OPTIMIZATION OF REALISTIC REFRIGERATION PLANT UNDER FIXED TOTAL THERMAL CONDUCTANCE CONSTRAINT

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Received November 2006, Accepted June 2007
No. 06-CSME-48, E.I.C. Accession 2967

ABSTRACT

This study analyzes the internal irreversibility of a realistic refrigeration plant under the design constraint of a fixed total thermal conductance. The internal heat losses are determined using a heat by-pass model. The optimal thermal conductance allocation and optimal coefficient of performance are derived from a series of detailed analyses and formulations. The numerical results indicate that the optimal thermal conductance ratio of the hot end of a realistic refrigeration plant is slightly higher than 0.5.

OPTIMISATION D'USINE REALISTE DE REFRIGERATION SOUS LA CONTRAINTE D'UNE CONDUCTANCE THERMIQUE TOTALE DETERMINEE

RESUME

Cette étude analyse l'irréversibilité interne d'une usine réaliste de réfrigération dans l'hypothèse d'une contrainte de conductance thermique totale déterminée. Les pertes de chaleur internes sont déterminées selon un modèle de by-pass de chaleur. L'allocation de conductance thermique optimale et le coefficient de performance optimal sont dérivés d'une série d'analyses et de formulations détaillées. Les résultats numériques indiquent que le rapport de conductibilité thermique optimal de l'extrémité chaude d'une usine de réfrigération réaliste est légèrement supérieur à 0.5.

INTRODUCTION

In analyzing the thermal efficiency of a heat engine and the coefficient of performance (*COP*) of a refrigeration plant, finite-time thermodynamics provides a more practical approach than classical thermodynamics. The literature contains many examples of the use of finite-time thermodynamics to investigate the optimal performance of endoreversible Carnot thermal cycles. The consequences of incorporating finite-time heat transfer processes into an otherwise ideal thermodynamic cycle were elegantly demonstrated by Curzon and Ahlborn [1] for the case of finite heat transfer rates to and from a Carnot heat engine. For an endoreversible refrigeration plant, Bejan [2, 3] proposed a theory for heat transfer in an external irreversible refrigeration plant. In recent years, various optimization studies have been performed for heat engines of different types [4, 5]. A realistic refrigeration plant has both external irreversibilities and internal irreversibilities. The former are induced by the temperature differences between the hot end and the ambient environment and between the cold end and the refrigerated space, respectively, while the latter are induced by friction forces between the moving components of the refrigeration plant. Many researchers have attempted to optimize refrigeration plants under various objective functions using endoreversible or irreversible models [6-10]. Typically, these studies have chosen the power generation rate, the cooling rate, the thermal efficiency, the *COP*, the exergy efficiency, and so forth.

When designing a refrigeration plant for real-world applications, many design constraints must be satisfied, including those of space, weight, material compatibility, accessibility, cost of maintenance, and so on. However, the overall implementation cost of the refrigeration plant is perhaps one of the biggest constraints imposed on the designer. Broadly speaking, the cost of the refrigeration plant depends primarily on the cost of the high- and low-end heat exchangers, respectively, together with that of the compressor. The bulk of the cost of the two heat exchangers is constituted by that of the condenser and evaporator units within them. The cost of these units increases dramatically as the scale of the corresponding heat exchanger increases. As a result, improving the thermal conductance (*UA*) of heat exchangers is highly expensive. In designing a realistic refrigeration plant, the total attainable thermal conductance is constrained by the imposed cost and size considerations, and thus the total thermal conductance cannot be specified arbitrarily, but must be carefully divided between the hot and the cold ends of the plant in such a way as to optimize its performance.

Under the assumptions of a fixed total thermal conductance and a constant cooling rate, this study derives a series of analytical formulations to determine the optimal thermal conductance allocation within a realistic refrigeration plant and the corresponding *COP*. In performing the analysis, the internal irreversibilitues within the refrigeration plant caused by frictional heating and heat leakage are modeled using a heat by-pass model. If the internal irreversibility is neglected, the optimal thermal conductance allocation obtained by our formulations is consistent with that of Bejan [2, 3].

ANALYSIS

Figure 1 presents the current model of a steady-state refrigeration plant operating between two thermal reservoirs at temperatures T_H and T_C, respectively. From the second law of thermodynamics for a refrigeration plant, the relationship between the *COP* of the actual refrigeration plant and the *COP* of the Carnot refrigerator can be written as

$$COP \leq (COP)_C, \tag{1}$$

where $(COP)_C$ is the *COP* of a Carnot refrigerator operating between the same temperatures (T_H, T_C).

From Fig. 1, Eq. (1) can be rewritten as

$$COP = \frac{\dot{Q}_{in}}{\dot{W}} \leq (COP)_C = (\frac{T_H}{T_C} - 1)^{-1}. \tag{2}$$

Note that the equal condition in the $\leq$ sign in Eq. 2 is applicable only in the case of an ideal refrigeration cycle, in which all of the thermodynamic processes are assumed to be reversible.

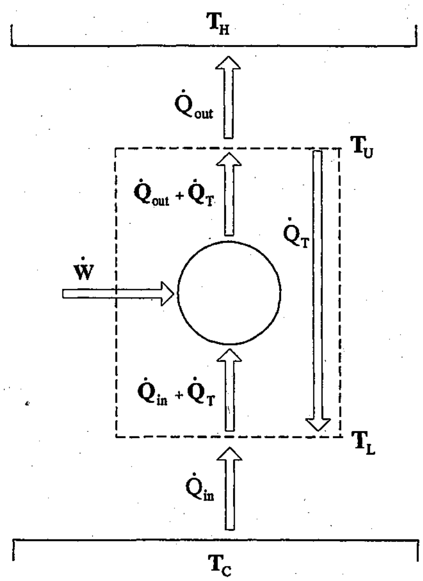

Fig. 1 Model of realistic refrigeration plant

However, in an actual refrigeration plant, heat transfer occurs under a finite temperature difference. The lowest temperature of the refrigerator's working fluid, T_L, is lower than the temperature of the refrigerated space, T_C. The relationship between the rate of the resulting heat transfer into the cold fluid and the temperature difference between the fluid and the refrigerated space is expressed by

$$\dot{Q}_{in} = U_C A_C (T_C - T_L). \tag{3}$$

Similarly, the highest temperature of the working fluid, T_U, is greater than the ambient temperature, T_H. The rate of the resulting heat dissipation from the hot fluid to the ambient surroundings is proportional to the local temperature difference and is given by

$$\dot{Q}_{out} = U_H A_H (T_U - T_H). \tag{4}$$

Since $T_U > T_H$ and $T_C > T_L$, the processes which take place as the refrigeration plant absorbs and dissipates heat are no longer reversible. Equations (3) and (4) describe the heat transfer rates across the refrigerator's cold and hot boundaries, respectively. Hence, these equations essentially represent the refrigerator's external irreversibilities.

When external irreversibilities are taken into account, the hot and cold end working fluid temperatures are denoted as T_U and T_L, respectively, and from an inspection of the dashed lines in Fig. 1, it can be shown that the *COP* of the refrigeration plant should be written in the form

$$COP = \frac{\dot{Q}_{in}}{\dot{W}} \le \left(\frac{T_U}{T_L} - 1\right)^{-1}. \tag{5}$$

Note that the equal condition in the $\le$ sign in Eq. (5) corresponds to the case where the cycle executed by the working fluid is assumed to be an internally reversible (i.e. endoreversible) cycle.

However, in an actual refrigeration plant, the losses caused by friction and various heat leakages within the system render this endoreversible assumption invalid. These losses are inevitable and act as a heat by-pass, i.e. this heat wastes the refrigerator's work and therefore reduces the *COP*.

In this study, the overall heat losses are denoted by $\dot{Q}_T$, where

$$\dot{Q}_T = U_T A_T (T_U - T_L). \tag{6}$$

From Fig.1, it can be seen that $\dot{Q}_{in} + \dot{Q}_T$ corresponds to the amount of heat actually absorbed by the refrigeration plant. However, the net cooling rate is only $\dot{Q}_L$. Therefore, the inner *COP* of the refrigeration plant is given by

$$\frac{\dot{Q}_{in} + \dot{Q}_T}{\dot{W}} = \left(\frac{T_U}{T_L} - 1\right)^{-1}. \tag{7}$$

Equation (7) implies that $\dot{Q}_T$ represents the overall internal heat gain. In the particular case where $\dot{Q}_T = 0$, the cycle is internally reversible and is the same as that reported by Bejan [2,3].

The input power ($\dot{W}$) required for a refrigeration plant is directly related to the absorption heat transfer rate ($\dot{Q}_{in}$) and the dissipation heat transfer rate ($\dot{Q}_{out}$). Equations (3) and (4) represent the fundamental heat transfer models for the actual heat exchangers at the cold and hot ends of the refrigeration plant, respectively. In these equations, the heat transfer rates are proportional to the corresponding temperature differences and thermal conductances, i.e. $U_C A_C$ and $U_H A_H$, where A is the area of the respective heat transfer surface and U is the overall heat transfer coefficient based on A.

As described in the Introduction, the cost of a refrigeration plant is a major design constraint. Furthermore, increasing the size of the condenser and evaporator units in the heat exchangers in order to achieve a higher thermal conductance is very expensive. As a consequence, the process of designing a practical refrigeration plant generally involves satisfying a set of cost and size objectives subject to the constraint of a given total thermal conductance. In other words, the sum of $U_C A_C$ and $U_H A_H$ can be regarded as a constant, i.e. the total size of the heat exchanger surface is fixed. This constraint can be expressed as

$$U_H A_H + U_C A_C = UA = Constant. \tag{8}$$

The ratio of the hot-end conductance to the total conductance is denoted by X and is given by

$$X = \frac{U_H A_H}{UA}.$$

Therefore, from Eq. (8), it can be shown that

$$1 - X = \frac{U_C A_C}{UA}. \tag{9}$$

Regarding the constraint given in Eq. (8), a decision must be made regarding the optimal allocation of the constant thermal conductance between $U_H A_H$ and $U_C A_C$ such that the *COP* of the refrigeration plant is maximized.

The internal irreversibility, ξ, of a realistic refrigeration plant is defined as the ratio of the overall internal heat loss, $\dot{Q}_T$, to the heat actually absorbed by the refrigerator's working fluid, $\dot{Q}_{in}$, i.e.

$$\xi = \frac{\dot{Q}_T}{\dot{Q}_{in}} \geq 0$$

or

$$Q_T = \xi Q_{in} . \tag{10}$$

Equation (7) can then be rewritten as

$$\frac{\dot{Q}_{in}}{\dot{W}} = \left((\xi+1)\left(\frac{T_U}{T_L} - 1 \right) \right)^{-1} . \tag{11}$$

From Eqs.(3), (4) and (9), Eq.(11) can be expressed as

$$\frac{\dot{Q}_{in}}{\dot{W}} = \left[(\xi+1)\left(\frac{T_H + \dot{Q}_{out} /XUA}{T_C - \dot{Q}_{in} /(1-X)UA} - 1 \right) \right]^{-1} . \tag{12}$$

Since $\dot{Q}_{out} = \dot{Q}_{in} + \dot{W}$, replacing $\dot{Q}_{out}$ by $\dot{Q}_{in} + \dot{W}$, Eq.(12) becomes

$$COP = \frac{\dot{Q}_{in}}{\dot{W}} = \frac{T_C\left(X - X^2\right)UA + \dot{Q}_{in}(X\xi - \xi)}{(\xi+1)\left[(T_H - T_C)\left(X - X^2\right)UA + \dot{Q}_{in} \right]} . \tag{13}$$

Since UA is assumed to be constant, under a fixed ambient temperature (T_H) and a fixed refrigeration space temperature (T_C), the heat absorbed per unit work input varies as a function of ξ and X only. The internal irreversibility, ξ, is generally dependent on the structure of the refrigeration plant and its value is relatively constant for a plant operating under fixed T_H and T_C.

The optimal *COP* of an operating refrigeration plant can be obtained by taking the derivative of Eq.(13) with respect to X and setting it equal to zero, i.e.

$$\left(1 - \frac{T_C}{T_H}\right)\left[-\xi X_{opt}^2 + (\xi+1)\left(2X_{opt} - 1\right)\right] - \frac{T_C}{T_H}\left(1 - 2X_{opt}\right) - \frac{\dot{Q}_{in}}{UAT_H}\xi = 0 . \tag{14}$$

From Eq.(14), the optimal heat transfer allocation ratio, X_{opt}, can be derived by specifying appropriate values for the parameters $\dot{Q}_{in} /UAT_H$, T_C/T_H, and the internal irreversibility, ξ.

Substituting X_{opt} from Eq.(14) into Eq.(13), the corresponding optimal *COP* is given by

$$COP_{opt} = \frac{\frac{T_C}{T_H}\left(X_{opt} - X_{opt}^2\right) + \frac{\dot{Q}_{in}}{UAT_H}\left(X_{opt}\xi - \xi\right)}{(\xi+1)\left[\left(1-\frac{T_C}{T_H}\right)\left(X_{opt} - X_{opt}^2\right) + \frac{\dot{Q}_{in}}{UAT_H}\right]}. \tag{15}$$

RESULTS AND DISCUSSIONS

Equation (14) yields the optimal thermal conductance ratio X_{opt} for a realistic refrigeration plant. For the case of an internally reversible refrigeration plant (when ξ=0 or $\dot{Q}_T = 0$, also referred to as an endoreversible cycle), Eq.(14) gives X_{opt}=0.5, which is consistent with the optimal thermal conductance ratio obtained by Bejan [2,3] for an endoreversible refrigeration plant.

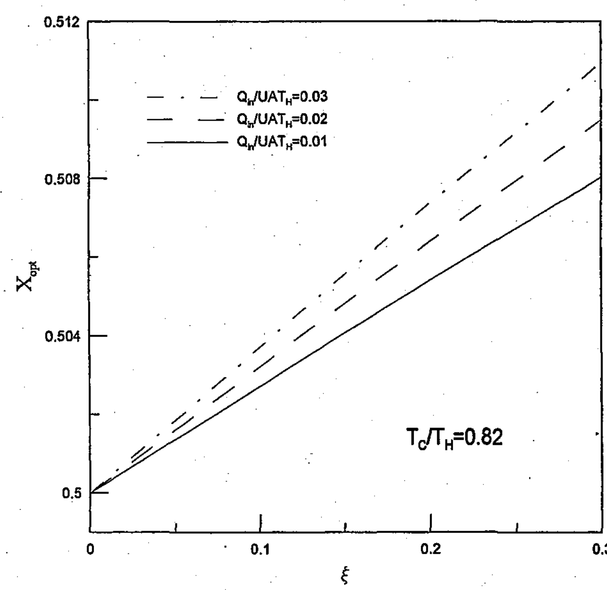

Fig. 2 Optimal allocation of thermal conductance for typical freezing system.

For a general refrigeration system, the value of T_C/T_H is approximately 0.82 (i.e. $T_C = -20°C$, $T_H = 35°C$) for a typical freezing system, and approximately 0.93 (i.e. $T_C = 12°C$, $T_H = 35°C$) for a typical air-conditioning system. Figures 2 and 3 plot X_{opt} against ξ for a typical freezing system and a typical air-conditioning system, respectively. Note that three different values of $\dot{Q}_{in}/UAT_H$ are plotted in each figure for comparison purposes. Many results and observations can be drawn from these two figures. Firstly, a larger value of $\dot{Q}_{in}/UAT_H$ induces a larger X_{opt}, which implies that the thermal conductance of the hot end should be increases if the refrigeration plant is designed with a larger value of $\dot{Q}_{in}/UAT_H$. Secondly, a larger value of T_C/T_H results in a smaller value of X_{opt}. Therefore, the thermal conductance of the hot end should be decreases if a higher cold end temperature is specified. Thirdly, the X_{opt} values are all slightly larger than 0.5 when $\xi > 0$. This indicates that the thermal conductance of the hot end should be slightly larger than that of the cold end in a realistic refrigeration plant. This result confirms the appropriateness of the design value $U_HA_H=U_CA_C$ commonly used in many application areas.

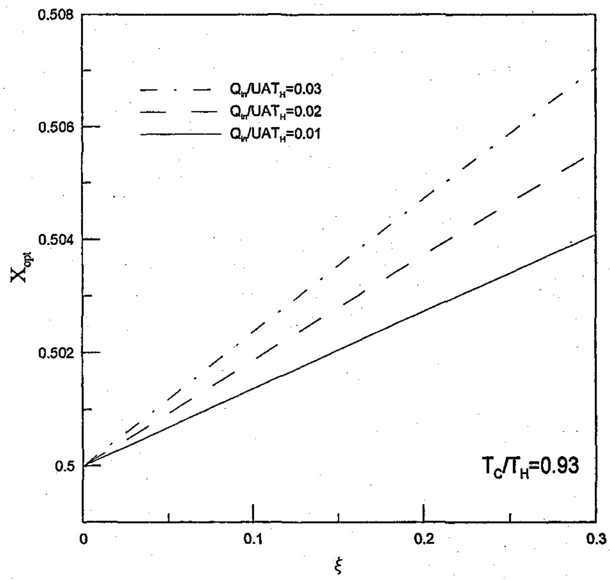

Fig. 3 Optimal allocation of thermal conductance for typical air-conditioning system.

Figure 4 plots the relationship between COP_{opt} and ξ for a typical freezing system. Again, three values of $\dot{Q}_{in}/UAT_H$ are plotted for comparison purposes. For the case of $\dot{Q}_{in}/UAT_H = 0.01$, the COP_{opt} of an internal reversible cycle (i.e. $\xi = 0$) is 3.73, but reduces to 2.85 when $\xi = 0.3$. Both of these values are lower than that of $(COP)_C$ (i.e. 4.60). Figure 5 plots COP_{opt} against ξ for a typical air-conditioning system. It can be seen that COP_{opt} decreases from a value of 8.45 (when $\xi = 0$) to 6.46 ($\xi = 0.3$) for the case of $\dot{Q}_{in}/UAT_H = 0.01$. Again, both values are lower than that of $(COP)_C$ (i.e. 12.40).

Figures 4 and 5 show that a larger value of $\dot{Q}_{in}/UAT_H$ induces a lower value of COP_{opt}. For a given refrigeration system (i.e. UA, T_H and T_C are all assumed to be constant), the lowest temperature of the refrigerator's working fluid, T_L, should be reduced in order to absorb a greater $\dot{Q}_{in}$, while the highest temperature of the working fluid, T_U, should be increased in order to dissipate a greater $\dot{Q}_{out}$. As a result, the corresponding value of COP_{opt}

will be reduced. Conversely, the figures show that a higher value of COP_{opt} can be obtained by specifying a higher T_C/T_H ratio.

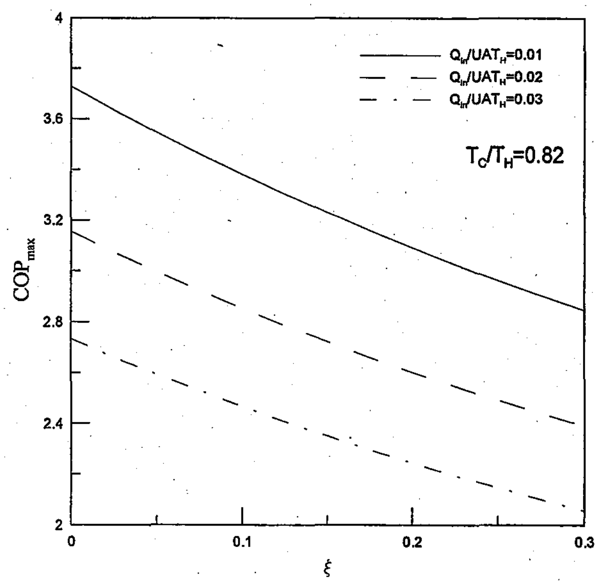

Fig. 4 Optimal coefficient of performance for typical freezing system.

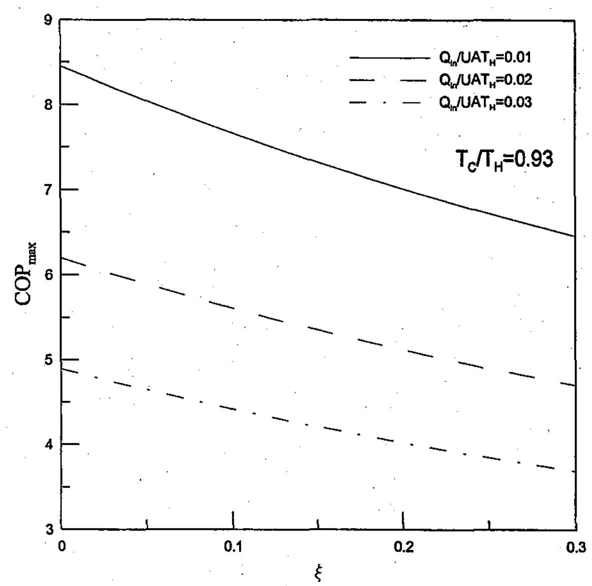

Fig. 5 Optimal coefficient of performance for typical air-conditioning system.

CONCLUSION

Using a heat by-pass model to represent internal irreversibilities, this study has derived general formulations for the optimized thermal conductance allocation and corresponding optimal coefficient of performance (*COP*) of a realistic refrigeration plant with a fixed total thermal conductance. The numerical results have indicated that the optimal thermal conductance ratio, X_{opt}, at the hot end is slightly higher than 0.5. This result verifies the design value of $U_H A_H = U_C A_C$ commonly used in many application areas. For the particular case of $\xi = 0$, the value of X_{opt} is found to be 0.5, which is consistent with the result presented by Bejan for an endoreversible refrigeration plant. Finally, the results have shown that the degree of internal irreversibility penalizes the plant's *COP*.

ACKNOWLEDGEMENT

This study was partially supported by the National Science Council of Taiwan under contract NSC95-2221-E-218-056.

REFERENCE

[1] Curzon, F. L., and Ahlborn, B., " Efficiency of a Carnot Engine at Mavimum Power Output," Amer. J. Phys, Vol.43, No.1 (1975), pp.22-24.

[2] Beian, A., "Theory of Heat Transfer-Irreversible Refrigeration Plants," Int. J. Heat Mass Transfer, Vol.32 (1989) , pp.1631-1639.

[3] Bejan, A., Tsatsaronis, G., and Moran, M., *Thermal Design & Optimization*, John Wiley & Sons (1996) , pp.319-320.

[4] Hsieh, Y. C. and Chiou, J. S., " Optimal Allocation of Heat Transfer Area for A Heat Engine," J. of Mechanics, Vol.21, No.1 (2005) , pp.1-4.

[5] Chang, T. B., " Exergetic Efficiency Optimization for an Irreversible Carnot Heat Engine", J. of Mechanics, Vol.23, No.2, (2007), pp.181-186.

[6] Wu, C., Chen, L. and Sun, F., "Optimization of Steady Flow Refrigeration Cycles," Int. J. Ambient Energy, Vol.17 (1996) , pp.199-206

[7] Chen, L., Wu, C. and Sun, F., "Finite Time Thermodynamic Performance of an Isentropic Closed Regenerated Brayton Refrigeration Cycle," Int. J. Energy Environment Economics, Vol.4 (1997) , pp.261-274.

[8] Chen, L., Sun, F., Wu, C. and Kiang, R. L., " A Generalized Model of a Real Refrigerator and Its Performance," Applied Thermal Engineering, Vol.17, No. 4 (1997) , pp.401-412.

[9] Chen, C. K. and Su, Y. F.," Exergetic Efficiency Optimization for an Irreversible Brayton Refrigeration Cycle", Int. J. of Thermal Sciences, Vol.44 (2005) , pp.303-310.

[10] Chang, T. B., "Internal Irreversibility Analysis and Optimization of a Refrigeration Plant," JSME International Journal Series B: Fluids and Thermal Engineering, Vol.49, No.4 (2006) , pp.1260-1265.

NOMENCLATURE

A	total heat transfer area $A=A_H+A_C$
A_H	heat transfer area of refrigerator's hot end
A_C	heat transfer area of refrigerator's cold end
COP	coefficient of performance
COP_{opt}	optimal coefficient of performance
$\dot{Q}_{in}$	heat transfer rate to refrigeration plant
$\dot{Q}_{out}$	heat rejection rate from refrigeration plant
$\dot{Q}_T$	overall internal heat loss
T_C	temperature of refrigerated space
T_H	temperature of ambient surroundings
T_L	lowest temperature of refrigerator's working fluid
T_U	highest temperature of refrigerator's working fluid
U	total heat transfer coefficient based on A
U_H	total heat transfer coefficient based on A_H
U_C	total heat transfer coefficient based on A_C
U_T	total heat transfer coefficient based on A_T
$\dot{W}$	input power of refrigeration plant
X	ratio of hot-end conductance to total conductance
X_{opt}	optimal value of X

Greek symbols

ξ	internal irreversibility $\dot{Q}_T/\dot{Q}_{in}$