

ON THE KINEMATIC SYNTHESIS OF JET PROPULSION MECHANISMS FOR BIONIC JELLYFISHES WITH 6 LINKS

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ABSTRACT

The purpose of this paper is to present a systematic approach for the kinematic synthesis of the jet propulsion mechanisms with 6-link for bionic jellyfishes. The structural synthesis is carried out based on the methodology of creative mechanism design, and the dimensional synthesis is performed by the loop closure method. First, the design requirements and design constraints are specified based on the kinematics characteristic of jellyfishes. Then, the joint assortments with respect to the (6, 7) and (6, 8) chains are derived by number synthesis, and the atlas of feasible chain with one degrees of freedom is generated. After that, the atlas of specialized feasible chain is obtained. Furthermore, the atlas of the new design is obtained through the particularization process. Finally, a design example selected from the atlas is given for illustration, dimensional synthesis is performed at first, and the feasibility of the design is verified by conducting computer simulation using ADAMS software. The result shows that the proposed design can effectively mimic the contraction and relaxation motion of a jellyfish.

SUR LA SYNTHÈSE CINÉMATIQUE DES MÉCANISMES DE PROPULSION PAR RÉACTION POUR LES MÉDUSES BIONIQUES AVEC 6 LIENS

RÉSUMÉ

Le but de ce document est de présenter une approche systématique pour la synthèse cinématique des mécanismes de propulsion par réaction avec le lien 6 pour les méduses bionique. La synthèse structurale est effectuée à base sur la méthodologie de la conception créatrice de mécanisme, et la synthèse dimensionnelle est exécutée par la méthode de fermeture de boucle. D'abord, les conditions de conception et les contraintes de conception sont spécifiées ont basé sur la cinématique caractéristique des méduses. Puis, les assortiments communs en ce qui concerne (6, 7) et (6, 8) des chaînes sont dérivés par la synthèse de nombre, et l'atlas de la chaîne faisable avec un degrés de liberté est produit. Après ce, l'atlas de la chaîne faisable spécialisée est obtenu. En outre, l'atlas de la nouvelle conception est obtenu par le processus de particularisation. En conclusion, un exemple de conception choisi parmi l'atlas est donné pour l'illustration, la synthèse dimensionnelle est exécutée au début, et la praticabilité de la conception est vérifiée en conduisant la simulation sur ordinateur using le logiciel d'ADAMS. Le résultat prouve que la conception proposée peut effectivement imiter le mouvement de contraction et de relaxation d'une méduse.

INTRODUCTION

Since the ancient time till now, people always have dreams to soar in the sky like a bird, and to swim in the ocean like a fish. Therefore, various aircrafts and ships were invented by the stimulation of the dreams. However their efficiencies and performances are far behind the natural creature. For example, although the most widely used way of transportation on the water is by using a propeller, its efficiency of propulsion is not good enough in underwater due to the shielding by the body of the vehicle.

Due to the drag of fluid, underwater propulsion is a transportation of energy consuming. The way of underwater swimming can be classified as body casual fin (BCF), median paired fin (MPF), and jet propulsion. A jellyfish swims by jet propulsion. The jellyfish compels water out of its medusa by quick contracting of its muscle, and it produces a water jet to propel the jellyfish. Then, the strain energy stored in the muscle of the medusa is released, the muscle is relaxed and water flows back into the medusa. Among different types of jet propulsions, the propulsion for the jellyfish is the simplest; therefore it might be the best way to explore jet propulsion by investigating the motion of a jellyfish.

As far as the studies of biomechanics for jellyfishes are concerned, in 1983, Daniel [1] established a mechanics model of jet propulsion for jellyfishes, and by way of the theoretical analysis, it was found the propulsion of a jellyfish is highly unsteady, and the acceleration reaction, not drag, is the dominant fluid force for the swimming of a jellyfish. In 2003, Matthew [2] studied the effect of the change in size, behavior, and shape on the hydrodynamics of jet propulsion in the jellyfish *Aurelia aurita*. About the related researches on artificial jellyfish, Takara Company developed a series of jellyfish toys [3-6]. They used electromagnets to create magnetic fields and actuate the jellyfish to swim in the water. Takaya [7] of Takara Company attempted firstly to design a jellyfish toy using a mechanism, as shown in Fig. 1. Its thrust force is mainly produced by the propeller, not the rocker. Not only its motion does not mimic a jellyfish, but also it uses both a DC motor and a gear reducer to operate the toy, its energy efficiency can be vastly improved. From the literature reviewed above, there are only some investigations on biological mechanics of jellyfishes, no bibliography on bionic jellyfishes has been found. Moreover, a bionic jellyfish can be applied to the exploration of underwater resource, the use of military, and the amusement of aquariums.

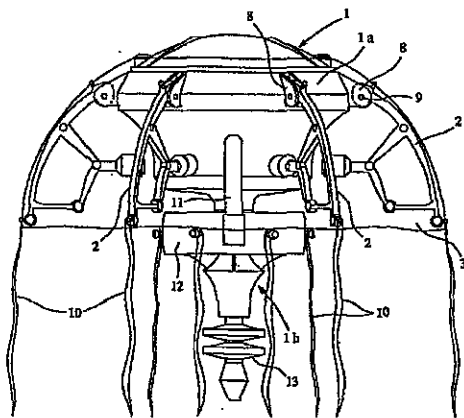


Fig. 1 A jellyfish toy [7]

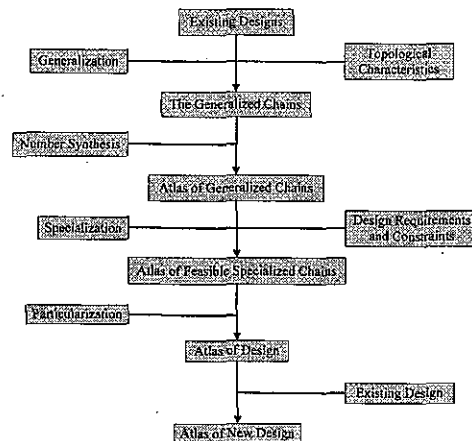


Fig. 2 Creative mechanism design methodology

CREATIVE MECHANISM DESIGN METHODOLOGY

Fig. 2 shows the flow chart of the creative design Mechanism methodology [8-10] for creating new designs. The approach consists of six steps:

Existing Designs (or called original designs)

To search existing design models or study an available new design model with required specifications, and to establish the topological structure of these models are the first step of the methodology. The goal of this step is to select some of these models for researching their equivalent mechanism skeletons and kinematic chains for developing the new designs.

Generalization

The above original designs are transformed individually into their corresponding generalized chains (kinematic chains). The generalized chain will be involved in various types of members (edges) and joints (vertices, or said kinematic pairs) for all possible assembly in the following steps.

Number Synthesis

The atlas of generalized chains and kinematic chains, respectively, with the required numbers of links and joints, are synthesized for obtaining all possible generalized chains that have the same number of links (N_L) and joints (N_J) as the original generalized chain.

Specialization

Specialization is to assign specific types of members and joints to every available generalized chain subject to certain design requirements to obtain the specialized chains. Among, design requirements are determined based on the concluded topological structure of the original designs.

Particularization

Particularization is the reverse process of generalization. Once a feasible specialized chain is obtained, it is particularized into its corresponding mechanical device in a skeleton drawing.

Atlas of New Designs

Every feasible specialized chain is particularized into its corresponding mechanical device in a skeleton drawing. Therefore, the last step is to identify all non-existing designs from the atlas of designs as the new designs.

EXISTING MECHANISM

The only existing mechanism found in the literature is the jellyfish toy, shown in Fig. 1. It is an open loop mechanism with 2 links, one is the frame, and the other is the output link (the rocker). It is driven by a propeller, but not the rocker, therefore it is not adopted as an existing mechanism. The design requirements and design constraints are specified based on the characteristics of the jellyfish joy and a real jellyfish. It is well known that the output motion of a four-bar linkage is simple; therefore a mechanism with six links will be used. Considering the difficulty in dimensional synthesis, a planar mechanism will be preferably synthesized. In addition, to limit the number of new designs, both the input link and the output link are limited to be ternary links.

Design Requirements

- (1) It is a planar mechanism with six links.
- (2) It has one degree of freedom; therefore it has an input link.
- (3) It has an output link with oscillating motion.
- (4) It has a frame (ground link) to support or constrain other links.

Design Constraints

- (1) The input link must be a ternary link and adjacent to the frame.
- (2) The output link must be a ternary link and adjacent to the frame.
- (3) All the joints incident to the output link must be revolute pairs.
- (4) The input link, the output link, and the frame, must be assigned on different link and in different loops.
- (5) A binary link must be adjacent to the output link with a revolute pair, and to the frame with a sliding pair.
- (6) For positive drive, rolling pairs must not be used.
- (7) For simplicity, gear pairs must not be applied.

GENERALIZATION AND NUMBER SYNTHESIS

Since there is no suitable existing mechanism, no generalization is necessary. The modified Kutzbach-Grübler criteria [11] for the degree of freedom of a planar mechanism can be expressed as:

$$F = 3(N-1) - 2J_1 - J_2 \quad (1)$$

where F is the number of freedoms, N is the number of the link, and J_i is the number of the joint with i ($i=1-2$) degree of freedoms. Moreover, the number of the joints in the mechanism is

$$J = J_1 + J_2 \quad (2)$$

According to design requirements (1) and (2), the mechanism has one degree of freedom and six links. Substituting them into Eqs. (1) and (2), then it yields J equals to 7 or 8. A (N, J) chain is used to denote a kinematic chain with N links and J joints, therefore, $(6, 7)$ and $(6, 8)$ chains can be obtained. In addition, based on the theory of number synthesis, there are three $(6, 7)$ and nine $(6, 8)$ chain types [12], as shown in Fig. 3.

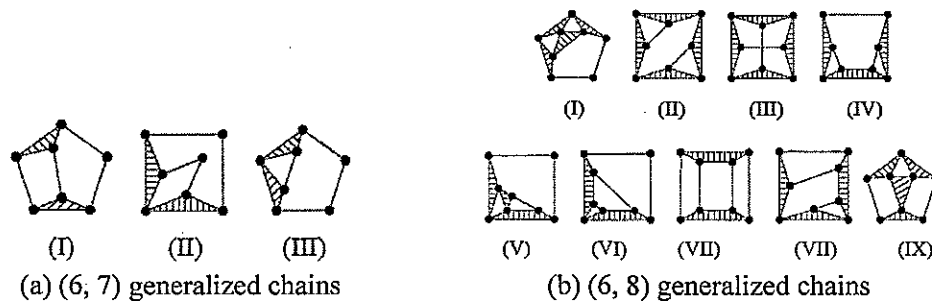


Fig. 3 Atlas of generalized chains with 6 links

SPECIALIZATION

The objective of specialization is to obtain the whole atlas of specialized chains by

assigning various types of members and joints to each available generalized chain subject to the design requirements and design constraints specified above. Here, (6, 7) and (6, 8) chains are selected as examples for illustrating the detailed process of specialization. According to these design requirements, the specialized members and joints include:

STEP 1: Input Link (member 1)

According to design requirement (2) and constraint (1), respectively, there is an input link, and it must be a ternary link. The results are shown in step (i) of Fig. 4 and Fig. 5, obtained by assigning the input link to Fig. 3(a) and Fig. 3(b), respectively.

STEP 2: Output Link (member 2)

Based on design requirement (3) and constraint (2), there is an output link, and it must be a ternary link. In addition, the input link, output link, as well as ground link, must be assigned on different link, and a binary link must adjacent to it, according to design constraints (4) and (5), respectively, the kinematic chains, as shown in step (ii) of Fig. 4 and Fig. 5, are generated by assigning the output link to those in step (i) of Fig. 4 and Fig. 5, respectively.

STEP 3: Frame (member 3)

Since there must be a frame as indicated in design requirement (4), and the frame must be adjacent to both the input link and the output link, as stated in design constraints (1) and (2), respectively, the chains, as shown in step (iii) of Fig. 4 and Fig. 5, meet the above requirement and constraints.

STEP 4: Joints

Based on design constraint (3), all the joints incident to the output link are revolute pairs. Moreover, there are neither cam pairs nor gear pairs according to design constraints (6) and (7). In addition, there is a sliding pair incident to the frame based on design constraints (8). The feasible specialized chains, 4 for (6, 7) chain and 40 for (6, 8) chain are shown in step (iv) of Fig. 4 and 5, generated by assigned the joints.

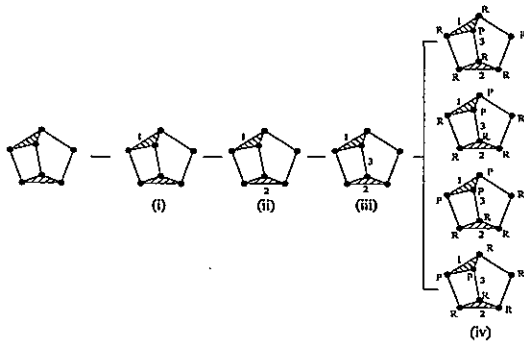


Fig. 4 Specialization - (6, 7) - (I) chain

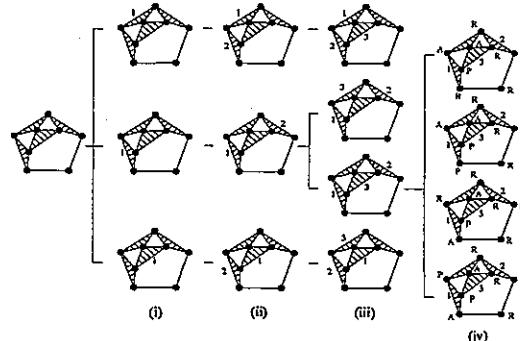


Fig. 5 Specialization - (6, 8) - (I) chain

PARTICULARIZATION

The next step of the creative design methodology is to particularize each feasible specialized chain, by applying the generalizing rules backwards to obtain the corresponding schematic diagram. Since no existing mechanism qualifies, all the mechanisms in the feasible specialized chain are new designs (feasible mechanisms), and shown in Table 1 and 2.

Table 1 Atlas of feasible specialized chain and new design - (6, 7) chain

6-1-1	6-1-2	6-1-3	6-1-4

Table 2 Atlas of feasible specialized chain and new design - (6, 8) chain

6-2-1	6-2-2	6-2-3	6-2-4	6-3-1	6-3-2	6-3-3
6-3-4	6-3-5	6-4-1	6-4-2	6-4-3	6-4-4	6-4-5
6-5-1	6-5-2	6-5-3	6-5-4	6-5-5	6-5-6	6-5-7
6-5-8	6-5-9	6-6-1	6-6-2	6-6-3	6-6-4	6-6-5

Table 2 Atlas of feasible specialized chain and new design - (6, 8) chain (continued)

6-6-6	6-6-7	6-6-8	6-6-9	6-6-10	6-6-11	6-6-12
6-6-8	6-6-9	6-6-10	6-6-11	6-6-12	6-6-13	6-6-14
6-6-15	6-6-16	6-6-17				

DESIGN EXAMPLE

After the atlas of new designs has been obtained, a detailed design can be carried out by selecting one from the atlas. Here, Mechanism 6-1-2 in Table 1 is selected as a design example, and a linear actuator is selected as the power input. Then, dimensional synthesis is performed to mimic the motion of a jellyfish. Finally, computer simulation is conducted to verify the design. The detailed process for the design is summarized as follows:

Motion Requirement

The motions in a cycle of a jellyfish are contraction, dwell, relaxation, and dwell, respectively. Table 3 indicates the duration time for each motion in a cycle [13]. Fig. 6(a) and (b) show a jellyfish in fully contracted and fully relaxed positions. It can be easily found the part of the head remains unchanged. Therefore the umbrella is divided into a fixed part OA and a moveable part AB as shown in Fig. 7. Furthermore $\theta = \angle B_0AB$ is defined as the output angular displacement of a jellyfish. In addition, it is assumed that the change of volume $V(t)$ inside the medusa is linearly in proportional to the output angular displacement $\alpha(t)$, and can be expressed as

$$\frac{V(t) - V_{\min}}{V_{\max} - V_{\min}} = \frac{\theta(t) - \theta_{\min}}{\theta_{\max} - \theta_{\min}} \quad (3)$$

where the subscript max and min denote the volume and the output angular displacement in fully

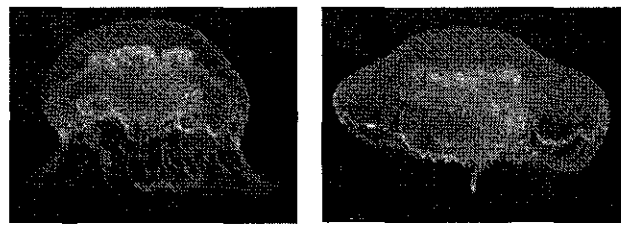
relaxed position and fully contracted position, respectively. Table 3 indicates the duration time for each motion in a cycle, and Fig. 8(a) shows the volumetric change (taken from [13]) inside the umbrella of a jellyfish, it can be found from the figure that V_{max} and V_{min} are about 4.04 cm^3 and 1.7 cm^3 , respectively. Moreover, it is assumed that θ_{max} and θ_{min} are 66° and 30° , respectively. Then, the corresponding change of the output angular displacement can be obtained by substituting the data points in Fig. 8(a) into Eq. (3), and is shown in Fig. 8(b). Therefore, the motion requirement of the design is to pass through the specified output angles, it is a problem of function generation with prescribed timing.

Dimensional Synthesis

The mechanism 6-1-2 in Table 1 is selected as the design example for dimensional synthesis. It is a 6-link mechanism with 5 revolute pairs and 2 sliding pair, and its dimensional synthesis is performed by using the vector loops method [14-16].

Table 3 Motion duration of a cycle

Time(sec)	Motion
0~0.1	Contraction
0.1~0.14	Dwell
0.14~0.37	Relaxation
0.37~0.4	Dwell



(a) Full contraction (b) Full relaxation

Fig. 6 Motion of a jellyfish

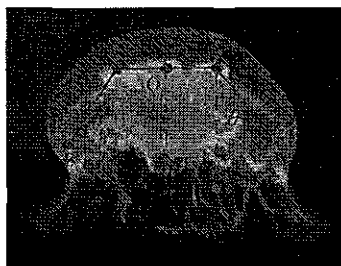
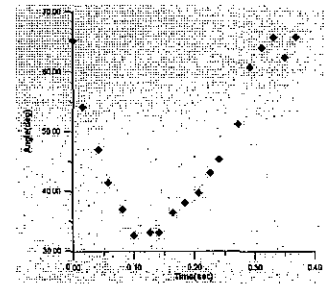
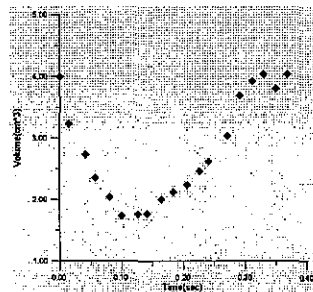


Fig. 7 Structure of a jellyfish



(a) Volumetric change [11] (b) Output angular displacement

Fig. 8 Motion requirement

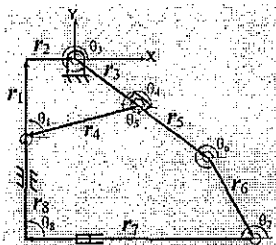


Fig. 9 Vector loops

Table 4 Accuracy points

No. of accuracy points	1	2	3	4	5	6
No. of equations	2	4	6	8	10	12
No. of unknowns	7	8	9	10	11	12
Free choice of design parameters	5	4	3	2	1	0

First, let \vec{r}_i represents the vectors shown in Fig. 9, the two vector loops of the mechanism are

$$\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4 = 0 \quad (4)$$

$$-\vec{r}_4 + \vec{r}_5 + \vec{r}_6 + \vec{r}_7 + \vec{r}_8 = 0 \quad (5)$$

Then, resolving Eq. (4) into its x and y parts, we have

$$r_1 \cos \theta_1 + r_2 \cos \theta_2 + r_3 \cos \theta_3 + r_4 \cos \theta_4 = 0 \quad (6)$$

$$r_1 \sin \theta_1 + r_2 \sin \theta_2 + r_3 \sin \theta_3 + r_4 \sin \theta_4 = 0 \quad (7)$$

Similarly, the x and y parts of Eq. (5) are

$$-r_4 \cos \theta_4 + r_5 \cos \theta_5 + r_6 \cos \theta_6 + r_7 \cos \theta_7 + r_8 \cos \theta_8 = 0 \quad (8)$$

$$-r_4 \sin \theta_4 + r_5 \sin \theta_5 + r_6 \sin \theta_6 + r_7 \sin \theta_7 + r_8 \sin \theta_8 = 0 \quad (9)$$

where r_i denotes the length of the vector \vec{r}_i , and θ_i represents the direction of \vec{r}_i , measured counterclockwise from the positive x axis. Based on these definitions, then it can be found that $\theta_1 = 90^\circ$, $\theta_2 = 0^\circ$, $\theta_7 = 180^\circ$, $\theta_8 = 90^\circ$, and $\theta_3 = \theta_5$ from Fig. 5. By substituting these into Eqs (6) - (9), one obtains

$$r_2 + r_3 \cos \theta_3 + r_4 \cos \theta_4 = 0 \quad (10)$$

$$r_1 + r_3 \sin \theta_3 + r_4 \sin \theta_4 = 0 \quad (11)$$

$$-r_4 \cos \theta_4 + r_5 \cos \theta_3 + r_6 \cos \theta_6 - r_7 = 0 \quad (12)$$

$$-r_4 \sin \theta_4 + r_5 \sin \theta_3 + r_6 \sin \theta_6 + r_8 = 0 \quad (13)$$

To eliminate the unknown variables θ_4 and r_7 , transforming θ_4 and r_7 to the right sides of Eqs. (10) - (11) and Eqs. (12) - (13), respectively, we have

$$r_2 + r_3 \cos \theta_3 = -r_4 \cos \theta_4 \quad (14)$$

$$r_1 + r_3 \sin \theta_3 = -r_4 \sin \theta_4 \quad (15)$$

$$r_5 \cos \theta_3 + r_6 \cos \theta_6 - r_7 = r_4 \cos \theta_4 \quad (16)$$

$$r_5 \sin \theta_3 + r_6 \sin \theta_6 + r_8 = r_4 \sin \theta_4 \quad (17)$$

By firstly squaring Eqs. (14) and (15), and then combining them based on the trigonometric identity $\cos^2 \theta + \sin^2 \theta = 1$, we obtain

$$2r_2r_3 \cos \theta_3 + 2r_1r_3 \sin \theta_3 + r_1^2 + r_2^2 + r_3^2 - r_4^2 = 0 \quad (18)$$

Similarly, Eqs. (16) and (17) can be combined into

$$r_5^2 + r_6^2 + r_7^2 + r_8^2 + 2r_5r_6 \cos(\theta_3 - \theta_6) - 2r_5r_7 \cos \theta_3 - 2r_6r_7 \cos \theta_6 + 2r_5r_8 \sin \theta_3 + 2r_6r_8 \sin \theta_6 = r_4^2 \quad (19)$$

Moreover, substituting Eq. (14) into Eq. (16) and simplifying, we have

$$r_7 = r_2 + r_3 \cos \theta_3 + r_5 \cos \theta_3 + r_6 \cos \theta_6 \quad (20)$$

Finally, substituting Eq. (14) and Eq. (20) into Eq. (19), and rearranging it, we obtain

$$\begin{aligned} & (r_3^2 - r_5^2) \cos 2\theta_3 - r_6^2 \cos 2\theta_6 + 4r_2r_3 \cos \theta_3 + 4r_5r_6 \sin \theta_3 \sin \theta_6 \\ & + 4r_5r_8 \sin \theta_3 + 4r_6r_8 \sin \theta_6 + 2r_2^2 + r_3^2 - 2r_4^2 + r_5^2 + r_6^2 + 2r_8^2 = 0 \end{aligned} \quad (21)$$

Eq. (18) and Eq. (21) are the loop closure equations for the dimensional synthesis of the design. It

can be found that there are seven unknowns including six unknown constants ($r_2, r_3, r_4, r_5, r_6,$ and r_8) and one unknown variable (θ_6). In addition, r_1 is the input variable and θ_3 is the output variable, and both are known. Therefore, we have 2 equations and 7 unknowns. In function generation, the function generated can only exactly pass a limited number of points, these points are called accuracy points. The number of accuracy points that can be synthesized are summarized in Table 4, and it can be found the maximum number of accuracy points is six.

Firstly, it is assumed that the relative displacement of θ_6 with respect to θ_3 is small, therefore the output angular displacement (volumetric change) of the design can be approximated by $\theta_6 = \theta_3 + 270^\circ$. Moreover, three accuracy points $(r_1, \theta_{31}) = (0.05, 330^\circ)$, $(r_1, \theta_{32}) = (1, 300^\circ)$, and $(r_1, \theta_{33}) = (0.1, 336^\circ)$ are selected from Fig. 8(b), then substituting into Eq. (18) and Eq. (21), we have six equations and nine unknowns, hence three free choices have to be specified. Since the displacements of θ_6 have to be approximately identical to that of θ_3 , Let $\theta_{61}=326^\circ$, $\theta_{62}=288^\circ$, and $\theta_{63}=326^\circ$, and then substituting three accuracy points and three free choices into Eq. (18) and Eq. (21), we have a system of six equations with six unknown parameter. Finally, by numerical methods, these parameters can be easily solved to be $r_2=1.28152$, $r_3=0.365228$, $r_4=1.6159$, $r_5=0.988508$, $r_6=0.60683$, and $r_8=0.743439$, respectively.

Simulation

Once all the kinematic dimensions are synthesized, the proposed solid model is established on a scale of 100 times by CATIA software. Then the model is introduced into ADAMS software for kinematic simulation. Fig. 10(a) and Fig. 10(b) show the fully contracted position and the fully relaxed position of the proposed design, respectively. Fig. 11 gives a comparison between the output angular displacement of the proposed design and that of the real jellyfish in Ref. [13], and it is apparent that they agree closely. Therefore the proposed design can effectively mimic the contraction and relaxation motion of a real jellyfish.

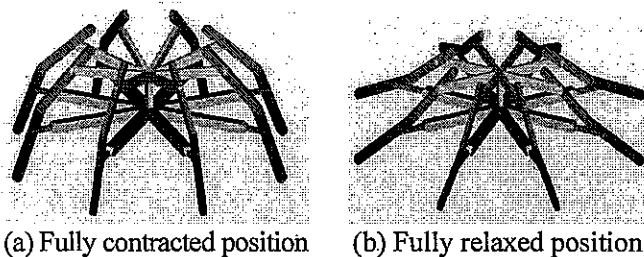


Fig. 10 Kinematic simulation

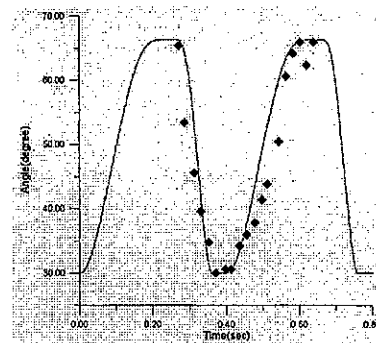


Fig. 11 Comparison

CONCLUSIONS

In this paper, the novel concept of designing a bionic jellyfish using a mechanism has been proposed for the first time. And the systematic approach for kinematic synthesis of the jet propulsion mechanism has been presented. Firstly, the design requirements and design constraints have been summarized based on the characteristics of the jellyfish toy and real jellyfishes. Then, specializing with respect to (6, 7) and (6, 8) generalizing chains, the atlas of specialized feasible

chain has been generated. Finally, the atlas of new designs has also been synthesized through the process of particularization. All of these are presented in a systematic methodology process. In addition, the feasibility of the design has been verified by dimensional synthesis and kinematic simulation through an illustrative example, selected from the atlas of new designs. The result shows that the proposed design can successfully imitate the contraction and relaxation motion of a jellyfish.

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REFERENCES

1. Daniel, T. L., "Mechanics and Energetics of Medusan Jet Propulsion," *Canadian Journal of Zoology*, Vol. 61, 1983, pp. 1406-1420.
2. Matthew, J. M. and Jason J., "The Ontogenetic Scaling of Hydrodynamics and Swimming Performance in Jellyfish (*Aurelia aurita*)," *The Journal of Experimental Biology*, Vol. 206, 2003, pp. 4125-4137.
3. Ikenaga, N., and Inomata, N. "Swimming Element Enjoying System," *US. Patent No. 6564484*, 2003.
4. Koltz, K. M., "Artificial Aquarium Having Magnetic and Water Pump Drive System," *US. Patent No. 6665964*, 2003.
5. Choh, J. and Chog, J., "Toy Robotic Jellyfish," *US. Patent No. D446830*, 2001.
6. Choh, J. and Chog, J., "Toy Robotic Jellyfish," *US. Patent No. D449083*, 2001.
7. Takaya, M., "Jellyfish Type Underwater Swimming Toy," *US. Patent No. 6422910*, 2002.
8. Yan, H. S., "A Methodology for Creative Mechanism Design," *Mechanism and Machine Theory*, Vol. 27, No. 3, 1992, pp.235-242.
9. Yan, H. S., *Creative Design of Mechanical Devices*, Springer-Verlag, Singapore, 1998.
10. Hsieh, W. H. and Chang, T. S., "Creative Design of Metal Can Crushers with Eight Links," *Materials Science Forum*, Vol. 505-507, 2006, pp. 925-930.
11. Grübler, M. "Allgemeine Eigenschaften der Zwangläufigen ebenen Kinematischen Ketten, Part II," *Verh. Ver. Bef. Gew.*, Vol. 64, 1885, pp. 179-223.
12. Mayourrian, M. and Freudenstein, F. "The Development of an Atlas of the Kinematic Structures of Mechanisms," *Transaction of ASME, Journal of mechanisms, Transmissions, and Automation in Design*, Vol. 106, 1984, pp. 458-461.
13. Daniel, T. L., "Cost of Locomotion: Unsteady Medusan Swimming," *The Journal of Experimental Biology*, Vol. 119, 1985, pp. 149-164.
14. Freudenstein, F., "An Analytical Approach to the Design of Four-Link Mechanisms," *Transactions of the ASME*, Vol. 76, 1954, pp. 483-92.
15. Shigley, J. E., *Kinematic Analysis of Mechanisms*, McGraw-Hill Book Company, New York, 1969.
16. Erdman, A. G. and Sandor, G. N., *Mechanism Design: Analysis and Synthesis*, 4th Edition, Prentice-Hall Inc., New York, 2001.