NEURAL MODELING AND CONTROL OF DYNAMIC SYSTEMS WITH HYSTERESIS

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ABSTRACT

A hysteretic operator is proposed to set up an expanded input space so as to transform the multi-valued mapping of hysteresis to a one-to-one mapping so that the neural networks can be applied to model of the behavior of hysteresis. Based on the proposed neural modeling strategy for hysteresis, a pseudo control scheme is developed to handle the control of nonlinear dynamic systems with hysteresis. A neural estimator is constructed to predict the system residual so that it avoids constructing the inverse model of hysteresis. Thus, the control strategy can be used for the case where the output of hysteresis is unmeasurable directly. Then, the corresponding adaptive control strategy is presented. The application of the novel modeling approach to hysteresis in a piezoelectric actuator is illustrated. Then a numerical example of using the proposed control strategy for a nonlinear system with hysteresis is presented.

MODELISATION A L'AIDE DE RESEAUX DE NEURONES ET COMMANDE DE SYSTEMES DYNAMIQUES AVEC HYSTERESIS

RESUME

Cet article propose un opérateur basé sur les réseaux de neurones pour la modélisation de l'hystérésis. Une approche dite de pseudo commande adaptative est proposée pour les systèmes non linéaires avec hystérésis, tels les actionneurs piézoélectriques. La méthode basée sur les réseaux de neurones permet d'éviter l'inversion du modèle de l'hystérésis comme c'est souvent le cas avec d'autres techniques de commande. La pseudo commande permet donc de contrôler les systèmes pour lesquels la sortie de l'hystérésis est non mesurable. Des simulations numériques illustrent l'efficacité de la méthode de commande.

1. INTRODUCTION

Hysteresis exists in smart materials such as piezoceramic materials and shape memory alloys that are used as smart actuators in ultra-precision manufacturing systems. It may cause undesirable inaccuracy or oscillations [1]. Therefore, the compensation for the effect of hysteresis inherent in piezoelectric actuator is very important for high precision resolutions in manufacturing process. As hysteresis is a non-smooth non-linearity with multi-valued mapping, it is a challenge to model the behavior of hysteresis so that the corresponding control strategy can be developed.

There have been several schemes proposed to describe the hysteresis. The Preisach model [2, 3] is one of the most popular methods since it can describe the basic features of the hysteresis phenomena in an elegant way. Although the Preisach model is useful to describe the behavior of hysteresis, it is not convenient to determine the values of the distribution functions of the model. [4] also gave some other models for hysteresis. However, those models were first principles based models so that they were not convenient for implementation in real time control.

Recently, neural networks have been proved to be able to model the behavior of hysteresis. [5] investigated the rate-independent memory property of hysteresis. Based on the analysis of mutilayer feedforward recurrent and reinforcement learning neural networks, it was found that neural networks with only computational nodes and links could not describe hysteresis simulators. Therefore, a propulsive neural unit is developed to construct hysteresis memory[5]. Several propulsive neural units with distinct sensible ranges were used to form a model. [6] as well as [7] proposed a neural network based method to identify the Preisach-type hysteresis. In their method, neural network was actually used to mimic the weight function of the Preisach model. The above-mentioned neural network based methods were useful for modeling hysteresis. However, the implementation of the modeling procedure is rather complicated.

Several approaches are proposed to control the systems with hysteresis. In order to compensate for the effect of hysteresis, one of the often used methods is to construct the inverse model of hysteresis. [8] presented a parameterized hysteresis model and developed an inverse model of hysteresis. Then an adaptive controller with an adaptive inverse model was designed for the plant with unknown backlash-like hysteresis. [9] developed a PID feedback control with inverse Preisach model in the feedforward loop. Their experimental results showed that the tracking performance was obviously improved. [10] also proposed an adaptive inverse controller based on the parameterized KP model to eliminate the effect of hysteresis. Since the construction procedure of the inverse model of hysteresis is very complicated, in this paper, a control strategy is proposed to avoid using inverse model to suppress the effect of hysteresis.

In this paper, a hysteretic operator is proposed. The proposed hysteretic operator is utilized to construct an additional coordinate in order to form an expanded input space so as to uniquely determine the output of the hysteresis based on the constructed input space. Then the neural model can be implemented based on the expanded input space containing the proposed hysteretic operator to describe the characteristic of the hysteresis. The advantage of the proposed neural networks based model is that it is convenient to tune the weights of neural networks to adapt the changes of the operating conditions such as the parts wear and tear in the system. Furthermore, the proposed hysteretic operator has a rather simple architecture and can be easily determined. Then, in terms of the proposed modeling approach for hysteresis, a neural network based pseudo controller is developed for such a system with hysteresis. In this method, considering the case where the output of hysteresis is unable to be measured directly, a neural network is used to estimate the dynamic error of the system in order to avoid constructing the hysteresis inverse to cancel the effect of hysteresis. Finally, the application of the novel modeling approach to hysteresis in a piezoelectric actuator will be illustrated. Then a numerical example of using the proposed control strategy for a nonlinear system with hysteresis will be presented.

2. SYSTEM DESCRIPTION

Consider a SISO nonlinear plant preceded by hysteresis characteristic $H[.]$

$$\Sigma_o : \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \cdots \\ \dot{x}_{n-1} = x_n \\ \dot{x}_n = f_o(x) + g_o(x)u \end{cases} \tag{1}$$

$$u = H(v) \tag{2}$$

$$y = x_1 \tag{3}$$

where the output is $y(t)$; $x \square [x_1, x_2 \cdots x_n]^T$ is the state vector; $v(t)$ is the control input; $u(t)$ is the actuator output. It is assumed that $f_o(x)$ and $g_o(x)$ are sufficiently smooth unknown functions and satisfy $\frac{\partial f_o}{\partial u} \neq 0$ and $\frac{\partial g_o}{\partial u} \neq 0$. Moreover, assume that f_i is invertible. Notation $H[.]$ denotes that the hysteresis operator is dependent on the trajectory, i.e. $v(t) \in C^0[0,t]$, but not on an instantaneous value of $v(t)$. The control objective is to design a control law for $v(t)$ to force the plant output, i.e. $y(t)$ to follow a smooth prescribed trajectory $y_d(t)$. The desired state vector is defined as $x_d(t) = [y_d, \dot{y}_d, \cdots y_d^{(n-1)}]^T$ where $y_d^{(n-1)}$ means the $(n-1)^{th}$ order derivative. Moreover, the tracking errors are defined as $e = x - x_d$. It is assumed that the desired states are bounded, i.e. $\|x_d\| \leq X_d$.

Define the filtered tracking error as

$$\tau = [\lambda_1, \lambda_2 \cdots \lambda_{n-1}, 1]e = [\Lambda^T, 1]e \tag{4}$$

where $\Lambda = [\lambda_1, \lambda_2 \cdots \lambda_{n-1}]^T$ is a parameter vector to be designed. Suppose

$$s^{n-1} + \lambda_{n-1} s^{n-2} + \cdots + \lambda_1$$

is Hurwitz. Differentiating (4) and using (1), it results in

$$\dot{r} = \dot{x}_n - y_d^n + [0, \Lambda^T]e = f_o(x) + g_o(x)u - y_d^n + [0, \Lambda^T]e + \xi \tag{5}$$

As u is the output of hysteresis and is usually unknown, an invertible function $\hat{f}(x,v)$ is introduced to approximate $f_o(x) + g_o(x)u$. Adding and subtracting $\hat{f}(x,v)$ to and from the right hand side of (5), it yields

$$\begin{aligned} \dot{r} &= \delta + f_o(x) + g_o(x)u - \hat{f}(x,v) - y_d^{(n)} + [0, \Lambda^T]e + \xi \\ &= \delta + F(x,u) - \hat{f}(x,v) - y_d^{(n)} + [0, \Lambda^T]e + \xi = \delta + \tilde{f}(x,v,u) - y_d^{(n)} + [0, \Lambda^T]e + \xi \end{aligned} \tag{6}$$

where $\delta = \hat{f}(x,v)$ is the so called pseudo-control [11, 12], $F(x,u) = f_o(x) + g_o(x)u$, and

$\tilde{f}(x,v,u) = F(x,u) - \hat{f}(x,v)$.

Suppose that $\hat{f}(x,v)$ is invertible with respect to v and satisfies

$$1.\ \mathrm{sgn}\frac{\partial F}{\partial u}\frac{\partial u}{\partial v} = \mathrm{sgn}\frac{\partial \hat{f}}{\partial v}, \tag{7}$$

and

$$2.\ \left|\frac{\partial \hat{f}}{\partial v}\right| > \frac{1}{2}\left|\frac{\partial F}{\partial u}\frac{\partial u}{\partial v}\right| > 0. \tag{8}$$

In order to design the corresponding control strategy, the approximation of the nonlinear residual $\tilde{f}(x,v,u)$ is required. Neural networks would be one of the recommended alternatives to model this residual. However, $\tilde{f}(x,v,u)$ involves the characteristic of hysteresis, the traditional identification approaches such as neural modeling technique usually cannot be directly applied to the modeling since the hysteresis is a kind of non-linearity with multi-valued mapping [5].

3. MODELING OF SYSTEM ERROR WITH HYSTERESIS

It is known that neural networks cannot be directly used to model the behavior of hysteresis since neural networks can only be used to describe the behavior of a continuous system with a one-to-one mapping [5]. To model hysteresis using neural networks, it is necessary to derive an input space, in which the mapping between the output and the input of hysteresis is transformed into a one-to-one mapping, that is, the derivative of the output of the hysteresis with respect to its input, which represents the moving direction of the hysteresis, should be included into the neural model. This way, the neural network can be used to describe the characteristic of hysteresis. However, the derivative of the output of the hysteresis with respect to its input cannot be obtained at the non-smooth points of hysteresis. In this section, a hysteretic operator is proposed to extract the change tendency such as the rising, falling and turning of hysteresis. Based on this hysteretic operator, an expanded input space is constructed. Therefore, the expanded input space does not depend on the derivative of the output of the hysteresis with respect to its input. This way, the neural networks can be used for modeling of hysteresis based on the proposed hysteretic operator based expanded input space. The proposed hysteretic operator $h(x)$ is defined as:

$$h(x) = (1 - e^{-|x-x_p|})(x - x_p) + h(x_p), \tag{9}$$

where x is the current input, $h(x)$ is the current output, x_p is the dominant extremum that is adjacent to the current input x, $h(x_p)$ is the output of the operator when the input is x_p.

Lemma 1: Let $x(t) \in C(R^+)$, where $R^+ = \{t \mid t \geq 0\}$ and $C(R^+)$ are the sets of continuous functions on R^+. If there exist two time instants t_1, t_2 and $t_1 \neq t_2$, such that $x(t_1) = x(t_2)$, $x(t_1)$ and $x(t_2)$ are not the extrema, then $h[x(t_1)] \neq h[x(t_2)]$.

Proof:

For $x(t)$ decreases or increases monotonically, (9) becomes

$$h(x) = \begin{cases} h_{in}(x) = [1 - e^{-(x-x_p)}](x - x_p) + h(x_p) & \dot{x}(t) > 0 \\ h_{de}(x) = (1 - e^{x-x_p})(x - x_p) + h(x_p) & \dot{x}(t) < 0 \end{cases} \tag{10}$$

and

$$h'_{in}(x) = e^{-(x-x_p)} \cdot (x - x_p) + [1 - e^{-(x-x_p)}] = 1 - [1 - (x - x_p)]/e^{x-x_p}$$
$$> 1 - 1/e^{x-x_p} > 0 \tag{11}$$

Therefore, $h_{in}(x)$ is monotonic. Similarly it can be obtained that $h_{de}(x)$ is also monotonic. It is noted that $h_{in}(x)$ is obtained from $h_{in0}(x) = (1 - e^{-x})x \ (x \geq 0)$ that its origin moves from $(0,0)$ to $(x_p, h(x_p))$. Similarly $h_{de}(x)$ is obtained from $h_{de0}(x) = (1 - e^{x})x \ (x \leq 0)$ that its origin moves from $(0,0)$ to $(x_p, h(x_p))$. As $h_{in0}(-x) = -h_{de0}(x)$, it implies that $h_{in}(x)$ and $h_{de}(x)$ are antisymmetric.

Thus it can be concluded that $h_{in}(x)$ and $h_{de}(x)$ intersect each other only at extrumum point $(x_p, h(x_p))$. That is, if $x(t_1)$ and $x(t_2)$ are not the extrema, $x(t_1) = x(t_2)$, then $h[x(t_1)] \neq h[x(t_2)]$.

Remark: It is noted that when $h(x)$ and $H[\cdot]$ are fed with the same input $v(t)$, the curve of $h[v(t)]$ exhibits similarity to that of $H[v(t)]$ such as rising, turning and falling. Moreover, since $x(t_1) = x(t_2)$, $x(t_1)$ and $x(t_2)$ are not the extrema, $h[x(t_1)] \neq h[x(t_2)]$, the pair $(v(t), h[v(t)])$ uniquely correspond to an output of hysteresis $H[v(t)]$.

Lemma 2: If there exist two time instants t_1, t_2 and $t_1 \neq t_2$, such that $h[x(t_1)] - h[x(t_2)] \to 0$, then $x(t_1) - x(t_2) \to 0$.

Proof:

$$\frac{h_{in}[x(t_1)] - h_{in}[x(t_2)]}{x(t_1) - x(t_2)} = k \quad k \in (0, +\infty), \tag{12}$$

and

$$x(t_1) - x(t_2) = \frac{h_{in}[x(t_1)] - h_{in}[x(t_2)]}{k} \tag{13}$$

It is clear that if $h_{in}[x(t_1)] - h_{in}[x(t_2)] \to 0$, then $x(t_1) - x(t_2) \to 0$. Similarly, it is obtained that if $h_{de}[x(t_1)] - h_{de}[x(t_2)] \to 0$, then $x(t_1) - x(t_2) \to 0$.

Thus, it leads to the following theorem, i.e.:

Theorem 1: For any hysteresis, there exists a continuous one-to-one mapping $\Gamma\ R^2 \to R$, such that $H[v(t)] = \Gamma(v(t), h[v(t)])$.

Proof:

Case 1: $v(t)$ is not the extrema. Based on lemma 1, if there exist two time instants t_1, t_2 and $t_1 \neq t_2$, then $(v(t_1), h[v(t_1)]) \neq (v(t_2), h[v(t_2)])$. Therefore the pair $(v(t), h[v(t)])$ uniquely corresponds to an output of hysteresis $H[v(t)]$.

Case 2: $v(t)$ is the extrema, i.e.

$(v(t_1), h[v(t_1)]) = (v(t_2), h[v(t_2)])$, according to the principle of the classical Preisach modeling, $H[v(t_1)] = H[v(t_2)]$, then the pair $(v(t), h[v(t)])$ uniquely corresponds to an output of hysteresis $H[v(t)]$.

Combining two situations, there exists a mapping $\Gamma: R^2 \to R$ such that $H[v(t)] = \Gamma(v(t), h[v(t)])$. In theorem 1 the obtained mapping $\Gamma(.)$ is a continuous function. According to lemma 2, from $v(t_1) - v(t_2) \to 0$, it leads to $H[v(t_1)] - H[v(t_2)] \to 0$. Also, from $h[v(t_1)] - h[v(t_2)] \to 0$, it yields $v(t_1) - v(t_2) \to 0$ and then results in $H[v(t_1)] - H[v(t_2)] \to 0$. Therefore, it is derived that Γ is a continuous function.

Theorem 1 indicates that a novel expanded input space is constructed so that the multi-valued mapping of hysteresis can be transformed to a one-to-one mapping. Moreover, theorem 1 also proves that the mapping is a continuous function:

Let $T = [t_0, \infty) \in R$, $V = \{v \mid T \xrightarrow{v} R\}$ and $F = \{h \mid T \xrightarrow{h} R\}$ are the input sets. Given $t_i \in T$, it is obvious that $v(t_i) < +\infty$ and $h[v(t_i)] < +\infty$, so that $(v(t_i), h[v(t_i)]) \in R^2$. Thus, it is obtained that $\Phi = \{(v(t_i), h[v(t_i)]) \mid v(t_i) \in V, h[v(t_i)] \in F\}$ is a compact set.

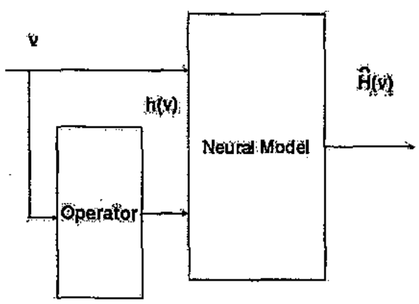

Fig.1. Expanded input space for modeling hysteresis

The architecture of using the corresponding expanded input space for modeling of hysteresis is shown in Fig. 1. Therefore, it provides a premise to apply neural networks to modeling of the behavior of hysteresis. Based on the proposed hysteretic operator, a feedforward neural network is used to approximate the residual $\tilde{f}(\mathrm{x}, \mathrm{v}, \mathrm{u})$, i.e.

$$\tilde{f}(\mathrm{x},\mathrm{v},\mathrm{u}) = \mathrm{F}[\mathrm{x}(\mathrm{t}), \mathrm{v}(\mathrm{t}), \Gamma(\mathrm{v}(\mathrm{t}), \mathrm{h}(\mathrm{v}(\mathrm{t})))] + \varepsilon \tag{14}$$

where F(.) is the neural network based mapping to describe the system residual, ε is the neural model residual, and $\|\varepsilon\| \le \varepsilon_N$, $\varepsilon_N > 0$.

4. ADAPTIVE CONTROL SCHEME

In the procedure of approximation of $\tilde{f}(x,v,u)$, u is the output of hysteresis. It is also the variable of $\tilde{f}(x,v,u)$. Based on theorem 1, input u can be designed as $u = \Gamma(v, h[v])$. Thus it can be obtained that $\tilde{f}(x,v,u) = \tilde{f}(x,v,\Gamma(v,h[v]))$. Based on [11], it can be approximated by a neural network. Therefore, the control scheme will not depend on whether the output of hysteresis is measurable or not. Hence, the corresponding input of the neural model can be chosen as

$$x_{nn} = (x^T, v(t), \cdots v(t-md), h[v](t), \cdots h[v](t-md))$$

where $d > 0$ is the time delay, and $m \ge 1$.

Assumption 1: Weights V and W are bounded from above by V_p and W_p , i.e. $\|W\|_F \le W_p$ and $\|V\| \le V_p$, where $W_p > 0$ and $V_p > 0$, where $\|\cdot\|_F$ represents Frobenius norm.

The corresponding pseudo-control is chosen as

$$\delta = y_d^{(n)} - K\tau - [0, \Lambda^T]e - v_{ad} + v_r \tag{15}$$

where v_r is the term for robust design, K is a parameter to be determined, v_{ad} is the output of the neural estimator which is utilized to estimate the residual $\tilde{f}(x,v,u)$, i.e.. $v_{ad} = \hat{W}^T \sigma(\hat{V}^T x_{nn})$, where $\hat{W}$ and $\hat{V}$ are the estimated values of W and V respectively.

From (9) and (14), notice that $\tilde{f}(x,v,u)$ depends on v_{ad} through δ , whereas v_{ad} has to be designed to eliminate the effect of $\tilde{f}(x,v,u)$. This should assume that the mapping $\delta_{ad} \mapsto \tilde{f}$ is a contraction over the entire input domain of interest. It has proven by [12] that the assumption is held when (15) is satisfied. It is known that (6) can be rewritten as

$$\dot{\tau} = -K\tau - \hat{W}^T \sigma(\hat{V}^T x_{nn}) + W^T \sigma(V^T x_{nn}) + v_r + \varepsilon + \xi \tag{16}$$

Define

$$\tilde{V} = V - \hat{V} , \quad \tilde{W} = W - \hat{W} \tag{17}$$

The Taylor series expansion of $\sigma(Vx_{nn})$ for a given x_{nn} can be written as

$$\sigma(Vx_{nn}) = \sigma(\hat{V}x_{nn}) + \sigma'(\hat{V}x_{nn})\tilde{V}x_{nn} + o(\tilde{V}x_{nn})^2 \tag{18}$$

where $\sigma'(\hat{z}) = d\sigma(z)/dz|_{z=\hat{z}}$ and $o(\tilde{z})^2$ denotes the higher order term. Denoting $\sigma = \sigma(V^T x_{nn})$, $\hat{\sigma} = \sigma(\hat{V}^T x_{nn})$, $\hat{\sigma}' = \sigma'(\hat{V}^T x_{nn})$, then it leads to

$$\dot{\tau} = -K\tau + \tilde{W}^T(\hat{\sigma} - \hat{\sigma}'\hat{V}^T x_{nn}) + \hat{W}^T\hat{\sigma}'\tilde{V}^T x_{nn} + v_r + \varepsilon + \xi + w \tag{19}$$

where

$$w = W^T(\sigma - \hat{\sigma}) + W^T\hat{\sigma}'\hat{V}^T x_{nn} - \hat{W}^T\hat{\sigma}'V^T x_{nn} \tag{20}$$

The upper bound for w can be presented as follows:

$$\|w\| \le \|W\|_1 + \|W\|\left\|\hat{\sigma}'\hat{V}^T x_{nn}\right\| + \|V\|_F \left\|x_{nn}\hat{W}^T\hat{\sigma}'\right\|_F \tag{21}$$

or

$$\|w\| \le \rho_w \vartheta_w(\hat{W}, \hat{V}, x_{nn}) \tag{22}$$

where $\vartheta_w = 1 + \left\|\hat{\sigma}'\hat{V}^T x_{nn}\right\| + \left\|x_{nn}\hat{W}^T\hat{\sigma}'\right\|_F$ and $\rho_w = \max(\|W\|_1, \|W\|, \|V\|_F)$.

Theorem 2: Let the desired trajectory be bounded. Considering system (1), (2), and (3), if the control law and the adaptive law are given by

$$v = \hat{f}^{-1}(x, \delta) \tag{23}$$

$$\delta = y_d^{(n)} - K\tau - [0, \Lambda^T]e - v_{ad} + v_r \tag{24}$$

$$\dot{\hat{W}} = F[(\hat{\sigma} - \hat{\sigma}'\hat{V}^T x_{nn})\tau - k\hat{W}\|\tau\|] \tag{25}$$

$$\dot{\hat{V}} = R[x_{nn}\hat{W}^T\hat{\sigma}'\tau - k\hat{V}\|\tau\|] \tag{26}$$

$$\dot{\hat{\phi}} = \gamma[\|\tau\|(\vartheta_w + 1) - k\|\tau\|\hat{\phi}] \tag{27}$$

$$v_r = \begin{cases} -\hat{\phi}(\vartheta_w + 1)\dfrac{\tau}{\|\tau\|} & \|\tau\| \ne 0 \\ 0 & \|\tau\| \ne 0 \end{cases} \tag{28}$$

where $F = F^T > 0$, $R = R^T > 0$, $\gamma > 0$, $\phi = \max[\rho_w, (\varepsilon_N + \xi_N)]$, $\tilde{\phi} = \phi - \hat{\phi}$. Then e, $\hat{W}$, $\hat{V}$, and $\hat{\phi}$ in the closed-loop system are ultimately bounded.

Proof:

Consider the Lyapunov function candidate

$$L = \frac{1}{2}\tau^2 + \frac{1}{2}tr(\tilde{W}^T F^{-1}\tilde{W}) + \frac{1}{2}tr(\tilde{V}^T R^{-1}\tilde{V}) + \frac{1}{2}\tilde{\phi}^T\gamma^{-1}\tilde{\phi} \tag{29}$$

The derivative of L will be

$$\dot{L} = \tau\dot{\tau} + tr(\tilde{W}^T F^{-1}\dot{\tilde{W}}) + tr(\tilde{V}^T R^{-1}\dot{\tilde{V}}) + \tilde{\phi}^T \gamma^{-1}\dot{\tilde{\phi}} \tag{30}$$

Substituting (19) into (30), it yields

$$\begin{aligned}\dot{L} = &-K\tau^2 + \tau v_r + \tau(w+\varepsilon+\xi) + \tilde{\phi}^T\gamma^{-1}\dot{\tilde{\phi}} \\ &+ tr\tilde{W}^T[F^{-1}\dot{\tilde{W}} + (\hat{\sigma} - \hat{\sigma}'\hat{V}^T x_{nn})\tau] + tr\tilde{V}^T(R^{-1}\dot{\tilde{V}} + x_{nn}\tau\hat{W}^T\hat{\sigma}')\end{aligned} \tag{31}$$

Considering $\dot{\tilde{W}} = -\dot{\hat{W}}$, $\dot{\tilde{V}} = -\dot{\hat{V}}$ and substituting (25) and (26) into (31), it can be rewritten as

$$\dot{L} = -K\tau^2 + \tau v_r + \tau(w+\varepsilon+\xi) + \tilde{\phi}^T\gamma^{-1}\dot{\tilde{\phi}} + k|\tau|[tr(\tilde{W}^T\hat{W}) + tr(\tilde{V}^T\hat{V})] \tag{32}$$

Using (22) and $\phi = \max[\rho_w, (\varepsilon_N + \xi_N)]$, it can obtain

$$\dot{L} \le -K\tau^2 + \tau v_r + |\tau|\phi(\vartheta_w + 1) - \tilde{\phi}\gamma^{-1}\dot{\hat{\phi}} + k|\tau|[tr(\tilde{W}^T\hat{W}) + tr(\tilde{V}^T\hat{V})] \tag{33}$$

Substituting (27) and (28) into (33), it results in

$$\dot{L} \le -K\tau^2 + k\|\tau\|[tr(\tilde{W}^T\hat{W}) + tr(\tilde{V}^T\hat{V}) + \tilde{\phi}^T\hat{\phi}] \tag{34}$$

Define

$$\tilde{Z} = \begin{bmatrix}\tilde{W} & 0 & 0\\ 0 & \tilde{V} & 0\\ 0 & 0 & \tilde{\phi}\end{bmatrix}, \quad \hat{Z} = \begin{bmatrix}\hat{W} & 0 & 0\\ 0 & \hat{V} & 0\\ 0 & 0 & \hat{\phi}\end{bmatrix} \text{and} \quad Z = \begin{bmatrix}W & 0 & 0\\ 0 & V & 0\\ 0 & 0 & \phi\end{bmatrix},$$

then, (34) can be written as

$$\dot{L} \le -K\tau^2 + k\|\tau\| tr(\tilde{Z}^T\hat{Z}). \tag{35}$$

Considering $tr(\tilde{Z}^T\hat{Z}) \le \|\tilde{Z}\|_F\|Z\|_F - \|\tilde{Z}\|_F^2$, it leads to

$$\dot{L} \le -K\tau^2 + k\|\tau\|(\|\tilde{Z}\|_F\|Z\|_F - \|\tilde{Z}\|_F^2). \tag{36}$$

Hence

$$\dot{L} \le -\|\tau\|[K\|\tau\| + k(\|\tilde{Z}\|_F - \frac{\|Z\|_F}{2})^2 - \frac{k\|Z\|_F^2}{4}]. \tag{37}$$

As long as either $\|\tau\| > \frac{k\|Z\|_F^2}{4K}$ or $\|\tilde{Z}\|_F > \|Z\|_F$, $\dot{L}$ will be negative. This will lead to τ , $\tilde{W}$, $\tilde{V}$, $\tilde{\phi}$ are ultimately bounded [13]. According to assumption 1 and the definition of τ and ϕ, it can be concluded that the variables e , $\hat{W}$, $\hat{V}$ and $\hat{\phi}$ in the closed-loop system are ultimately bounded.

5. EXPERIMENTAL RESULT AND SIMULATION

In this section, an example of using the proposed modeling method for hysteresis in a piezoelectric actuator will be illustrated. Then the developed control strategy is applied to a numerical nonlinear system with hysteresis.

Example 1: The proposed approach is applied to the identification of the hysteresis in a piezoelectric actuator (PZT-753.21C). The comparison of the performance between the obtained neural model and the KP model is also presented.

The actuator has a nominal expansion of 0-25 μm under the input voltage within 0-100V. In this experiment, it is excited with 5 Hz exponential decayed sinusoidal voltages. The sampling frequency is 1000Hz. After filtered, 1200 pairs of samples are selected to construct the training set and normalized so that the data set is located within $[-1,1]$. The data used for experiment is separated into two parts. One part of them is used for model identification and another part is for model validation. With 463 epochs, the training procedure is finished.

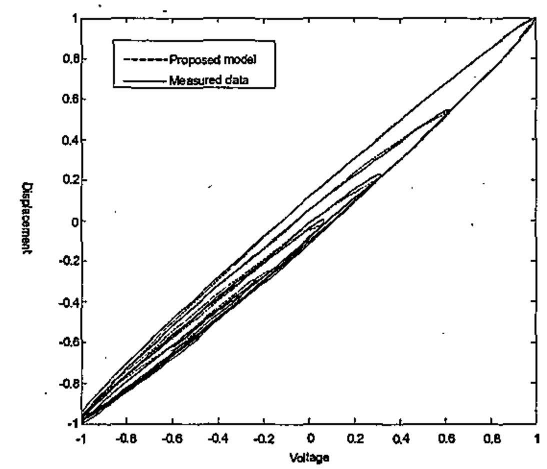

Fig. 2 The validation result of the proposed model

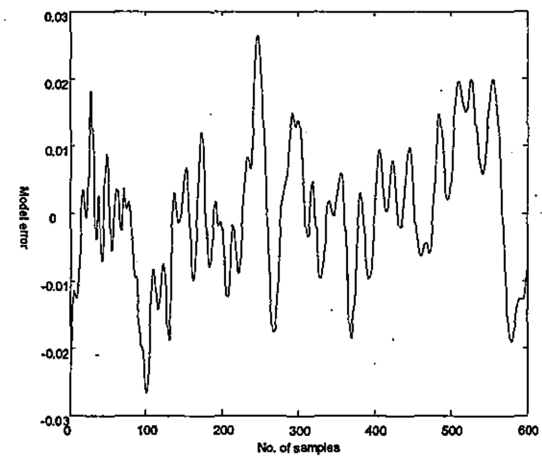

Fig. 3 The validation error of the proposed model

The architecture of the neural model consists of 2 input nodes, 12 hidden neurons and 1 output neuron. The input of the neural model is $(v(t), h[v(t)])$. The sigmoid and linear functions are respectively used as the activation function in the hidden layer and in the output layer. The conjugate gradient algorithm with Powell-Beale restarted method [14] is employed to train the model. Fig. 2 demonstrates the result of model validation. Fig. 3 presents the model validation error. It is shown that the resulted maximum error is 0.026735 and the mean square error is 0.01027.

In order to make a comparison, a KP model [10] is also employed to model this system. The KP model consists of 210 KP operators. The derived maximum error of the model is 0.0363 and the mean square error is 0.013. The model validation result of the KP model and the model validation error are respectively shown in Fig. 4 and Fig. 5. Compared with KP model, the proposed neural model shows better modeling performance.

Example 2: The developed control strategy is applied to the control of a nonlinear system with hysteresis. In this system, the hysteresis is simulated by a sum of backlash operators, i.e.

$$u = H[v(t)] = \sum_{i=1}^{N} u_i ,$$

$$\dot{u}_i = \begin{cases} \dot{v}(t) & \dot{v}(t) > 0, u_i(t) = v(t) - \frac{d_i}{2} \\ \dot{v}(t) & \dot{v}(t) < 0, u_i(t) = v(t) + \frac{d_i}{2}; \\ 0 & \text{otherwise} \end{cases}$$

where u_i and d_i are respectively the output and the dead-band width of i-th backlash operator ($i = 1,2,\cdots N$) where N is a positive integer. In this example, it chooses $N = 50$. The values of the dead-band width are evenly distributed within $[0.02,1]$. All the initial outputs are set to zero.

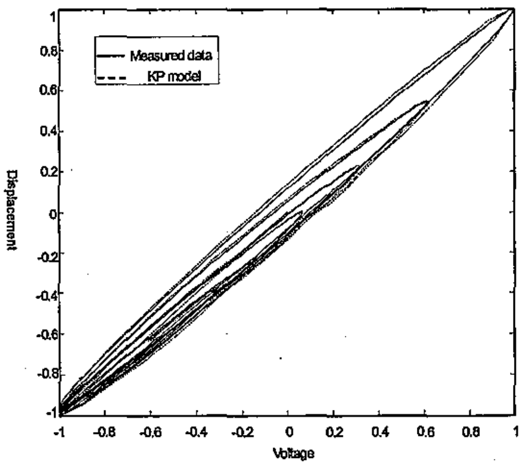

Fig.4 The validation result of the KP model

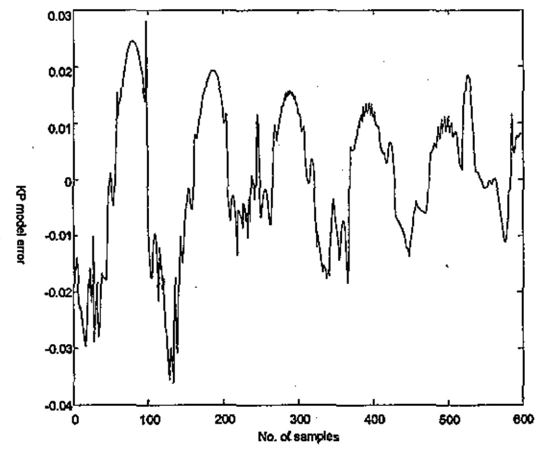

Fig. 5 The validation error of the KP model

The dynamic system to be controlled is given by:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = (1 - x_1^2)x_2 - x_1 + u \end{cases},$$

$$y = x_1 .$$

Firstly, the neural estimator used to approximate $\tilde{f}(x,v,u)$ is constructed with 4 input nodes, 35 hidden neurons and 1 output neuron. The conjugate gradient algorithm with Powell-Beale restarted method [14] is used to train the neural estimator. Then, the other parameters are respectively chosen as $\lambda_1 = 2$, $K = 200$, $k = 0.001$, $\gamma = 0.1$ $F = 8I$, $R = 5I$, $\hat{f}(x,v) = v$, $d = 0.3$, $m = 1$, where I is a unit matrix.

For comparison, a PID controller of the form is used:

$$v(t) = -22e_1 + \int_0^t e_1 dt - 13e_2 ,$$

where $e_1 = y - y_d$, $e_2 = \dot{y} - \dot{y}_d$ are also applied. The control parameters are selected based on the trial and error about the initial values obtained from Ziegler-Nichols method. The desired output is

$$y_d(t) = 0.1\pi[\sin 2t - \cos t].$$

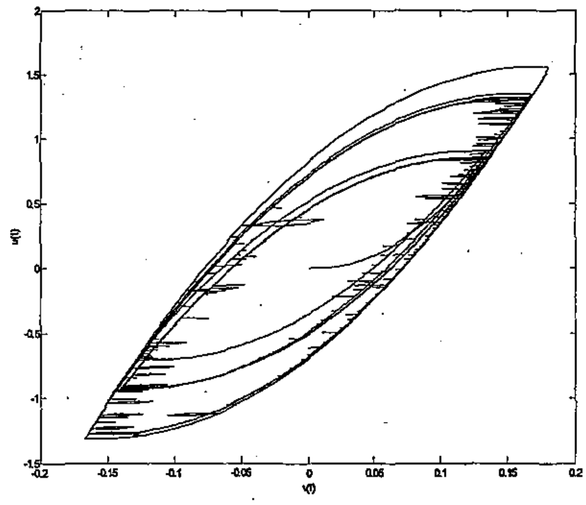

Fig. 6 The response of the hysteresis in the system

Fig. 6 shows the response of the hysteresis contained in the system. Fig. 7 illustrates the tracking performance when the proposed controller is applied to the system. Also, Fig. 8 illustrates the tracking performance of the PID controller. Moreover, Fig. 9 shows the comparison of tracking error between the two control schemes. As there is no hysteresis compensator in the PID control scheme, there is a larger oscillation caused by the hsteresis in the tracking error signal. However, the proposed control strategy obtained better control performance due to the neural estimator can compensate for the effect of residual caused by hysteresis. It can obviously derive more accurate tracking results.

6. CONCLUSIONS

In this paper, a neural network based control strategy is proposed to compensate for the effect of the hysteresis in a nonlinear dynamic system. In order to handle the case where the output of hysteresis is unmeasurable directly, which is often met in engineering practice, a neural network based estimator should be developed. As hysteresis is a non-smooth nonlinear function with multi-valued mapping, the traditional modeling technique even neural network is very hard to tackle it. Thus a hysteretic operator is proposed to construct an expanded input space to transform the multi-valued mapping of hysteresis into a one-to-one mapping so as to enable the neural networks to model the behavior of hysteresis.

In order to avoid constructing the inverse model for hysteresis, an adaptive control strategy with the pseudo-control technique is developed for the system embedded with hysteresis. Considering the case where the output of the hysteresis element cannot be directly measured, a neural estimator based on the proposed hysteretic operator is presented to estimate the modeling residual so that the control scheme will not rely on whether the output of hysteresis is measurable or not. The control law and adaptive law are derived in terms of Lyapunov stability theorem, so that the ultimate boundedness of the closed-loop system is guaranteed. The developed modeling technique has been applied to model hysteresis in a piezoelectric actuator and obtained satisfied result. The proposed control scheme is also used for a numerical system with hysteresis. Simulation results have illustrated that the proposed control scheme is rather promising.

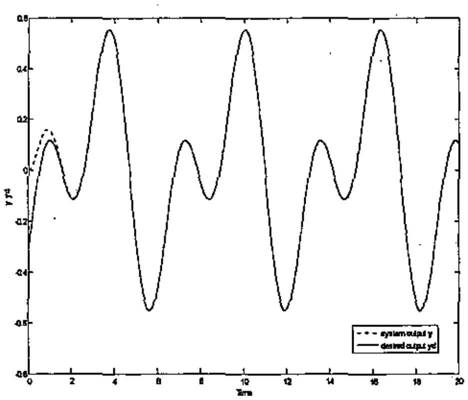

Fig. 7 The system response controlled by the proposed method

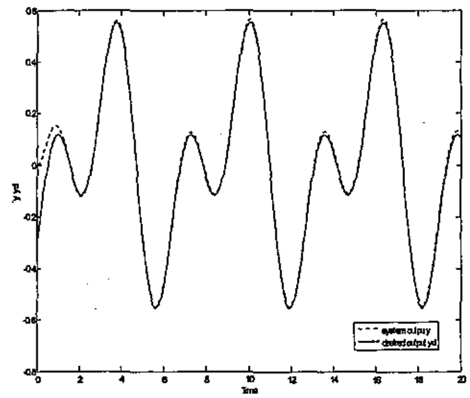

Fig. 8 The system response controlled by PID method

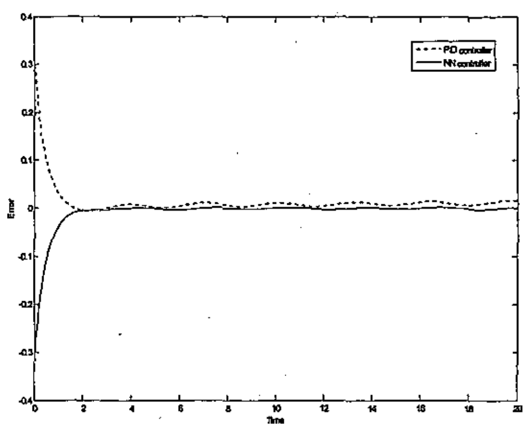

Fig. 9 Comparison of tracking errors between the two control schemes

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