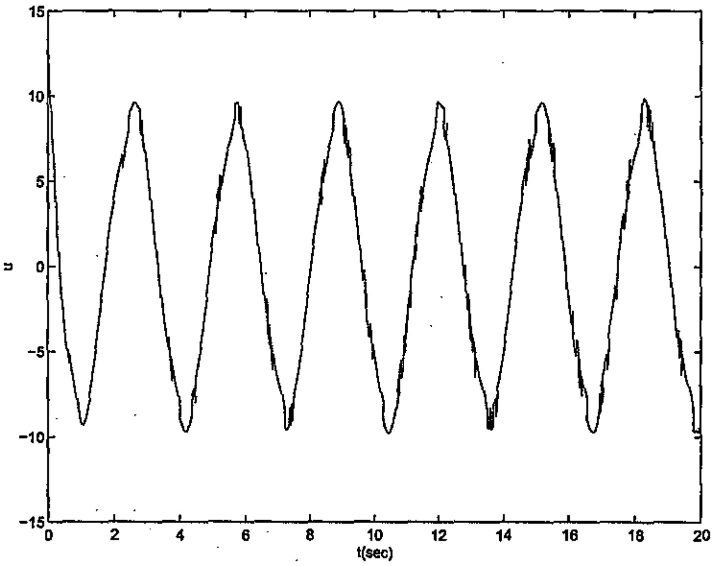

Figure 5: Time history of control input $u(t)$ with Scheme II

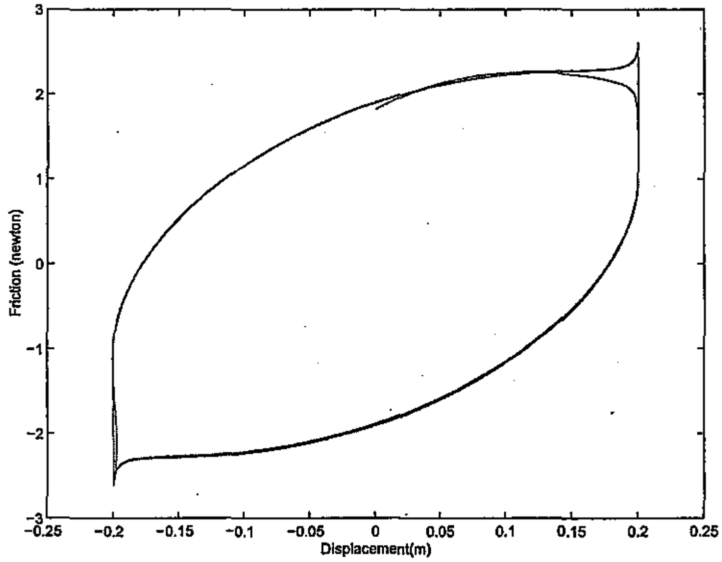

Figure 6: Hysteresis identification

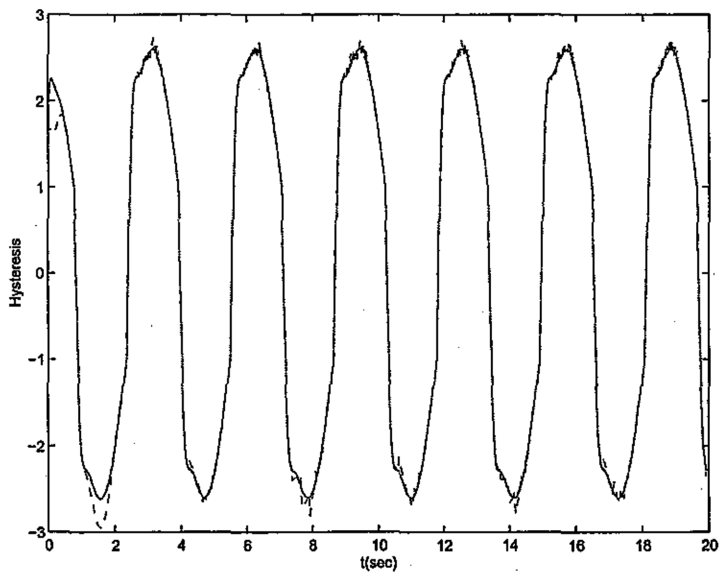

Figure 7: Time history of hysteresis F_H (solid) and $\hat{F}_H$ (dashed) with Scheme I