INVESTIGATING THE EFFECT OF THE ORTHOTROPIC PROPERTY OF PIEZOELECTRIC PVDF

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ABSTRACT

The applications of the piezoelectric Polyvinylidene Fluoride, PVDF, integrated with the beams, plates, and membranes, performing as sensor, actuator or combination have been received considerable attention in the recent years. However, not much work has been reported on the influence of the PVDF's orthotropic behavior, particularly the effect of the orientation of the PVDF film in the host structure, on the performance of the system. In the present study, the effect of the piezoelectric PVDF film orientation on the output voltage, the actuation force, and the dynamic response of the integrated structures has been studied using the finite element method. In the sensory mode, the difference between the output voltages obtained from the biaxial piezoelectric PVDF film and uniaxial one, when the orientation of the film varies from 0 to 90 degree, is investigated. In each case the proportion contributions of the involved piezoelectric coefficients including d_{31}, d_{32} and d_{33} are studied. Alternatively, in the actuation mode, the effect of orthotropic behavior of the actuator on the nodal displacements has been taken into consideration. The influence of the material orthotropic property of the transducer on the free undamped response of the system is also investigated. Moreover an effective Young's modulus and effective Poisson ratio for the uniaxial PVDF film has been introduced using an optimization procedure to minimize the error caused by isotropic assumption of uniaxial PVDF film.

RECHERCHE SUR L'EFFET DE LA PROPRIÉTÉ ORTHOTROPE DU PVDF PIÉZOÉLECTRIQUE

RESUMÉ

Au cours des dernières années, une attention toute particulière a été portée à l'application de polyfluorure de vinylidène piézoélectrique (PVDF) intégré aux poutres, plaques et membranes et agissant comme capteur, actuateur ou connexion. Cependant, il y a très peu de travaux de recherche qui ont été présentés sur l'influence du comportement orthotrope du PVDF sur la performance du système, et plus particulièrement sur l'effet de l'orientation de la pellicule de PVDF dans la structure. La présente étude, qui a été réalisée à l'aide de la méthode des éléments finis, porte sur l'effet de l'orientation de la pellicule de PVDF piézoélectrique sur la tension de sortie, la force motrice et la réponse dynamique des structures intégrées. En mode sensoriel, une étude a été faite sur la différence entre les tensions de sortie obtenues à partir d'une pellicule de PVDF piézoélectrique biaxe et uniaxe lorsque que l'orientation de la pellicule varie entre 0 et 90 degré. Dans chaque cas, les contributions proportionnelles des coefficients piézoélectriques impliqués, incluant d31, d32 et d33, ont également été étudiées. En mode action, l'effet du comportement orthotrope de l'actuateur sur les déplacements nodaux a été pris en considération. L'influence de la propriété orthotrope du matériau du transducteur sur la réponse non amortie du système a aussi été examinée. De plus, un module de Young et un coefficient de Poisson effectifs pour la pellicule de PVDF uniaxe ont été introduits en utilisant une méthode d'optimisation afin de minimiser les erreurs causées par l'hypothèse d'une pellicule de PVDF uniaxe isotrope.

1. INTRODUCTION

Since 1970's, when the enhanced piezoelectricity of PolyVinylidene DiFluoride, PVDF, was achieved [1], due to its combined merits of high elasticity, high processing capacity, and high piezoelectricity, the investigation on its properties and applications has been increasingly grown.

The successful applications of PVDF in beams, plates, and membranes for the vibration control [2, 3], damage detection or structure health monitoring [4, 5], shape and motion control [6], force and pulse sensing applications [7], among many others are increasingly reported in the literature. However, the orthotropic properties of PVDF and their influence on the performance of the constructed systems are often overlooked. The discrepancy observed between few reported values for the material properties of the orthotropic PVDF, which has root in dissimilar mechanical and electrical production process, is one of the obstacles in considering the anisotropic properties of the piezoelectric PVDF in the studies. Various electrical and mechanical properties for the uniaxial and biaxial PVDF film are reported by different manufacturers are given in Table 1 [8-10].

Table 1. PVDF properties reported by manufacturers. Piezoelectric and Elastic coefficients are given in pC/N and GPa respectively. The d_{3h}, represents the hydrostatic piezoelectric coefficient.

	Uniaxial PVDF							Biaxial PVDF		
	d_{31}	d_{32}	d_{33}	d_{3h}	E_{11}	E_{22}	E_{33}	$d_{31}=d_{32}$	d_{33}	$E_{11}=E_{22}$
Piezoflex	14	2	-34	-18	2.5	2.1	0.9	-	-	-
Goodfellow	18-20	2	-20	-6	1.8-2.7	1.7-2.7	-	8	15-16	2
Piezotech	18	3	-20	-	-	-	-	7	-24	-

Discrepancy between the reported piezoelectric properties is evident from Table 1. Furthermore, the material properties of PVDF have been measured using different techniques and used by different researchers [11, 12]. Nevertheless, among the published data a satisfactory consistency has not yet been observed. In order to implement the numerical simulations, in the present investigation the data measured and reported in reference [12] is adopted.

The piezoelectric PVDF, which is commonly prepared in the form of thin films, is a semi crystalline polymer with a crystal volume fraction of about 50-60 % after melt extrusion. Having quenched at a temperature below 150 °C, PVDF crystallizes in phase α (or form II). As a result of a mechanical stretching normally up to four times of initial length [13] at about 60 °C, the α-phase film undergoes transition to the β-phase (or form I), which exhibits highly piezoelectric sensitivity [14]. Among four stable crystal structures at room temperature, a PVDF film in β phase, polarized at elevated temperatures, shows very strong piezoelectric and pyroelectric properties. The mechanical stretch tends to align the 1-axis of the crystals parallel to the stretching direction, giving the crystals the desired orientation. Then an induced electric field above 100 V/μm under controlled temperature aligns the dipoles [15]. It seems that the field is more effective in aligning the dipoles during the mechanical processing stage in which the crystals are reformed into oriented lamellae [16].

Although the above PVDF manufacturing method is popular, it is not the only way of preparing this piezoelectric polymer. Some other techniques such as spin coating, and types of deposition methods are also investigated [17, 18], particularly in conjugation with the Micro-Electro Mechanical Systems, MEMS technology. However, due to the complexity of the in-house PVDF depositions, the pre-manufactured PVDF in different applications including MEMS devices are frequently reported [19-21].

In many applications the anisotropic characteristics of the PVDF film might play an important role or at least affect the accuracy of the results, therefore, it is necessary to investigate the influence of the anisotropic behavior of PVDF on the voltage output in the sensing mode, on the deflection in the actuation mode, and also on the frequency response of the system.

As mentioned earlier, the piezoelectric PVDF is mostly manufactured in the thin film form with the thickness between 9-110 μm. Due to the very small thickness of the PVDF film, in many applications the mechanical material properties in the thickness direction can be ignored. However, ignoring the in-plane anisotropic properties of the PVDF film may lead to erroneous results. Although the transduction properties of PVDF in the principle direction are dominant, nevertheless, in many applications using beams, plates, and membranes the orientation of the force with respect to the affixed PVDF film may vary. Therefore, a comprehensive knowledge of the performance of the piezoelectric system, when the applied force is not co-axial with the material principle axis, is required. In other words, in cases that both principle and transverse directions of PVDF film might contribute in the output, the in-plane anisotropic properties of the PVDF film cannot be ignored.

In this research study, the anisotropic behavior of the PVDF film and particularly the effect of the orientation of the PVDF film with respect to the problem's coordinate system are investigated. For the numerical simulations in this study, to avoid unnecessary complexity and concentrating primarily on the PVDF film behavior, a cantilever beam equipped with the piezoelectric PVDF film has been selected. To compare the influence of the mechanical and electrical anisotropic properties, both biaxial and uniaxial piezoelectric PVDF films have been taken into account.

In the following, first general characteristics and governing constitutive equations of the PVDF films are briefly described. This is followed by an abstract of the finite element formulation. Finally illustrative numerical examples are presented to demonstrate the influence of the PVDF orientation on both the sensing and actuation modes as well as on the dynamic response of the system.

2. THE CHARACTERIUSTICS OF THE PIEZOELECTRIC PVDF

The anisotropic behavior of the uniaxial PVDF film, indeed originates from the aforementioned process history, as the large degree of microscopic order resulting from the orientation reduces the in plane 1-axis randomness and make the macroscopic piezoelectric behavior more consistent with that of the *mm2* point group crystal symmetry.

Of the thirty-two crystal classes, PVDF with *mm2* symmetry is known to exhibit direct piezoelectricity with the permittivity, g (a 3×3 matrix with nonzero component on diagonal), piezoelectric, d (a 3×6 matrix with nonzero $d_{15}, d_{24}, d_{31}, d_{32}, d_{33}$), and the stiffness matrix, s (given for orthotropic material) [22]. Due to the production difficulties associated with the orientation and polarization of the PVDF, it is commercially produced substantially in thin films. The uniaxial film is the result of mechanical drawing of the film in one direction. Alternatively, stretching the film in both in-plane axes yields the biaxial PVDF film with lower but laterally isotropic piezoelectric properties. Due to the remarkable difference between d_{31}, the piezoelectric coefficient in the 1-axis direction, and d_{32}, the coefficient in the 2-axis direction, where $d_{32} \approx d_{31}/10$, the uniaxial PVDF film is used in length extensional mode (1-axis) in over 90% of the practical applications [23].

This study aims at the elaborating the effects of anisotropic behavior of both the uniaxial and biaxial PVDF film on the output voltage in the sensory mode, on the amount of the deflection in the actuation mode and the dynamic response of the system. The behavior of the PVDF film can be attributed to the mechanical orthotropic characteristic, which affects the stress-strain relation as well as non-equality of d_{31} and d_{32} in the electro-mechanical relations. The response of the sensation and actuation modes varies as a function of deviation angle θ, the angle between the material coordinate system in which all the material properties are defined and the global coordinate system.

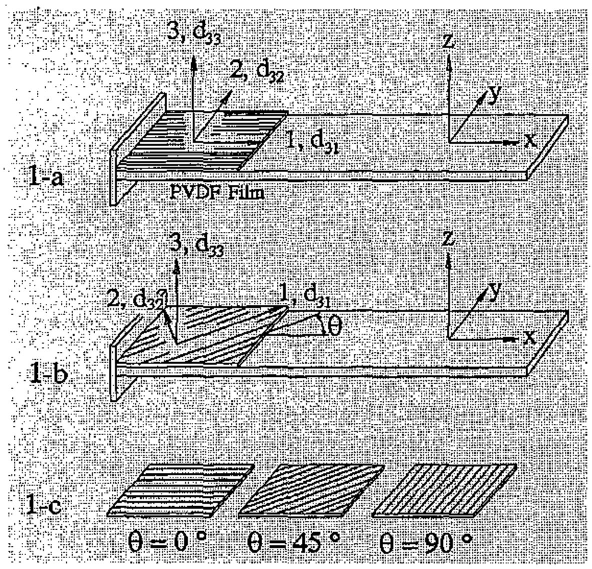

Figure 1. The PVDF material coordinate system (1, 2, 3) versus the global coordinate system (x, y, z).

Figure 1-a, shows the case where the PVDF film is adhered to the beam in such a way that the material coordinate system and the global coordinate system have the same orientations, while Figure 1-b, represents the case that the PVDF 1-axis is rotated with the amount of θ with respect to the global coordinate system (*x, y, z*). As it is emphasized in Figure 1-c, the edges of the piezoelectric patch remain parallel with the global axes. However the drawn direction, i.e. 1-axis rotates with respect to the global axis.

When the developed stress due to the applied load is in the x-axis direction, the best sensing and actuation response will be achieved from a piezoelectric, which its 1-axis is oriented in the global x direction. However, in some cases the orientation of the applied load might result in a two-dimensional stress profile and thus finding the best orientation to attach the piezoelectric film is important. The rotation between the material property and the global coordinate system can also occur inadvertently during the manufacturing phase of a sensor. Obviously by simply considering PVDF as an isotropic material, no alterations in the responses in terms of deviation angle would be observed.

3. THE CONSTITUTIVE EQUATIONS

The linear constitutive equations for the orthotropic piezoelectric crystals in the principle material coordinates assuming a constant temperature (i.e. $\Delta T \approx 0$) are given by Reddy [24]:

$$\{\sigma\} = [\overline{C}]\{\overline{\varepsilon}\} - [\overline{e}]^T\{E\} \tag{1}$$

$$\{D\} = [\overline{e}]\{\overline{\varepsilon}\} - [\overline{g}]\{E\} \tag{2}$$

where $[\overline{C}]_{6\times6}$ is the stiffness matrix, $[\overline{e}]_{6\times3}$ the piezoelectric coupling matrix, $\{\overline{\varepsilon}\}_{6\times1}$ the strain field and $[\overline{g}]_{3\times3}$ the permittivity matrix. The bar sign indicates the transformed quantities from material axis to global axis which is obtained using $[\overline{C}] = [T_\sigma]^{-1}[C][T_\varepsilon]$, in which $[T_\sigma]$ and $[T_\varepsilon]$ represent the transformation matrices corresponding to stress vector, σ and strain vector ε. $\{\sigma\}_{6\times1}$ represents the stress vector, $\{D\}_{3\times1}$ the electric displacement and $\{E\}_{3\times1}$ the electric field. The piezoelectric coupling matrices $[\overline{e}]$, (C/m^2), and $[\overline{d}]$,(C/N), are related by $[\overline{e}] = [\overline{C}][d]$. Equation (1) describes the inverse piezoelectric effect while Equation (2) describes the direct piezoelectric effect. It is noted that components of the stiffness matrix $[\overline{C}]_{6\times6}$ are functions of deviation angle θ. For instance the first and last components of $[\overline{C}]_{6\times6}$ are described as:

$$\overline{C}_{11} = C_{11}\cos^4\theta - 4C_{16}\cos^3\theta\sin\theta + 2(C_{12} + 2C_{66})\cos^2\theta\sin^2\theta - 4C_{26}\cos\theta\sin^3\theta + C_{22}\sin^4\theta$$

$$\vdots$$

$$\begin{aligned}\overline{C}_{66} &= 2(C_{16} - C_{26})\cos^3\theta \ldots \sin\theta + (C_{11} + C_{22} - 2C_{12} - 2C_{66})\cos^2\theta\ \sin^2\theta \\ &+ 2(C_{26} - C_{16})\cos\theta\ \sin^3\theta + C_{66}\left(\cos^4\theta + \sin^4\theta\right)\end{aligned}$$

Due to the thinness of the films, one can avoid the complexity of a 3D analysis and still preserve the reasonable accuracy of the obtained results from the reduced in-plane formulation. Equations (1) and (2) can be derived for 2-D plate element simply by neglecting stresses through the thickness and compute the resultant components for stiffness matrix. Similarly a beam element can be obtained by considering only axial and transverse shear stress in Equation (1) and considering electrical displacement in thickness direction in Equation (2). For details regarding the piezoelectric plate and beam elements one may consult the book written by Reddy [24].

4. FINITE ELEMENT FORMULATION

The mechanical response of the piezoelectric material with the coupled mechanical and electrical properties is represented by the stress equation of motion [25]:

$$\sigma_{ij,i} + f_j = \rho \ddot{u}_j \tag{3}$$

while the electrical response is described by the electrostatics equation for the conservation of the electric flux,

$$D_{i,i} = 0 \tag{4}$$

where ρ is the density, f is the body force, superscript dots represents time derivatives and subscript commas represents differentiation with respect to space. Neglecting the body force and using divergence theorem, the combined electro-mechanical response of the piezoelectric material is determined as:

$$\int_V \left(\rho \ddot{u}_i \delta u_i + \sigma_{ij} \delta \varepsilon_{ij} - D_i \delta E_i\right) dV = \int_{\Gamma_t} t_i \delta u_i d\Gamma + \int_{\Gamma_p} q \delta \psi d\Gamma \tag{5}$$

where u_i are the generalized displacements, t_i are the surface tractions applied on the surface Γ_t and q is the electrical charge applied on the surface Γ_p of the piezoelectric material and V represents the whole volume including the piezoelectric and substrate materials.
As mentioned before, for the sake of simplicity, the finite element formulation is expressed for a beam model. It should be noted that the Equation (5) is a general equation and can be applied to obtain any kind of elements. The displacement fields for a Kirchhoff beam element are given by:

$$u(x,z,t) = u_o(x,t) - z\frac{\partial w_o}{\partial x}, \quad w(x,z,t) = w_o(x,t) \tag{6}$$

where $u_o(x)$ and $w_o(x)$ denote the displacements of the reference point at x, y and z axis, respectively. The finite element matrix equation of the piezoelectric beam element can be obtained by implementing Equation (6) into Equation (5) and collecting the coefficients as:

$$\begin{bmatrix} M & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \{\ddot{U}\} \\ \{\ddot{\Phi}\} \end{Bmatrix} + \begin{bmatrix} [K_{UU}] & [K_{U\Phi}] \\ [K_{\Phi U}] & [K_{\Phi\Phi}] \end{bmatrix} \begin{Bmatrix} \{U\} \\ \{\Phi\} \end{Bmatrix} = \begin{Bmatrix} \{F(t)\} \\ \{Q(t)\} \end{Bmatrix} \tag{7}$$

where $\{U\} = \{\{u\} \ \ \{w\}\}^T$ and the submatrices $[K_{UU}]$, $[K_{\Phi U}]$, $[K_{\Phi\Phi}]$ indicate the elastic, piezoelectric, and the permittivity matrices; $[M]$ is the mass matrix; $\{F\}$ are the applied forces; and $\{Q\}$ are the applied voltages. From Equations (1), (2) and (5), one may realize that the submatrices $[K_{UU}]$, $[K_{\Phi U}]$ and $[K_{\Phi\Phi}]$ are all functions of deviation angle θ. Equation (7) can be partitioned into the sensing and actuating components as follows:

$$\begin{bmatrix} M & 0 \\ 0 & 0 \end{bmatrix}\begin{Bmatrix} \{\ddot{U}\} \\ \{\ddot{\Phi}^s\} \end{Bmatrix} + \begin{bmatrix} [K_{UU}] & [K^{ss}{}_{U\Phi}] \\ [K^{ss}{}_{\Phi U}] & [K^{ss}{}_{\Phi\Phi}] \end{bmatrix}\begin{Bmatrix} \{U\} \\ \{\Phi^s\} \end{Bmatrix} = \begin{Bmatrix} \{F(t)\} - [K^{sa}_{U\Phi}]\{\Phi^a\} \\ \{Q^s(t)\} - [K^{sa}_{\Phi\Phi}]\{\Phi^a\} \end{Bmatrix} \tag{8}$$

where the superscripts s and a indicate the partitioned submatrices in accordance with the sensing and actuating modes, respectively. The left hand side indicates the unknown displacement and voltage at the sensor and the right hand side represents the known applied mechanical load and voltage on the actuator.

5. NUMERICAL EXAMPLES

In order to clarify the anisotropic behavior of the PVDF film governed by the above-mentioned equations, in this section, the effects of orthotropic properties on a variety of the output responses have been investigated. An Aluminum cantilever beam with surface bounded sensor/actuator, as shown in Figure 1, is considered for the numerical simulations. In all the following numerical results, the length and width of the beam are considered to be 0.245 m and 0.0245 m, respectively. The thickness of the beam for the sensing mode is considered to be 2 mm, while in the actuation mode it is varied to show the effect of the thickness ratio on the beam response. Both the uniaxial and biaxial 25-µm PVDF films are examined in the sensing as well as the actuation mode.

The stiffness coefficients for unidirectional PVDF are
$\{C_{11}, C_{12}, C_{13}, C_{22}, C_{23}, C_{33}, C_{44}, C_{55}, C_{66}\} = \{3.7, 1.47, 1.23, 3.2, 1.0, 1.51, 0.55, 0.59, 0.7\}$ GPa
and the piezoelectric coefficients

$$\{d_{15}, d_{24}, d_{31}, d_{32}, d_{33}\} = \{-27, -23, 18, 2, -20\} \times 10^{-12} \ C/m^2.$$

The material properties of Aluminum are given as: E=70 GPa, ν=0.30, ρ=2712 kg/m^3.

The beam is meshed with 20 equal-length elements and PVDF film is bonded at the first three elements. In the sensing mode, a downward F=10 N point load is exerted at the tip while in the actuation mode the applied voltage to the piezoelectric patch is 240 V and load is removed.

5.1. Sensing Mode: The Effect of Orientation on the Output Voltage

In the following simulations, due to the firm adherence of the piezoelectric patch, it is assumed that the strain in the PVDF film is the same as the strain developed in the beam due to the applied load. To show the difference between the isotropic (i.e. Biaxial PVDF) and the orthotropic (Uniaxial PVDF), simulations are carried out for both materials. In the following tests, as illustrated in Figure 1, the *deviation angle*, the angle between the 1-axis in material coordinate system and the x-axis in the global coordinate system, is varied from 0 to 90°.

5.1.1. Biaxial PVDF

The piezoelectric coefficients of the biaxial PVDF are equal in the 1-axis and the transverse direction, 2-axis. In other words, the ratio of d_{31}/d_{32}, for the biaxial PVDF film is unit. This equality of the piezoelectric coefficients is in compliance with the isotropic mechanical property of the biaxial PVDF film. Figure 2 shows the result for the output voltage versus orientation angle of the biaxial PVDF film. The output voltage is normalized with respect to the total output voltage, V_{total}. The variations of the contributing components in the total output voltage, namely V_{d31} and V_{d32} which are due to the piezoelectric coefficients d_{31} and d_{32} respectively, are clearly shown in this figure. It is noted that in order to compute the voltage component associated with each piezoelectric coefficient, the other coefficients are set in turn to be zero. It can be realized that in the biaxial PVDF film, the total output voltage is constant, hence as expected independent of the deviation angle.

The symmetry observed in Figure 2 is due to the equality of the d_{31} and d_{32} in the two perpendicular directions along with the isotropic material properties of the biaxial PVDF film. It is seen that in the zero degree deviation, the contribution of d_{32} in the total output is about 7%.

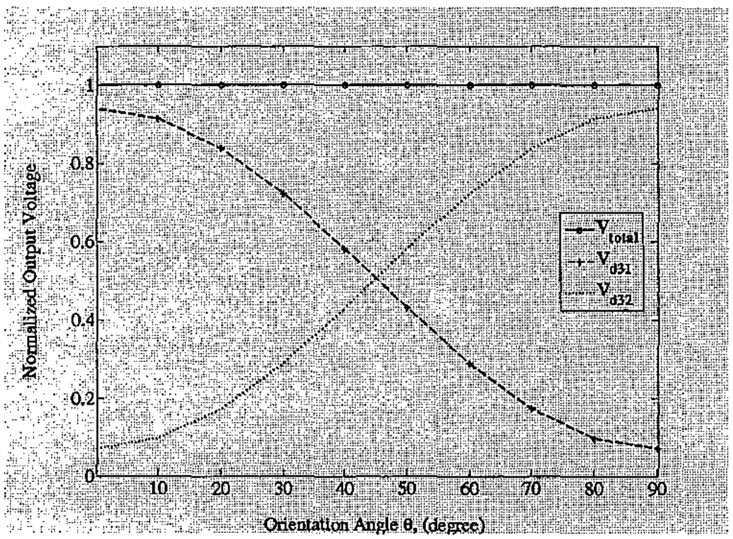

Figure 2. The normalized output voltage of the biaxial piezoelectric PVDF film.

It should be noted that the contribution of the d_{33} in total output voltage depends on the boundary conditions and when there is no constraint in the Z-axis, no stress will be developed in this direction, hence no contribution due to d_{33} in the total output will be seen.

5.1.2. Uniaxial PVDF

As mentioned earlier, due to the nature of the manufacturing process the piezoelectric PVDF film undergoes, the mechanical anisotropic behavior associated with the difference between the d_{31} and d_{32} is resulted. Thus, the behavior of the uniaxial PVDF film under the similar condition is different from that of the biaxial PVDF film. Figure 3 shows the normalized total output voltages of the uniaxial PVDF subjected to a set of forces, F, 2F and 3F in terms of the deviation angles.

The results show that for each force state, the variation of the voltage due to the orientation angle is nonlinear and significant (the voltage output at $\theta = 90°$ is dropped to 4% of its initial value at zero degree). Although the piezoelectric output voltage decreases nonlinearly with increasing the deviation angle, for each specific angle the output voltage increases linearly with increasing the applied force.

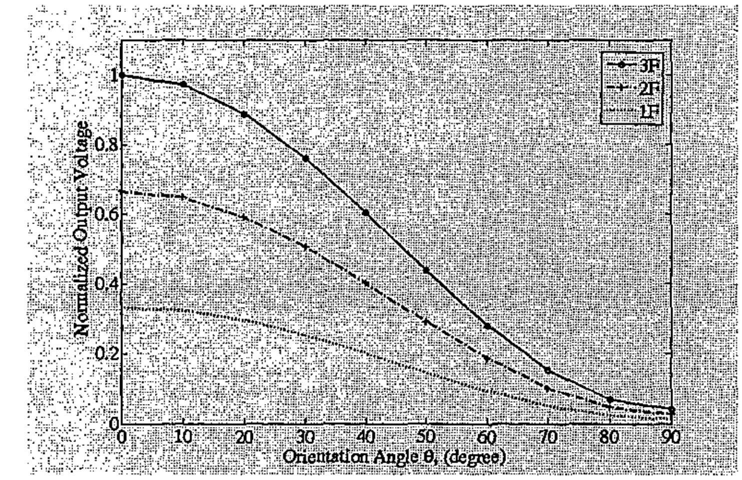

Figure 3. The normalized output voltage of the uniaxial piezoelectric PVDF film for different applied loads

Similar to the biaxial case, the contribution of each piezoelectric coefficient in the total response when the deviation angle varies between $\theta = 0°$ and $\theta = 90°$ is investigated and the results are shown in Figure 4. It can be realized that the total output voltage is mainly comprised of the voltage associated with d_{31}. For instance, at $\theta = 0°$, just 1.5% of the total voltage is due to d_{32}. The significant reduction of d_{32} contribution in the output voltage can be attributed to the combined effect of both mechanical anisotropic properties of the uniaxial PVDF film, as well as the significant difference between d_{31} and d_{32} values in the uniaxial PVDF. The lower Young's modulus in the transverse direction yields the lower stress, hence the lower charge output for the same strain value compared with the biaxial film.

It is seen that the maximum difference between the total output voltage and the voltage induced by d_{31} is at $\theta = 90°$ and is equal to 4.5 %. Similar to the biaxial case the voltage component resulted from d_{33} coefficient is close to zero. In many applications in which the uniaxial PVDF film experiences merely a uniaxial tension, it is preferred to adhere the film in such a way that its material coordinate system coincides with the global coordinate system, where the deviation angle is zero.

Nevertheless, practically the angle eventually resulted after fabrication might not be the one that was initially intended, the zero degree. Therefore, the influence of a probable deviation from the original orientation, $\theta = 0°$ on the output voltage is required to be studied. The simulation results for a deviation of $\pm$ 10° are shown in Figure 5. A maximum 3% drop in output voltage is calculated when the PVDF film is at $\theta = 10°$ with respect to the global coordinate system. This result is important and helpful when PVDF is used as a precise sensor transducer. Knowing the amplitude of the voltage for the zero degree, one is able to calculate the error in the orientation of the bounded piezoelectric film by measuring the reduction in the output voltage.

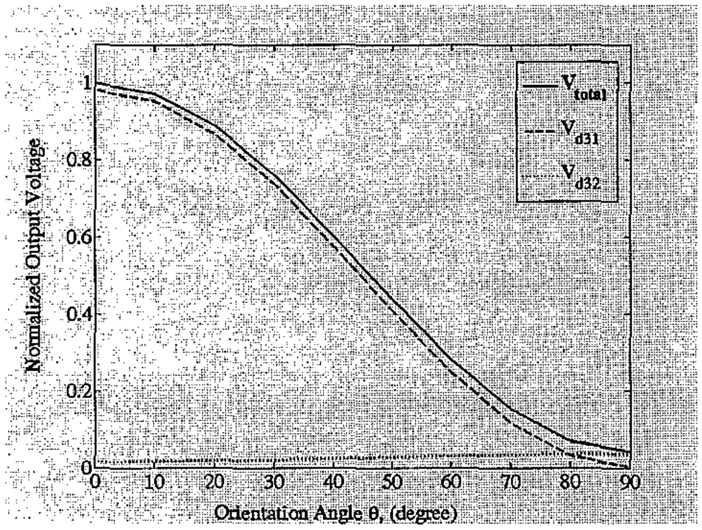

Figure 4. The normalized output voltage of the uniaxial piezoelectric PVDF film versus deviation angle.

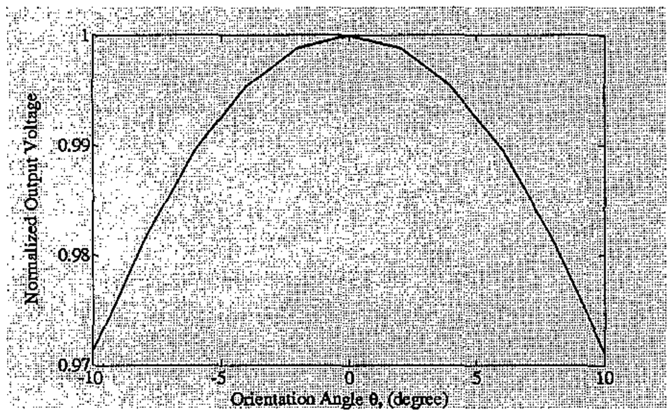

Figure 5. Variation of the normalized output voltage of uniaxial PVDF file versus ±10 degree deviation angle

5.2. <u>Actuation Mode: The Effect of Orientation on the Deflection</u>

In order to study the effect of orientation of the attached orthotropic PVDF actuator on the tip deflection, a cantilever beam described in Section 5 is considered. In the actuation mode, similar to the sensing mode, the tip deflection is constant for all deviation angles when biaxial PVDF is used. However for the uniaxial PVDF, the tip deflection is a function of the deviation angle. On the other hand, the deflection values also depend on the geometries of the PVDF and the beam. The variations of tip deflection versus rotation angle θ = 0 to 90 ° and for the different thickness ratios H_b/H_P (H_b is the thickness of the beam and H_p is the thickness of the PVDF) are shown in Figure 6. It should be noted that $H_b/H_P = 20$ curve is used as the reference normalized curve. It is observed that the maximum effect exists for the thinner beams (or the thicker PVDF films). For instance, when $H_b/H_P = 20$, the deflection at 90° drops to 12 % of its initial value at 0°. When the thickness of the PVDF film is much lower than the thickness of the beam, the sensitivity to the deviation angle reduces remarkably. For example for $H_b/H_p = 40$, the deflection at 90° is 10% of the 0° value. Therefore for some applications such as MEMS, where the thickness of the PVDF film and the beam could be of the same order of magnitude, the deviation angle plays an important role. Similar performance in the sensing mode and in the actuation mode can be observed by comparing the graphs in Figures 3 and 6.

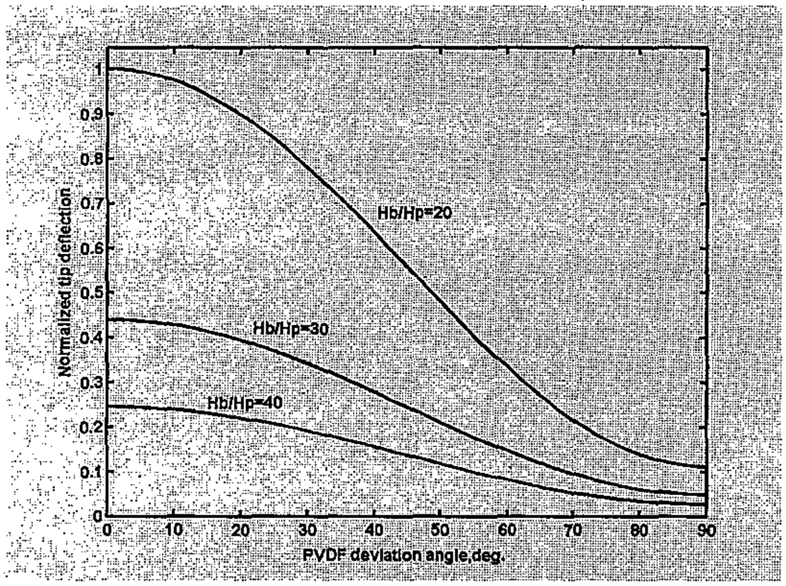

Figure 6. The normalized tip deflection of the beam versus the deviation angle of the uniaxial PVDF patch.

5.3. Effect of Orientation on the Dynamic Response

The effect of the uniaxial PVDF film orientation on the free undamped vibration response, using the similar cantilever beam and PVDF film explained in Section 5, is investigated. For free vibration response the equation (7) is reduced to

$$[M]\{\ddot{U}\} + \left([K_{UU}] - [K^{ss}_{\Phi U}][K^{ss}_{\Phi\Phi}]^{-1}[K^{ss}_{\Phi U}]\right)\{U\} = 0 \tag{9}$$

Considering $U = U_O e^{-i\omega t}$ and substituting in Equation (9) leads to the following eigenvalue problem:

$$\left([\hat{K}] - \lambda[M]\right)U_o = 0 \tag{10}$$

where $[\hat{K}] = [K_{UU}] - [K^{ss}_{\Phi U}][K^{ss}_{\Phi\Phi}]^{-1}[K^{ss}_{\Phi U}]$ and $\lambda = \omega^2$. As mentioned earlier the generalized stiffness $[\hat{K}]$ is a function of deviation angle θ. Equation (10) has been solved for angles $\theta = 0^o - 90^o$ and the variation of the normalized fundamental frequency (ratio of fundamental frequency at deviation angle to fundamental frequency at deviation angle of zero) with respect to ply orientation angle for the PVDF layer is demonstrated in Figure 7.

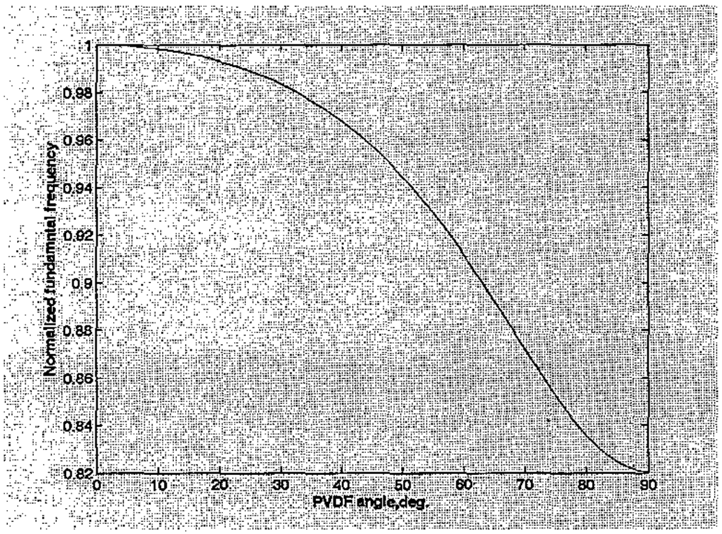

Figure 7. Normalized fundamental frequency of the cantilever beam against the deviation angle of adhered uniaxial PVDF film.

It can be realized that the fundamental frequency drops significantly for ply orientation angles greater than 10^{o}.

5.4. The Optimal Contribution of the Mechanical Anisotropic Coefficients

It is frequently observed that in many studies the uniaxial PVDF is considered as an isotropic material [10, 14, 26, 27], though the exact values of piezoelectric coefficients are usually used. Here an investigation has been done to better understand the influence of each factor namely anisotropic mechanical property, and anisotropic electrical property. It is found that the isotropic assumption for the uniaxial PVDF can produce errors up to 20%. This error however depends on the selected equivalent Young's modulus and the Poisson ratio. For a planar case there are two Young's modulus, E_1 and E_2 and two Poisson ratios ν_{12} and ν_{21}. To model the uniaxial PVDF film as an isotropic material, one should use the equivalent isotropic Young's modulus, E_{eq} along with the equivalent Poisson ratio ν_{eq}. In this study, optimal E_{eq} and ν_{eq} are found using optimization technique. Using these optimal values, the isotropic assumption generates minimum error. Especially for the applications in which the deviation angle might encounter, knowing the optimum values of the E_{eq} and ν_{eq} for the interval $\theta = 0$ -90 is helpful.

Here, the equivalent Young' modulus E_{eq} has been considered as a linear combination of E_1 and E_2 as the following expressions:

$$E_{eq} = \alpha E_1 + \beta E_2 \tag{9}$$

Alternatively for the equivalent Poisson ratio, the following relationship is used.

$$\nu_{eq} = \gamma \nu_{12} + \eta \nu_{21} \tag{10}$$

The optimization problem is to determine the values of coefficients $\alpha, \beta, \gamma,$ and η (design variables) in order to minimize the error between the deflection obtained based on the set of E_1, E_2, ν_{12} and ν_{21} and the deflection calculated based on the equivalent E_{eq} and ν_{eq} (objective function). Logically the Equivalent Young's modulus must be less than E_1 and greater than E_2. Thus the following constraint relationships for the Young's modulus and the Poisson ratios should be satisfied.

$$E_2 \langle E_{eq} \langle E_1, \quad \nu_2 \langle \nu_{eq} \langle \nu_1 \tag{11}$$

Therefore, the design variables $\alpha, \beta, \gamma, \eta$ are bounded in the interval [0, 1].

Since the deflection of cantilever beam is nonlinear in nature, the above optimization problem is a nonlinear constraint problem. An optimization algorithm based on the finite element formulation (analysis module) and the Sequential Quadratic Programming (SQP-optimization module [28]) has been developed to find the optimal values of coefficients $\alpha, \beta, \gamma,$ and η.

The cantilever beam with surface bounded piezoelectric patches described in Section 5 is considered for numerical illustration. The optimum values for $\alpha, \beta, \gamma, \eta$ are determined and the non-dimensional equivalent values for E_{eq} and ν_{eq} are presented in Table 2. Close observation of these results reveals that, to minimize the error for all angles, one may consider 95% of E_1 and 5% of E_2 and also 72% of ν_{12} and 28% of ν_{21}.

Table 2. The equivalent material properties for different deviation angles.

	PVDF orientation angle, Θ						
	0	15	30	45	60	75	90
α	0.8475	0.8495	0.8502	0.8499	0.8479	0.8482	0.8417
β	0.1524	0.1505	0.1498	0.1500	0.1527	0.1517	0.1518
γ	0.00094	0.0000	0.00066	0.00064	0.00097	0.00078	0.02005
η	0.99905	1.0000	0.99933	0.99936	0.99903	0.99921	0.97995
E_{eq}/E_1	0.95862	0.95923	0.95943	0.95927	0.95924	0.95881	0.95238
ν_{eq}/ν_{12}	0.72574	0.72549	0.72566	0.72566	0.72575	0.72569	0.73099
$\text{Error} \times 10^{-7}$	0.37	0.93	9.50	0.50	9.40	10.0	5.34

6. CONCLUSIONS

The influence of the sensor/actuator orientation on the system performance including output voltage in sensory mode, nodal displacements in the actuation mode and natural frequency has been investigated. Both biaxial and uniaxial piezoelectric PVDF films have been considered in this investigation when the deviation angle changes between zero to 90 degree. The outcome of this study indicates the same descending trend in the response amplitude for all three sets of simulations, namely, sensation, actuation and fundamental natural frequency of the structure.

In the sensory mode the contributions of piezoelectric coefficients including d_{31}, d_{32} and d_{33} have been studied. It is observed that the output of the biaxial PVDF film is independent on the deviation angle, hence is appropriate for the applications in which the constant output is required regardless of the force orientation. Conversely the output of the uniaxial PVDF film is highly dependent on the orientation angle. Having characterized the relationship between the deviation angle and the output response, one can compensate any attenuation due to probable misalignment. Furthermore this can be potentially helpful to find the orientation of the applied load in the force-position applications. To avoid computational complexity of the orthotropic materials, without sacrificing the accuracy of the problem, an investigation performed in order to find the optimum equivalent isotropic properties. These equivalent properties (E_{eq} and ν_{eq}) minimize the output error for the whole range of deviation angles. The equivalent material properties have been determined through a formal optimization procedure. It has been observed that considering 95% of axial and 5% of transverse material properties can provide results similar to orthotropic one with negligible error. These data potentially could be useful when uniaxial PVDF film is used and the orientation of the force respect to the film axes might vary, such as PVDF film attached to a membrane.

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