MODELING AND MANUFACTURING OF PP-TYPE SINGLE SCREW COMPRESSOR

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ABSTRACT

A geometric model and a mathematical model of a PP-type single screw rotor with planar gate rotor are derived from the gate-rotor generation process and gear theory. The teeth of gate rotor are planar. Based on the inverse envelope concept, the cutter required for machining the single screw rotor can be obtained using an inverse envelope of a one-parameter family of screw surfaces. The surface of the proposed screw rotor is analyzed using the developed mathematical model. A surface analysis, including stress analysis, of the design and manufacture of the screw compressor is presented. Finally, a numerical example demonstrates the geometric model of the PP-type single screw rotor with a compression ratio of 11:6.

MODELAGE ET FABRICATION DE COMPRESSEUR VIS UNIQUE DE TYPE PP

RÉSUMÉ

Le modèle géométrique et le modèle mathématique d'un rotor vis unique de type PPA avec un rotor femelle planaire dérivent du processus de génération du rotor femelle et de la théorie de l'engrenage. Les dents du rotor femelle sont planaires. Basé sur le concept de l'enveloppe inversée, le couteau nécessaire pour faire fonctionner le rotor vis unique peut être obtenu en utilisant une enveloppe inversée de la famille des un-paramètre des vis de surface. La surface du rotor à vis proposé est analysée à l'aide d'un modèle mathématique développé. Une analyse de la surface, analyse de stress incluse, du design et de la fabrication du compresseur à vis est présentée. Pour finir, un exemple numérique démontre le modèle géométrique du rotor vis unique de type PP avec un taux de compression de 11:6.

INTRODUCTION

A single-screw compressor is essential to oil-injected refrigeration and air conditioning compressors. Such a compressor is a positive displacement machine that increases the pressure of gas. The main components of the single-screw compressor are four gate rotors, one screw rotor, and the casing. The gas volume is reduced along the helical groove, thereby increasing the pressure. Based on the enhanced design of a single-screw compressor, the derivation of mathematical models of cutting curves associated with machining screw rotors is attempted in this work.

Fundamentally, a single screw compressor is a kind of worm gear. Conjugate worm-gear surfaces are extensively adopted in power transmission and in fluid compression or pumping applications. The contact line between the conjugate surfaces is a load-carrying belt; however, strength capacity and precision are the main problems. In fluid compression or pumping applications, the contact lines between the flat tip of the gate-rotor and the surface of the root of screw-rotor flutes represent a sealing line.

Four kinds of single screw compressor are adopted. Table 1 describes four basic configurations of the screw or gate rotor: pp-type, cc-type, cp-type and pc-type single-screw compressors. The pp-type has a planar screw and four planar gate rotors. The cc-type has a cylindrical screw and two cylindrical gate rotors. The cp-type has a cylindrical screw and two planar gate rotors. All of these compressors have some marked advantages. One is that the screw rotor drives the gate rotors with only a small friction loss during transmission from the screw rotor to the gate rotor, resulting in negligible wear between the components and thus eliminating the need for a lubricating liquid as the injection fluid. The performance of the screw compressor is, thus superior to reciprocating compressors [1, 2]. The first report of the cp-type and the cc-type single-screw compressors was published by Zimmern in 1960 [3]. Since then, some U.S. patents of the cp-type single screw compressor [4, 5, 6, 7] have been registered. However, Yang [8] presented the direct envelope concept and the geometric model of a pp-type single screw compressor with conical gate rotors. In this study, the inverse envelope concept is used instead and the gate rotor's teeth are planar. This inverse envelope is applied to grinding the groove of the manufacturing screw rotor cutter. The gate rotor requires less material compared to the conical gate rotor in reference [8]. In addition, compared to Ref. [8], the sealing portion (elastic material) can be more conveniently replaced. and the manufacture of the gate rotor in the study is quicker.

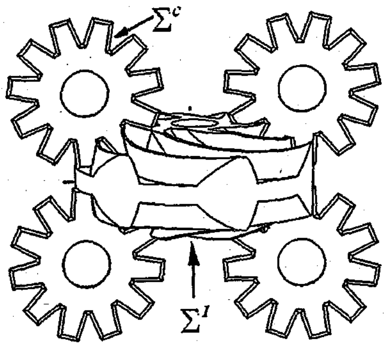
Typically, a pp-type single screw compressor has four gate rotors. As presented in Fig. 1a, the primary components and the contours of a pp-type single screw compressor are the gate rotors and the screw rotor. The gate rotors have many teeth that extend radially outwards from the axes of rotation and have side edges that match the sides of the flank of the helical flutes. As displayed in Fig. 1b, the gate rotors include a sealing portion made of plastic and supported by a rigid metal support portion. The support part does not come into contact with the screw rotor but carries the axial load.

At least one of the teeth of the gate rotor is typically meshed with the screw-rotor threads. The gate rotor is driven by the screw rotor. More importantly, the straight-line tips of a gate rotor match the infinite number of ruling line sealing segments of the roots and the helical flutes in their functional intermeshed position. A mathematical model of the pp-type single screw compressor with gate rotor and screw rotor can be established by applying the theory of [9] and differential geometry. Based on the gear theory, the mathematical model of the screw rotor is considered to be an envelope of the family of the gate-rotor surfaces. Since no cutter can be used to machine a screw rotor, in this work, the inverse envelope concept is adopted to develop a cutter for manufacturing screw rotors. Using the obtained envelope as the generating surface, and based on the inverse envelope concept, a cutting-edge curve can be treated as an envelope of the family of the generating surface when it rotates for a complete cycle. The profile of the cutting-edge curve in manufacturing a screw rotor can be easily determined by applying the inverse envelope concept. Concepts associated with the inverse problem are reviewed elsewhere [10, 11].

Computer software is used to show a complete profile of the pp-type single screw compression mechanism. One benefit of the herein method is its capacity to provide a rapid and simple geometric model of a pp-type single screw compression mechanism. The developed computer program can be applied to determine the geometric characteristics of a cutting-edge curve. Based on the above results, the developed cutter can be used in screw rotor machining. Finally, Table 2 presents a numerical example to demonstrate the geometric model of a pp-type single screw compressor with a compression ratio of 11:6.

Table 1 Four basic configurations of the screw and gate rotor

Type	Screw	Gate rotor
CP	Cylindrical	Planar
PC	Planar	Cylindrical
PP	Planar	Planar
CC	Cylindrical	Cylindrical



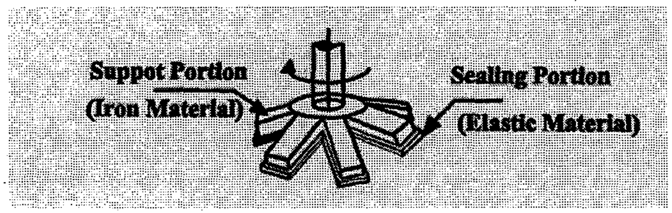


Fig. 1a PP-type single screw compressor

Fig. 1b Gate rotor with rigid support

2 DESIGN OF A GATE ROTOR

This section discusses the design of a gate rotor to generate the pp-type screw rotor. Figure 1b presents a gate rotor with rigid support. The gate rotor comprises a series of teeth, with each tooth having the same profile. As presented in Fig. 2, the profile of the gate rotor comprises four regions. Each gate-rotor tooth is trapezoidal, where, ξ is the taper angle of each tooth. Most compressors today have rectangular shaped teeth, where $\xi = 0$. This work develops a general mathematical model of the gate rotor. The radius of the gate rotor is *r*. Each tooth of the gate rotor has three straight edges that generate the helical flutes of the screw rotor. The straight edge of the gate rotor represented in the coordinate system $S_c(O_c,\ x_c,\ y_c,\ z_c)$ is expressed as regions $\overline{AB}$, $\overline{BD}$, and $\overline{DE}$ of the gate rotor. Region $\overline{AI}$ is a circular-arc curve.

REGIONS $\overline{AB}$ AND $\overline{DE}$ OF GATE ROTOR

Figure 2 shows the cross-section of a gate rotor. δ is the thickness of the gate rotor. Figure 2 does not include the term δ. $\beta_1 \sim \beta_3$ is the curvilinear parameters of the gate-rotor surface in regions $\overline{AB}$ and $\overline{DE}$. The tooth length is *h*. Parameter $\theta = \dfrac{\pi}{N}$ is the half angle of one tooth of the gate rotor. Herein, parameter *N* represents the number of teeth in the gate rotor. Regions $\overline{AB}$ and $\overline{DE}$ are adopted to generate the different sides of the helical flute of the screw rotor. Herein, a general equation is adopted to develop straight-edged tooth gate rotors. The equation in regions $\overline{AB}$ and $\overline{DE}$, respectively, in the $\boldsymbol{S}_c$ coordinate system can be written as follows.

$$\boldsymbol{R}_c^{AB} = \begin{bmatrix} -(r\sin\theta - \beta_2 \sin\xi) \\ -(r\cos\theta + h - \beta_2 \sin\xi) \\ \delta \end{bmatrix}, \quad 0 < \beta_2 < h/\cos\xi, \tag{1}$$

$$\boldsymbol{R}_c^{DE} = \begin{bmatrix} (r\sin\theta - \beta_3 \sin\xi) \\ -(r\cos\theta + h - \beta_3 \sin\xi) \\ \delta \end{bmatrix}, \quad 0 < \beta_3 < h/\cos\xi, \tag{2}$$

where β_2 is the surface parameter of gate rotors in region $\overline{AB}$, β_3 is the surface parameter of gate rotors in region $\overline{DE}$, and the tooth length is *h*. The vectors $\boldsymbol{R}_c^{AB}$ and $\boldsymbol{R}_c^{DE}$ in Eq. (1) and Eq. (2) are the position vectors, where superscript $\overline{AB}$ and $\overline{DE}$ represent regions $\overline{AB}$ and $\overline{DE}$, respectively. Subscript *c* denotes the position vectors are represented in coordinate system S_c.

REGION $\overline{BD}$ OF GATE ROTOR

Region $\overline{BD}$ of the gate rotor is used to form the bottom of the screw rotor. β_1 and δ are the surface parameters of the gate rotor for region $\overline{BD}$. The position vector in region $\overline{BD}$ of the gate rotor can be determined as follows.

$$\boldsymbol{R}_c^{BD} = \begin{bmatrix} -\beta_1 \\ -(h + r\cos\theta) \\ \delta \end{bmatrix}, \quad -(r\sin\theta - h\tan\xi) < \beta_1 < (r\sin\theta - h\tan\xi). \tag{3}$$

where *r* represents the pitch radius of the gate rotor. The form parameter of the gate rotor is $r\sin\theta - h\tan\xi$. Table 2 presents the values *r*, *h* and ξ.

REGION $\overline{AI}$ OF GATE ROTOR

Region $\overline{AI}$ is employed to represent the root of the gate rotor. The region $\overline{AI}$ in the gate rotor is to generate the top of the screw rotor. However, in the formation of the screw rotor, the screw rotor must be a part of the original cylindrical surface from which it is cut. Herein β_4 is the curvilinear parameter of the gate rotor for region $\overline{AI}$. The position vector in region $\overline{AI}$ of the gate rotor is as follows.

$$R_c^{AI} = \begin{bmatrix} -r\sin\beta_4 \\ -r\cos\beta_4 \\ \delta \end{bmatrix}, \ \theta < \ \beta_4 \ < \theta_s . \qquad (4)$$

where β_4 is the surface parameter of the gate rotors in region $\overline{AI}$. The form parameter of the gate rotor is $\theta_s = \theta + 2(\pi / N - \theta)$, and r represents the pitch radius of the gate rotor.

A computer software and Eqs. (1)-(4) were used to derive the complete contour of the gate rotor displayed in Fig. 3.

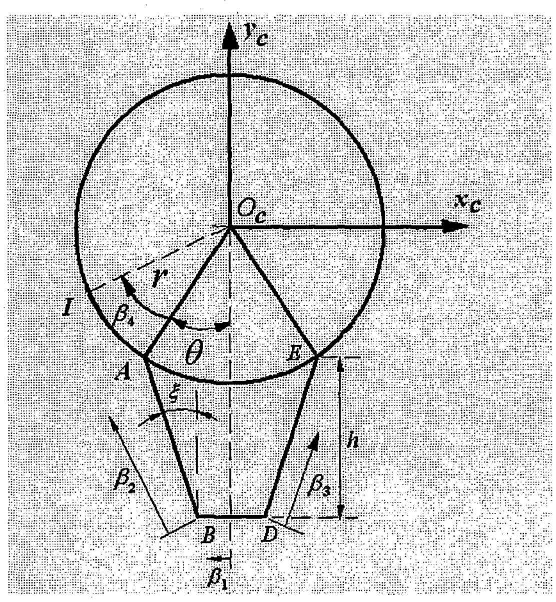


Fig. 2 Design of gate rotor

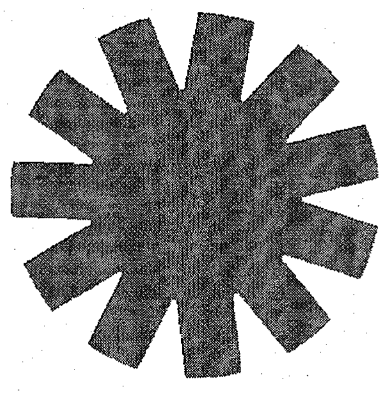


Fig.3 Complete profile of gate rotor.

GENERATION PROCESS OF A SCREW ROTOR

The profile of the screw rotor can be regarded as the envelope of the family of surfaces of the gate rotors during the rotation of the screw. Figure 4 shows the relationship between the gate rotor Σ^c and the screw rotor Σ^1. The gate rotor Σ^c is called the generating surface and the screw rotor Σ^1 is called the generated surface. Moveable co-ordinate systems, $S_c(X_c,\ Y_c,\ Z_c)$ and $S_1(X_1,\ Y_1,\ Z_1)$, are rigidly connected to the generating surface and the generated surface, respectively. The co-ordinate system, $S_f(X_f,\ Y_f,\ Z_f)$, is rigidly connected to the frame of the compressor. The Z_c, Z_1, and Z_f axes are set by right-handed co-ordinate system.

As presented in Fig. 4, the angles ϕ_c and ϕ_1 represent the rotary angles of Σ^c and Σ^1 about their centres, O_c and O_1, respectively. When one of the gate rotors rotates along the Z_c axis at a rotary angle ϕ_c, the screw rotates along the Z_1 axis to rotary angle ϕ_1. The relationship between the angles ϕ_c and ϕ_1 is given by $\phi_c = (s_n / g_n)\phi_1$. The symbol s_n is the number of threads on the screw rotor. The term g_n represents the number of teeth on the gate rotor. The distance between O_c and O_1 is decomposed into two components, C_x and C_y, which represent the horizontal and the vertical distances in coordinate system S_f, respectively. Subsequently, the change of the coordinate system from S_c to S_1 can be represented as.

$$M_{1c} = \begin{bmatrix} \sin\phi_1\cos\phi_c & \sin\phi_1\sin\phi_c & \cos\phi_1 & C_x\sin\phi_1 \\ \cos\phi_1\sin\phi_c & \cos\phi_1\sin\phi_c & -\sin\phi_1 & C_y\cos\phi_1 \\ -\sin\phi_c & \cos\phi_c & 0 & C_y \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{5}$$

Equations (1)-(4) and the coordinate transformation matrix M_{1c} can be used to obtain the one-parameter family of the gate-rotor surfaces, and can be represented in the coordinate system S_1 as,

$$R_1^i = M_{1c} R_c^i .\qquad (6)$$

where R_c^i is the position vector of the designed gate rotor, as discussed in Section 2. The upper sign of vector R_c^i refers to regions $\overline{AB}$, $\overline{BD}$, $\overline{DE}$ and $\overline{AI}$ of the gate rotor.

According to the gear theory, the meshing equation can be determined by the following equation:

$$\left(\frac{\partial R_1^i}{\partial \beta_j} \times \frac{\partial R_1^i}{\partial \delta}\right) \bullet \frac{\partial R_1^i}{\partial \phi_1} = 0 \qquad (7)$$

where β_j is the curvilinear parameters as represented in Section 2 (*j=1, 2, 3, 4*). The profile of the gate rotor is similar to a sealing line. For simplicity, the determination of the profile of the gate rotor can be treated as a two-dimensional problem, and thus gate rotor is represented as a plane. Based on Eqs. (1) – (4), the parameter δ can be set to zero. Substituting Eqs. (1) ~ (4) into Eq. (6) and setting δ to zero, the mathematical model of the pp-type single screw rotor $R_1^i(\beta_j,\ \phi_1) = \left[x_1^i,\ y_1^i,\ z_1^i\right]$ can be obtained.

The helical flute of the screw rotor is generated by region $\overline{AB}$ of the gate rotor and is represented as

$$\begin{cases} x_1^{AB} = -r\sin\phi_1 \sin(\theta+\phi_c) + \beta_2 \sin\phi_1 \sin(\xi+\phi_c) - h\sin\phi_1 \sin\phi_c + C_x \sin\phi_1 \\ y_1^{AB} = -r\cos\phi_1 \sin(\theta+\phi_c) + \beta_2 \cos\phi_1 \sin(\xi+\phi_c) - h\cos\phi_1 \sin\phi_c + C_y \cos\phi_1 \\ z_1^{AB} = -r\cos(\theta+\phi_c) + \beta_2 \cos(\xi+\phi_c) - h\cos\phi_c + C_y \end{cases} \qquad (8)$$

The helical flute of the screw rotor is generated by region $\overline{DE}$ of the gate rotor and is represented as

$$\begin{cases} x_1^{DE} = r\sin\phi_1\sin(\theta-\phi_c) - \beta_3\sin\phi_1\sin(\xi-\phi_c) - h\sin\phi_1\sin\phi_c + C_x\sin\phi_1 \\ y_1^{DE} = r\cos\phi_1\sin(\theta-\phi_c) - \beta_3\cos\phi_1\sin(\xi-\phi_c) - h\cos\phi_1\sin\phi_c + C_y\cos\phi_1 \\ z_1^{DE} = -r\cos(\theta-\phi_c) + \beta_3\cos(\xi-\phi_c) - h\cos\phi_c + C_y \end{cases} \tag{9}$$

The bottom of the screw rotor's groove is generated by region $\overline{BD}$ of the gate rotor and is represented as

$$\begin{cases} x_1^{BD} = \beta_1\sin\phi_1\cos\phi_c - (h + r\cos\theta)\sin\phi_1\sin\phi_c + C_x\sin\phi_1 \\ y_1^{BD} = \beta_1\cos\phi_1\cos\phi_c - (h + r\cos\theta)\cos\phi_1\sin\phi_c + C_y\cos\phi_1 \\ z_1^{BD} = -\beta_1\sin\phi_c - (h + r\cos\theta)\cos\phi_c \end{cases} \tag{10}$$

The top of the threads of the screw rotor is generated by region $\bar{AI}$ of the gate rotor and represented as

$$\begin{cases} x_1^{AI} = -r\sin\phi_1\sin(\beta_4+\phi_c) + C_x\sin\phi_1 \\ y_1^{AI} = -r\cos\phi_1\sin(\beta_4+\phi_c) + C_y\cos\phi_1 \\ z_1^{AI} = -r\cos(\beta_4+\phi_c) + C_y \end{cases} \tag{11}$$

where the subscript 1 refers to the generated surface (screw rotor). The relationship between the angles ϕ_c and ϕ_1 is generally expressed as $\phi_c = (6/11)\phi_1$. Table 2 shows the dimensional parameters of a pp-type single screw compressor with the composition ratio 11:6. The geometric model of the screw rotor with agate rotor can be obtained from Eqs. (8) - (11) and Table 2. The complete profile of the single screw compressor with compressor ratio 11:6 is plotted in Fig. 5.

Table 2 Major design parameters of a pp-type single screw compressor with a compression ratio's 11:6

Parameters	Screw Rotor	Gate Rotor
Number of Teeth	s_n =6	g_n =11
C_x and d_x	230 mm	230 mm
C_y and d_y	230 mm	230 mm
h		60
N		11
θ	π/N	π/N
r	108 mm	108 mm
ξ		$\pi/36$

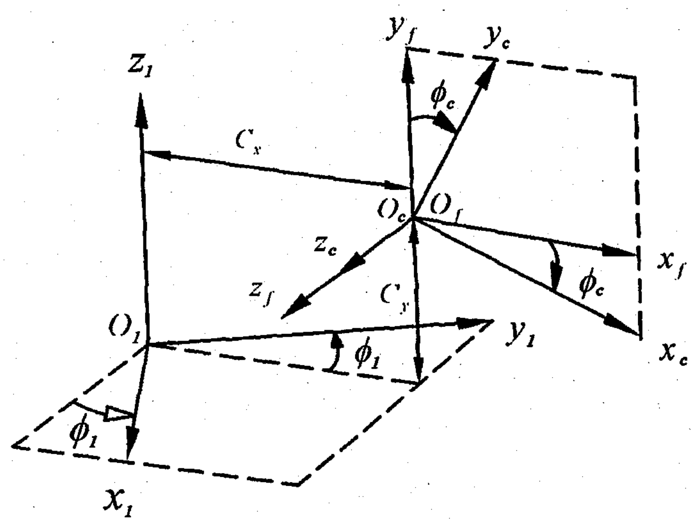


Fig. 4 Coordinate systems for generating screw rotor

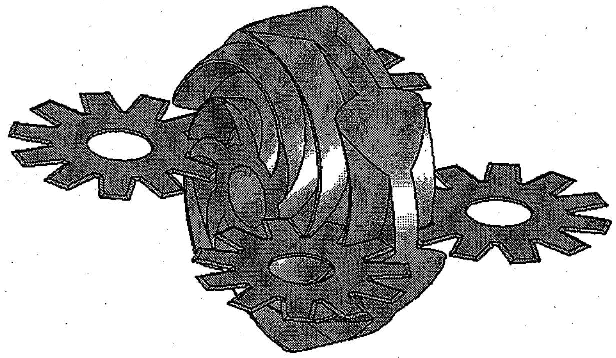


Fig. 5 PP-type single screw compressor with compressor ratio 11:6

MATHEMATICAL MODEL OF A CUTTING-EDGED CURVE

The design of the cutters is critical to the manufacture of a screw rotor. Hence, in this Section, an inverse envelope is adopted to determine a cutting-edge curve, which is used to machine the screw rotor. In Section 3, the geometric model of the screw rotor was obtained; however, the cutting-edge curve for machining the screw rotor was unknown.

The corresponding tool contour used in manufacturing the screw rotor can be determined by applying the inverse envelope concept [9, 10]. The developed mathematical model of the screw rotor in Section 3 now becomes the generating surface, and subsequently, the contour of the cutting edge curve is regarded as the generated surface. Figure 6 shows the three coordinate systems used to obtain a mathematical model of the cutting-edge curve. The coordinate systems $S_1(O_1, x_1, y_1, z_1)$ and $S_2(O_2, x_2, y_2, z_2)$ are rigidly attached to the screw rotor and the workpiece, respectively. z_1 is the rotation axis of the screw rotor. z_2 is the rotation axis of the workpiece. As presented in Fig. 6, the relationship between ϕ_1' and ϕ_2 is given by $\phi_2 = \phi_2(\phi_1')$. The Coordinate system $S_f'(O_f', x_f', y_f', z_f')$ is rigidly attached to the fixed frame.

The machine-tool setting error is assumed to be zero. The inverse envelope concept is used to determine the cutting-edged curve for grinding the groove of the manufacturing screw rotor cutter. Hence, the mathematical model of the cutting-edge curve is given by

$$\boldsymbol{R}_2^i(\beta_j, \phi_2, \phi_1) = \boldsymbol{M}_{21}(\phi_2)\boldsymbol{R}_1^i(\beta_j, \phi_1) \qquad (12)$$

$$N_f^1 \bullet V_f^{12} = f(\beta_j,\ \ \phi_2,\ \ \phi_1) = 0 \tag{13}$$

where $R_1^i(\beta_j,\ \ \phi_1)$ is described in Section 3. N_f^1 is the unit normal to the screw-rotor's surface and V_f^{12} is the relative velocity between the screw rotor and the workpieces. Vectors N_f^1 and V_f^{12} are represented in the S_f' coordinate system and written as follows;

$$N_f^1 = \frac{\partial R_1^i}{\partial \beta} \times \frac{\partial R_1^i}{\partial \phi_1} = \begin{bmatrix} (y_{1\beta_j}^i z_{1\phi_1}^i - y_{1\phi_1}^i z_{1\beta_j}^i)\cos\phi_1' - (x_{1\phi_1}^i z_{1\beta_j}^i - x_{1\beta_j}^i z_{1\phi_1}^i)\sin\phi_1' \\ (y_{1\beta_j}^i z_{1\phi_1}^i - y_{1\phi_1}^i z_{1\beta_j}^i)\sin\phi_1' + (x_{1\phi_1}^i z_{1\beta_j}^i - x_{1\beta_j}^i z_{1\phi_1}^i)\cos\phi_1' \\ x_{1\beta_j}^i y_{1\phi_1}^i - x_{1\phi_1}^i y_{1\beta_j}^i \end{bmatrix}, \tag{14}$$

$$\begin{aligned} V_f^{12} = & -\dot{\phi}_1'(x_1^i \sin\phi_1' + y_1^i \cos\phi_1')i_f + [\dot{\phi}_1'(x_1^i \cos\phi_1' - y_1^i \sin\phi_1') - \dot{\phi}_2(d_z - z_1^i)]j_f \\ & + \dot{\phi}_2(x_1^i \sin\phi_1' + y_1^i \cos\phi_1' - d_y)k_f \end{aligned} \tag{15}$$

Replacing the parameter ϕ_2 in Eq. (13) with β_j and ϕ_1, the cutting-edge curve is yielded. The distance between O_2 and O_1 is decomposed into two components, d_y and d_z, which represent the horizontal and the vertical distances, respectively, in coordinate system S_f'. Matrix M_{21} is the coordinate transformation matrix from S_1 to S_2. Hence, matrix M_{21} can be written as,

$$M_{21}(\phi_2) = \begin{bmatrix} \cos\phi_2 \sin\phi_1' & \cos\phi_2 \cos\phi_1' & -\sin\phi_2 & -d_y\cos\phi_2 + d_z\sin\phi_2 \\ \sin\phi_2 \sin\phi_1' & \sin\phi_2 \cos\phi_1' & \cos\phi_2 & -d_y\sin\phi_2 - d_z\cos\phi_2 \\ \cos\phi'_1 & -\sin\phi_1' & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{16}$$

To determine the envelope (cutting-edge curve) of the family of screw-rotor surfaces, the equation of meshing can be expressed in the coordinate system $S'_f(O'_f,\ x'_f,\ y'_f,\ z'_f)$ as follows;

$$f(\beta_j,\ \phi_2,\ \phi_1)=(d_z n-z_1^i h+mx_1^i)\sin\phi_1+(d_z\ell-\ell z_1^i+my_1^i)\cos\phi_1+ (\ell x_1^i\frac{\partial\phi_1}{\partial\phi_2}-ny_1^i\frac{\partial\phi_1}{\partial\phi_2}-mC_y)=0 \tag{17}$$

where $n=(y_{1\beta_j}^i z_{1\phi_1}^i-y_{1\phi_1}^i z_{1\beta_j}^i)$, $\ell=(x_{1\phi_1}^i z_{1\beta_j}^i-x_{1\beta_j}^i z^i{}_{1\phi_1})$, and $m=x_{1\beta_j}^i y_{1\phi_1}^i-x_{1\phi_1}^i y_{1\beta_j}^i$.

Section 3 gives the values of x_1^i, y_1^i and z_1^i in equation (17). Superscript i refers to $\overline{AB}$, $\overline{BD}$, $\overline{DE}$ or $\overline{AI}$.

Figure 7 displays the geometric model of the cutting-edge curve obtained from Eqs. (12) to (17). Based on this figure, the envelope of the family of the screw-rotor surfaces consists of lines and curves. Based on this envelope, the cutting-edge curve of a tool can be made.

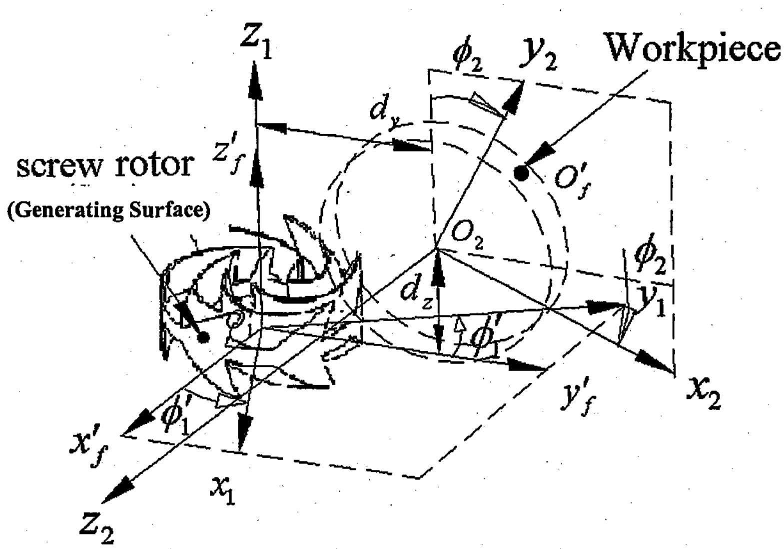


Fig. 6 Coordinate systems for determining the cutting-edge curve

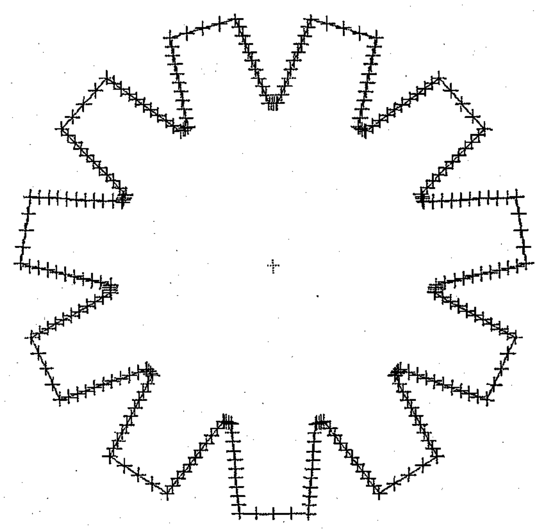


Fig. 7 The complete cutting-edge curve of cutter profile

CONCLUSION

This study developed a complete mathematical model of the creation of a pp-type single screw compressor with a gate rotor. A computer program that was based on the proposed mathematical model was developed. The developed computer program yields a geometric model of the screw rotor and the gate rotor via a computer-aided design.

The contours of a screw rotor and four gate rotors can be used in manufacturing technology to understand a developed pp-type single screw compressor. This procedure can be adopted as a fundamental part of a computer-aided design program to determine the cutters required for manufacturing a screw rotor on a numerically controlled milling machine.

A mathematical model of the cutting-edged curve for helical groove machining was developed using the engagement relationships between the screw rotor and the work piece. The mathematical model analysis developed herein for a pp-type single screw compressor should provide a valuable reference for the design and production of pp-type single screw compressors.

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References

1. Zhou, Lei and Jin, Guangxi, "Dynamic Characteristics of Gate Rotor for Single Screw Compressor", *Journal of Xi'an Jiaotong University China*, Vol. 23, No. 3, 1989, 7-15.

2. Duncan, Todd, The Rotary Screw Compressor, *ASHRAE Journal*, Vol. 41, No. 5, 1999, ASHRAE Atlanta GA USA, 34-36.

3. Zimmern, Bernard "Design and Operation Characteristics of the Zimmern Single Screw Compressor," *ICECP*, 1972, 96-99.

4. Zimmern, Bernard, "Method and a Screw Machine for Processing Fluid under High Pressures, with liquid injection between a sealing portion and a support portion of the gate rotor," *U.S. Patent 4900239, 1990.*

5. Zimmern, Bernard, "Single Screw Compressor with Liquid Lock Preventing Slides", *U.S. Patent 6106241, 2000.*

6. Aquino, Giovanni, Choroszylow, Ewan, "Mainrotor Machining Process and Apparatus," *U.S. Patent 5598618, 1997.*

7. Jensen, David L., " Method for Manufacturing Fluid Compression/Compressor Rotor," *U.S. Patent 6122824, 2000.*

8. Shyue-Cheng Yang, "Profile Generation and Analysis for a pp-Type Single Screw Compressor," *International Journal of Advanced Manufacturing Technolog*, v. 30, No. 9-10, September, 2006, 789-796.

9. Litvin, F. L., *Gear Geometry and Applied Theory*, 1994 (Prentice-Hall, New Jersey)

10. Kang, S. K, Ehmann, K. F. and Lin, C. "A CAD Approach to Helical Groove Machining-I. Mathematical Model and Model Solution," *Int. J. Mach. Tools Manufact*, 1996,Vol. 36, No. 1, 141-153.

11. Chyan, H. C. and Ehmann, K. F., "Tapered-web helical groove machining," *Journal of Mechanical Engineering Science Proceedings of the Institution of Mechanical Engineers*, Vol. 213, Part B, 779-785.

NOTATION

C_x and C_y, distance between the centre of screw rotor and the centre of gate rotor

d_y and d_z distance between the centre of screw rotor and the centre of workpiece

$f(\beta_j,\ \phi_2,\ \phi_1)$ equation of meshing

h	tooth length
r	pitch radius of gate rotor
M_{ij}	co-ordinate transformation matrix from co-ordinate system S_j to co-ordinate system S_i
N_f^1	unit normal to the generating surface
g_n	number of teeth of gate-rotor
s_n	thread number of screw rotor
R_1^i	position vector of generating surface ($i=\overline{AB}$, $\overline{BD}$, $\overline{DE}$ and $\overline{AI}$)
R_2^i	family of generating surfaces ($i=\overline{AB}$, $\overline{BD}$, $\overline{DE}$ and $\overline{AI}$)
R_c^i	position vector of gate rotor ($i=\overline{AB}$, $\overline{BD}$, $\overline{DE}$ and $\overline{AI}$)
$S_i(X_i,\ Y_i,\ Z_i)$	co-ordinate system where subscript *i=c, 1, 2, f.* c denotes gate rotor. *1* denotes generating surface, *2* denote generated surface, *f* is rigidly connected to the frame of reference
V_f^{12}	relative velocity between the screw rotor and the workpieces represented in S'_f co-ordinate system
β_j, δ	bi-parameter definition of generating surface
ϕ_1	rotary angle of screw rotor
ϕ'_1	rotary angle of generating surface
ϕ_2	rotary angle of generated surface (workpiece)
θ	half angle of one tooth
ξ	taper angle of tooth
Σ^1	screw rotor
Σ^c	gate rotor