

# DESIGN OF AN ACTIVE SUSPENSION CONTROL FOR A VEHICLE MODEL USING A GENETIC ALGORITHM

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## ABSTRACT

This paper presents the design of an active suspension controller for an automotive vehicle using a genetic algorithm as the optimization technique. A four-degree-of-freedom model is used to represent a vehicle with different front and rear axes characteristics. The suspension deflection, tire deflection, vertical and angular acceleration are the performance criteria optimized. Different filters are used to model the frequency sensitivity of these criteria and the weighting is based on a passive suspension reference system. Independent front and rear controller optimization is performed with a genetic algorithm. The controllers include a linear gain matrix and a single filter. Each controller is designed to work with a minimum number of sensors and a limited order filter. To adapt the passive suspension components to the active system, the stiffness and damping of the suspension are optimized with values limited to a realistic range. Results show the impact of the various filters used to specify the critical frequency range of the inputs and outputs. This is observable for ride and handling criteria that are known to be frequency dependant. There is 38% improvement in the global performance of the active system compared to the baseline passive system.

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## CONCEPTION D'UN SYSTÈME DE CONTRÔLE DE SUSPENSION ACTIVE D'UN VÉHICULE EN UTILISANT L'ALGORITHME GÉNÉTIQUE

### RÉSUMÉ

Cet article présente la conception d'un système de contrôle de suspension active pour un véhicule en utilisant l'algorithme génétique comme technique d'optimisation. Un modèle de quatre-degré-de-liberté est employé pour représenter un véhicule avec les différentes caractéristiques des axes avant et arrière. Les critères de performances sont représentés par la déflexion de la suspension, la déflexion du pneu et l'accélération verticale et angulaire. Différents filtres sont utilisés pour modéliser la sensibilité de ces critères et les résultats obtenus sont comparés à un système de suspension passif de base. L'optimisation du contrôleur des suspensions avant et arrière est obtenue en utilisant l'algorithme génétique. Le contrôleur inclus un filtre simple et une matrice de gain linéaire. Chaque contrôleur est conçu pour fonctionner avec un nombre minimum de sondes et un filtre d'ordre réduit. Pour adapter les coefficients de la suspension passive à la suspension active, les coefficients de raideur et d'amortissement sont optimisés avec des valeurs limitées et réalistes. Les résultats montrent l'impact des divers filtres utilisés pour indiquer la gamme de fréquence critique des entrées et des sorties. Ceci est particulièrement observé pour les critères du comportement dynamique qui sont connus pour être liés à la fréquence. Nous avons remarqué une amélioration globale de 38% du système actif comparé au système passif de base.

## INTRODUCTION

Modern automotive vehicles exploit suspension systems to provide stability and passenger comfort. With the advances of electronics and active systems it is now possible to add active elements to improve suspension performance. In such systems an electronic controller uses different sensors to control the force applied by an actuator. In the present study, different control strategies are proposed to maximize the performance of an active suspension, each one having advantages and disadvantages.

The controller developed is inspired by the LQR and  $H_{\infty}$  control methods. The LQR requires the measurement of the entire state variable to multiply a gain matrix. If one or more variables are not measured, an optimal estimator has to be used. In either case, neither the frequency content of the inputs nor the sensitivity of the performance criteria is defined. On the other hand, the  $H_{\infty}$  method allows the user to model both the frequency content of the inputs and performance measurements. The order of the controller depends on the order of the model and the different filters added. This results in a high order control filter that cannot be reduced without compromising stability [1].

The goal of this work is to design a simple controller using a gain matrix for predefined state variables and a low order filter. A simple model with 4 degrees of freedom is used to model the suspension. Filters are added to represent more accurately the spectral density of the inputs, the delay in the force applied by the actuators and the frequency sensitivity of the different performance measurements. A limited number of easily measured variables are used with a gain matrix to calculate the required force. A filter adds a frequency shaping to the controller. A genetic algorithm (GA) is used to optimize the controller parameters. The flexibility of the GA eliminates a predefined controller structure, allowing the use of a limited number of gains with a simple filter of predefined order. The performance function can take any form as long as it is related to the variable to optimize [2, 3 and 4]. These two properties make the GA more efficient for the presented work.

## VEHICLE MODEL

The vehicle model presented in this work has four degrees of freedom (DOF) and is depicted in Figure 1. Sprung and unsprung masses are separated by the actuator, the linear spring and the damper. The front and rear actuators are force sources where the forces are calculated by the controller. The tire is modeled by a linear spring. The values of the parameters are given in Table 1, [5]. Note that the speed of the vehicle is assumed to be constant. The front and rear unsprung masses, suspension stiffness and damping have different values to reflect the difference in weight distribution.

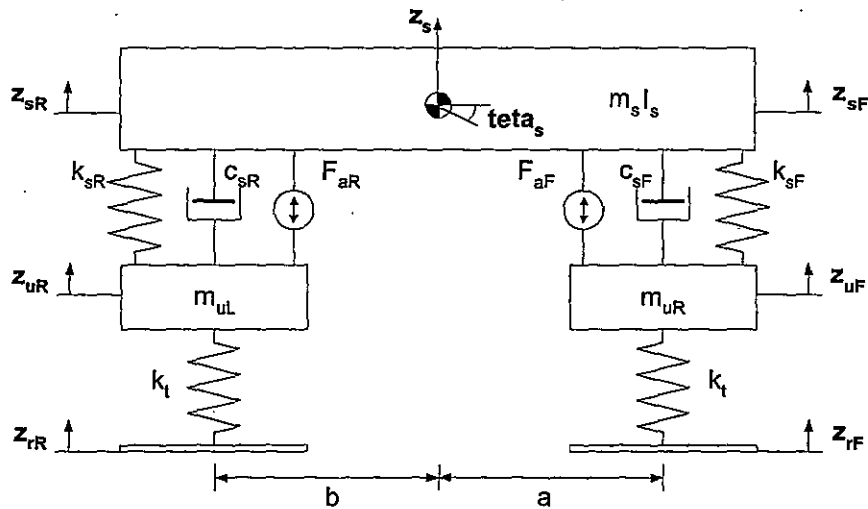


Figure 1: Vehicle model

Symbol	Signification	Value	Units
$m_s$	Sprung mass	2 040	kg
$I_s$	Sprung mass inertia	4 630	kg m <sup>2</sup>
$m_{uF}, m_{uR}$	Unsprung mass	44	kg
$k_{sF}$	Front suspension stiffness	13 000	N/m
$k_{sR}$	Rear suspension stiffness	25 000	N/m
$c_{sF}$	Front suspension damping	2 100	Ns/m
$c_{sR}$	Rear suspension damping	4 400	Ns/m
$k_{tR}, k_{tL}$	Tire stiffness	262 700	N/m
$a$	Distance front wheel to CG	2.03	m
$b$	Distance rear wheel to CG	1.01	m

Table 1: Model parameters

### CONTROLLER MODEL

In the present study, the model has four different inputs: front and rear road motion and actuator forces. The mathematical model uses the state space variable formulation to model the dynamics of the system. This model is defined by the following equations:

$$\dot{x} = Ax + B_{aF}u_{aF} + B_{aR}u_{aR} + L_{rF}z_{rF} + L_{rR}z_{rR} + L_d a_{long} \quad (1)$$

$$x = [z_{sR} - z_{uR} \quad z_{uR} - z_{rR} \quad \dot{z}_{uR} \quad z_{sF} - z_{uF} \quad z_{uF} - z_{rF} \quad \dot{z}_{uF} \quad \dot{z}_s \quad z_s \quad \dot{\theta}_s \quad \theta_s]^T \quad (2)$$

$$A = \begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & b & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{k_{sR}}{m_{uR}} & -\frac{k_t}{m_{uR}} & -\frac{c_{sR}}{m_{uR}} & 0 & 0 & 0 & \frac{c_{sR}}{m_{uR}} & 0 & \frac{bc_{sR}}{m_{uR}} & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & -a & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{k_{sF}}{m_{uF}} & -\frac{k_t}{m_{uF}} & -\frac{c_{sF}}{m_{uF}} & \frac{c_{sF}}{m_{uF}} & 0 & -\frac{ac_{sF}}{m_{uF}} & 0 \\ -\frac{k_{sR}}{m_s} & 0 & \frac{c_{sR}}{m_s} & -\frac{k_{sF}}{m_s} & 0 & \frac{c_{sF}}{m_s} & -\frac{c_{sF}}{m_s} - \frac{c_{sR}}{m_s} & 0 & \frac{ac_{sF} - bc_{sR}}{m_s} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ -\frac{bk_{sR}}{I_s} & 0 & \frac{bc_{sR}}{I_s} & \frac{ak_{sF}}{I_s} & 0 & -\frac{ac_{sF}}{I_s} & \frac{ac_{sF} - bc_{sR}}{I_s} & 0 & \frac{-a^2c_{sF} - b^2c_{sR}}{I_s} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad (3)$$

$$B_{aF} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \frac{-1}{m_{uF}} & \frac{1}{m_s} & 0 & \frac{-a}{I_s} & 0 \end{bmatrix}^T \quad (4)$$

$$B_{aR} = \begin{bmatrix} 0 & 0 & \frac{-1}{m_{uR}} & 0 & 0 & 0 & \frac{1}{m_s} & 0 & \frac{b}{I_s} & 0 \end{bmatrix}^T \quad (5)$$

$$L_{zF} = [0 \ 0 \ 0 \ 0 \ -1 \ 0 \ 0 \ 0 \ 0 \ 0]^T \quad (6)$$

$$L_{zR} = [0 \ -1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]^T \quad (7)$$

Note that the sprung mass position and angle are added to model the output of potential sensors. To increase the accuracy of our model, filters are added to model the frequency dependency of the inputs and outputs. The complete model is shown in Figure 2.

The dashed square represents the transfer function to be minimized. The road motion input  $z_R$  is assumed to be white noise and a filter  $W_{ZR}$  is used to reshape its frequency response. With this method, the model  $G(s)$  is excited with a frequency content close to reality. Each controller is composed of a different gain matrix  $K_i$  and filter  $W_{Ki}$ . A second filter is used to model the actuator dynamics, mainly the delay between the force required by the controller and the force applied by the actuator.

To evaluate the performance of a given controller, all the performance criteria are filtered according to the requirements of the designer. The filter shape is the opposite of the expected frequency response in order to increase the penalty in the most sensitive range. The mean square value of each criterion is weighted according to a reference model. The sum of these measurements represents the global performance to minimize.

The road input is known to have a more important spectral density in the low frequency range. A simple explanation is the higher amplitude of the longer wavelength bumps. Various studies have demonstrated that the power spectral density and the frequency content vary with the type of road: minor road, main road or highway [6, 7]. In accordance with these studies, a low pass filter  $W_z$  with a cutting frequency of 1Hz is used in this work. The output of this filter is directly applied at the front road vs. tire interface and a delay  $\Delta t$  is inserted before the application of the input at the rear wheel. Since the vehicle speed is constant, the delay is also constant and equals:

$$\Delta t = \frac{a+b}{V} \quad (8)$$

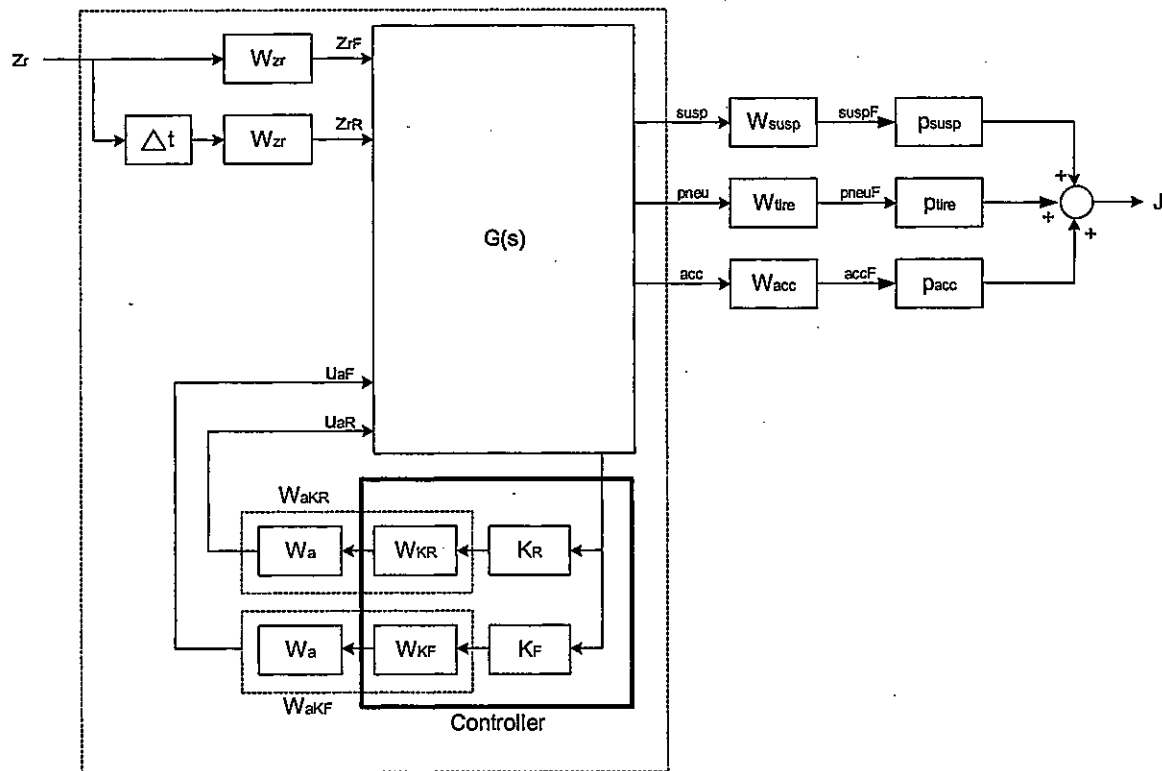


Figure 2: Complete model

A limiting factor in the design of most active suspensions is the maximum frequency response of the actuator. In this work, a delay of 50ms is approximated [8]. To include this characteristic in the model, a low pass filter  $W_a$  is used with a cutting frequency of 2.5Hz. Figure 3 shows the frequency response of the filter. As expected, the magnitude of the force is reduced for the higher frequencies and the delay of the filter matches the actuator delay for the main part of the frequency range. Since the sprung mass resonance is usually close to 1Hz, an improvement in ride and handling is expected around sprung mass resonance [9]. Due to the simplicity of this model, the accuracy of the dynamic response for high frequencies is not guaranteed. As a consequence, a dramatic improvement at a high frequency would be suspect and subject to further investigation.

In order to design a controller, it is critical to define the measurements available. Several sensors are available for this task, implying different cost and signal processing requirements. From all these sensors, the linear potentiometers are widely used at the moment to measure the distance between the sprung and the unsprung mass. Since these sensors are cheap, robust and reliable, the controller is designed on the assumption that both front and rear suspension displacements are measured. The relative velocity between the sprung and unsprung mass can be measured by differentiating the signal of each sensor.

As described previously, each controller is composed of two blocks. The first block is a gain matrix  $K_i$  multiplying the measurement vector  $y$  in a manner similar to a LQR controller. For the front and rear controller a different gain matrix is optimized.

The second block of the controller is the filter  $W_{Ki}$  to adjust the force according to the frequency. The optimal behaviour of such filters is unknown. Since the goal of this study is to design a simple controller, the optimization of the complete system is done with filters of the second and third order. Each filter is designed by optimizing the coefficients  $b_i$  and  $a_i$  of the filter transfer function. After

the optimization process, the results are compared to identify the common point of the optimal filters.

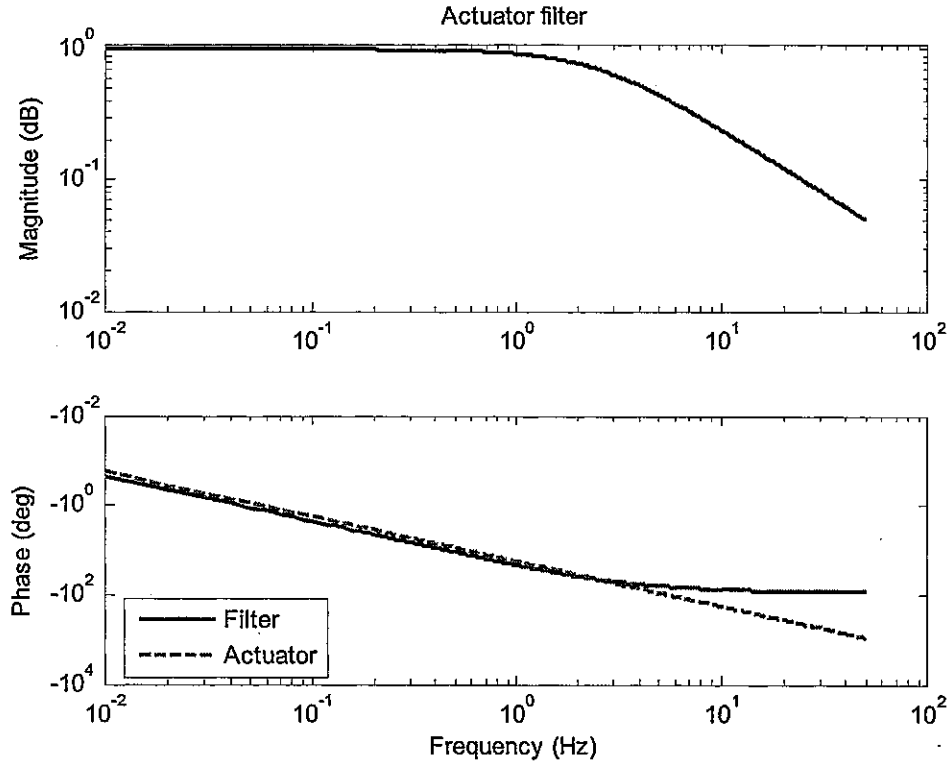


Figure 3: Frequency response of actuator filter

Once all the gains and filters are defined, matrices  $A_p$ ,  $L_{rFP}$ ,  $L_{rRP}$  and  $L_{aP}$  are created to model the complete system (equations 9 to 13). Note that the controller filter  $W_K$  and the actuator filter  $W_a$  are included in the filter  $W_{aK}$  to reduce the matrix size.

$$\dot{x}_p = A_p x_p + L_{rFP} z_{rF} + L_{rRP} z_{rR} + L_{aP} a_{long} \quad (9)$$

To create these matrices, we use the  $A_i$ ,  $B_i$ ,  $C_i$  and  $D_i$  matrices of the different filters where  $i$  represents each filter and  $x_p$  represents the state variable for the model.

$$A_p = \begin{bmatrix} A + B_{aF} D_{aK} K_F + B_{aR} D_{aK} K_R & L_{zF} C_{zr} & L_{zR} C_{zr} & L_a C_{al} & B_{aF} D_{aK} & B_{aR} D_{aK} \\ 0 & A_{zr} & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{zr} & 0 & 0 & 0 \\ 0 & 0 & 0 & A_{al} & 0 & 0 \\ B_{aK} K_F & 0 & 0 & 0 & A_{aK} & 0 \\ B_{aK} K_R & 0 & 0 & 0 & 0 & A_{aK} \end{bmatrix} \quad (10)$$

$$L_{rFP} = [L_{zF} D_{zr} \quad B_{zr} \quad 0 \quad 0 \quad 0 \quad 0]^T \quad (11)$$

$$L_{rRP} = [L_{zR} D_{zr} \quad 0 \quad B_{zr} \quad 0 \quad 0 \quad 0]^T \quad (12)$$

$$L_{aP} = [L_a D_{aL} \quad 0 \quad 0 \quad B_{aL} \quad 0 \quad 0]^T \quad (13)$$

In order to optimize the system, it is necessary to measure its performance. According to the literature, three main criteria have to be minimized to optimize suspension performance: sprung mass acceleration, tire deflection and suspension deflection [10; 11, 12 and 13].

Of all these criteria, the sprung mass acceleration is the most frequency dependant since it is related to the ride comfort of the passengers. Numerous studies have investigated the various ways of measuring human sensitivity to vibrations and most of them recommend acceleration as a representative measure of comfort. According to the Norm ISO-2631, humans are more sensitive to acceleration between 4 and 8 Hz. Sensitivity decreases outside this range. There is a human response filter (HRF) [14]. It is a second order filter with the same characteristics as the ISO 2631 standard, and is used to represent the sensitivity of humans to vibration as given in equation 14 (note that the numerator is changed to normalize the filter output).

$$\frac{30s}{s^2 + 30.02s + 901.3} \quad (14)$$

Other studies show that the angular acceleration is as important as the vertical acceleration. Unfortunately the sensitivity of humans to angular acceleration is not documented as well as the vertical acceleration. In the present work the filter represented by equation 14 is used to model human sensitivity to angular acceleration.

The minimization of acceleration results in improved ride comfort, but the optimization of these parameters will generally affect the vehicle handling capability. To insure that car handling is acceptable, tire deflection has to be minimized. A reduced tire deflection results in a more constant tire load and predictable tire grip, making the vehicle more stable and easier to drive. To maintain the stability of the car, both front and rear tire grip are essential. For this reason, only one criterion is used to measure vehicle stability in this work. At each frequency, the maximum value of the front and rear tire deflection is taken as the representative value for the frequency. No filters are added since a vehicle has to be stable in all conditions.

Another suspension performance requirement is making sure the maximum travel is realistic and to avoiding suspension bottoming. In order to satisfy this condition, the suspension deflection should be minimized. Usually both front and rear suspension maximum travel are similar. As discussed previously for tire deflection, the maximum suspension deflection between the front and the rear is the performance criterion for suspension working space.

Once all the transfer function amplitudes are filtered properly, the mean square (MS) value of the resulting frequency response  $H_y$  is computed using equation (15) where  $y$  represents one of the performance criteria [15]. The mean square value is largely used in the field of optimization. Even if the genetic algorithm allows the use of any performance function, the use of a mean square function guarantees a reduction of both area and peak amplitude, as desired in this work.

$$E[y^2] = S_0 \int |H_y(\omega)|^2 d\omega \quad (15)$$

For the present study, a classic weighting method has been used to solve a multi objective dynamic optimization problem. The value of each weighting  $\rho_i$  is determined from the reference passive suspension using values from Table 1. The road input is applied to the complete model and the

results are then used to calculate the value of each criterion. To define the weighting of criteria with a realistic order of magnitude, the inverse of  $J_{iref}$  is computed as in equation (16).

$$\rho_i = \frac{1}{J_{iref}} \quad (16)$$

During the optimization process, the global performance  $J$  of a defined controller is the sum of each weighting multiplied by the respective criterion value as in equation (17). The optimization process goal is to minimize this value, thus improving overall suspension performance.

$$J = \sum \rho_i J_i \quad (17)$$

### GENETIC ALGORITHM

Genetic algorithms (GA) are used to solve a wide variety of engineering problems. The main advantage of GA is its high level of flexibility. GAs do not rely on assumptions of linearity, convexity or differentiability like other optimization techniques. This property removes the constraints from the formulation of the system and the performance measurement functions, thus allowing the introduction of more realistic criteria, such as using maximum tire deflection instead of the sum of each tire deflection. The second advantage is the ability to do a global search without being trapped in a local optimum [2]. The ease of modification and the robustness of the method are other reasons why it is so popular today.

The genetic algorithm optimization method uses the “survival of the fittest” principle present in natural evolution. In each generation, the fittest individuals will have offsprings and the next generation will be better than the last one. The algorithm in Figure 4 shows the application of this method to an engineering optimization problem.

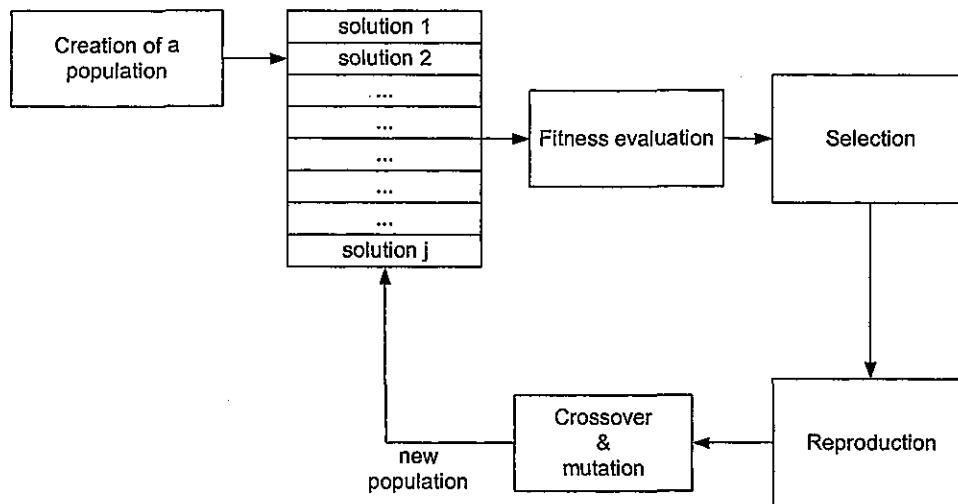


Figure 4: Genetic algorithm

The first step in the process is to create an initial population. A population is a group of individuals where each individual represents a solution to the problem. In this work, the problem is to optimize the value of different control parameters. A binary coding method is used to code each solution in a chromosome. For each parameter  $k_i$ , a minimum  $k_{imin}$  and a maximum boundary value  $k_{imax}$  are set. Each parameter is then coded on a finite number of bits, to form a binary number corresponding to a gene  $g_i$ . All the genes are connected to form a chromosome that represents a solution to the



problem. For a gene coded on  $m$  bits the number of different possibilities are  $2^m$ . To convert the gene to the numeric value of the parameter the following equation is used:

$$k_i = k_{i\min} + g_i \frac{k_{i\max} - k_{i\min}}{2^m - 1} \quad (18)$$

As stated previously, the controller is composed of two blocks. The gains of the first block are coded on 8 bits with values between -70000 and 0. The values are limited to negative numbers to guarantee the stability of the controller. The second block is a filter of a predefined order. The coefficients  $a_i$  and  $b_i$  defining the transfer function of the  $W_{Ki}$  filters have values between 0 and 100. They are coded on 10 bits since the filter response is sensitive to their value. The active suspension developed uses passive springs and dampers. In order to optimize the value of these passive components, the passive damping is added in the optimization parameters with a value ranging from 0 to 20000 Ns/m. The suspension stiffness is also optimized within a range of  $\pm 10\%$  based on the passive reference model [16].

The following step is the natural selection of the fittest individuals. The fitness of each solution corresponds to the global performance of equation 17. The association procedure gives more selection chances to the fittest individuals. The result is a new population composed of the fittest individuals of the last generation.

Crossover and mutation operations are done to create new solutions in the population. Crossover occurs with a probability of 35% for each individual. Once two individuals are selected, one part of the chromosome is exchanged with the other parent's and two new individuals are created as shown in Figure 5.

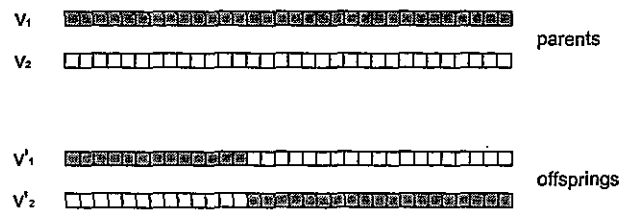


Figure 5: Crossover procedure

Mutation process occurs with a probability of 1.5% for each bit in the chromosome. The binary value of the bit changes if the bit is selected for a mutation.

Once all these steps are completed a new generation is created. The same process is repeated for the next generation until a limit condition is reached: the desired result or a maximum number of generations. In this work 800 generations have been created to insure algorithm convergence. Since the whole process includes stochastic values, the optimum can be different for the same problem. As it is recommended for heuristic optimization methods, many optimizations are run for the same problem and the results are compared to confirm the optimality of the different solutions.

## DISCUSSION AND RESULTS

As stated previously, the passive model of Table 1 is the baseline for evaluating the performance of the active system. The value of the performance J of the reference model is 4. Two different control systems have been optimized: a second and a third order filter. For each controller, eight optimization runs have been done to find the optimal gains, filter coefficient, passive damping and suspension stiffness. Table 2 shows the results for different runs and the reduction of the global performance J.

Run	Controller with 2nd order filter		Controller with 3rd order filter	
	Performance	Improvement	Performance	Improvement
1	3.649771286	8.76%	3.76905379	5.77%
2	3.621068012	9.47%	3.653945111	8.65%
3	3.657844742	8.55%	3.841907592	3.95%
4	2.869154363	28.27%	3.807015805	4.82%
5	3.731907658	6.70%	2.475439348	38.11%
6	4.203854762	-5.10%	3.76905379	5.77%
7	3.803164128	4.92%	3.653945111	8.65%
8	3.675141321	8.12%	3.841907592	3.95%

Table 2: Genetic optimization results

The best run with a second order controller gives an improvement of 28.27%, increasing to 38.11% with a third order controller. After comparing the optimized parameters it was noticed that the better improvements correspond to lower passive damping. Figure 6 shows the frequency response of the optimal filters of these two controllers. All filters have low pass behaviour with a cutting frequency close to the resonance frequency of the passive system and the cutting frequency of the actuator filter. This can be explained by the delay and smaller force put out by the actuator for frequencies above 2.5Hz. Since the force applied by the actuator is higher for low frequencies, it is expected that most of the performance improvements will occur in this frequency range. A resonance peak characterizes the third order rear filter. Since the peak does not match any resonance of the passive system, we suppose the better performance is a consequence of the reduced response at very low frequencies.

Equations 19 to 24 give the different coefficients for the best third order filter controller. The suspension stiffness is slightly increased at the front and reduced at the rear. This indicates that no major improvements could be achieved by changing the stiffness to facilitate the work of the active system, since the vehicle was not stiffly sprung and a minimum of stiffness is required to hold the static weight of the vehicle close to steady state at very low frequencies. On the other hand, the front and rear suspension damping are strongly reduced compared with the passive values. This reduction was expected, since the dampers are counteracting the force applied by the actuators, thus slowing them. A reduced damping increases the efficiency of the actuator. Since a minimum damping is required in the high frequency range where the actuator is not usable, the optimal damping is smaller than the passive reference model but higher than 0.

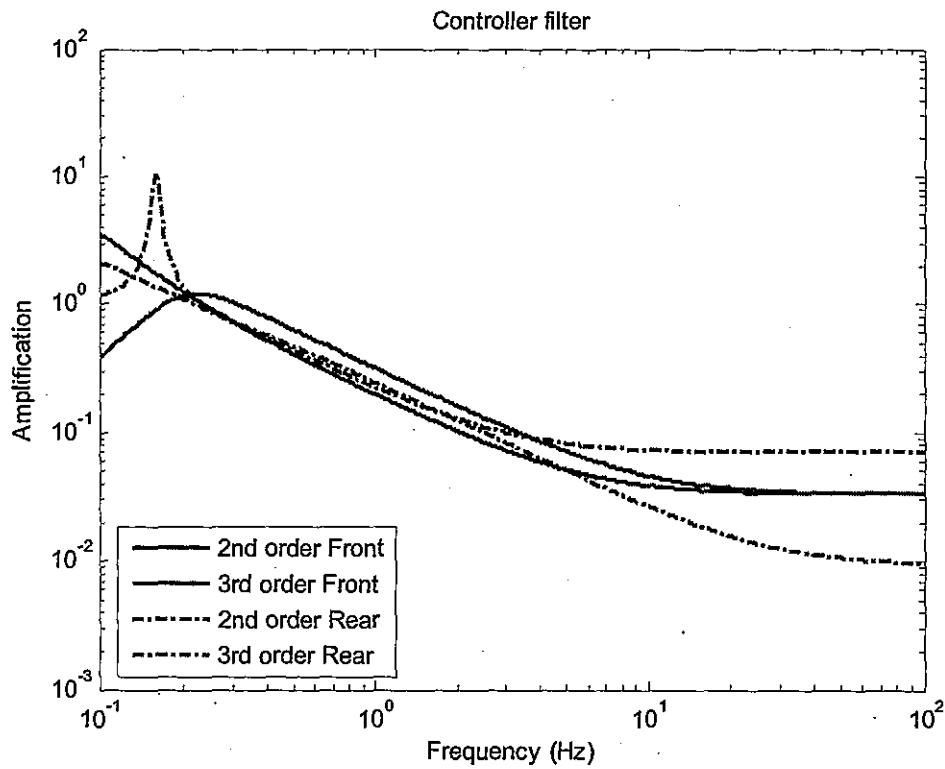


Figure 6: Frequency response of the controller filter

$$k_{sF} = 13903 \text{ N/m} \quad k_{sR} = 23886 \text{ N/m} \quad (19)$$

$$c_{sF} = 546 \text{ Ns/m} \quad c_{sR} = 312 \text{ Ns/m} \quad (20)$$

$$K_F = -51132(z_{sF} - z_{uF}) - 22148(z_{sR} - z_{uR}) - 14218(\dot{z}_{sF} - \dot{z}_{uF}) - 30078(\dot{z}_{sR} - \dot{z}_{uR}) \quad (21)$$

$$K_R = -46484(z_{sF} - z_{uF}) - 56601(z_{sR} - z_{uR}) - 45390(\dot{z}_{sF} - \dot{z}_{uF}) - 8750(\dot{z}_{sR} - \dot{z}_{uR}) \quad (22)$$

$$W_{KF} = \frac{1.46s^3 + 88.57s^2 + 42.77s + 9.76}{44.33s^3 + 10.44s^2 + 0.88s + 99.61} \quad (23)$$

$$W_{KR} = \frac{0.59s^3 + 98.44s^2 + 54.39s + 95.51}{62.79s^3 + 84.96s^2 + 67.29s + 82.52} \quad (24)$$

Figure 7 presents the frequency response of the controller to road input. The shape is similar for second and third order controller filters, with the exception of the peak of the 3<sup>rd</sup> order rear controller. The force is higher with the second order controller and the difference is more important between the rear controllers. It is observable that a greater force is applied at the front and rear ends for frequencies close to sprung mass resonance. The higher force requirement is used to counteract

the higher forces generated by the passive elements in resonance mode, making the actuator act as a frequency sensitive damper. Since the controller is frequency sensitive, the force is used to increase the damping at the resonance, requiring a smaller passive damping.

Figure 8 shows the frequency response for angular acceleration. Peaks correspond to frequencies where front and rear road motions are out of phase and troughs correspond to when they are in phase. More detailed explanation about this phenomenon can be found in [17]. The response between 1 and 10Hz is greatly reduced but the improvement is limited to the low frequency range. This result corresponds to the previous analysis of the actuator and filter dynamics. The second order controller shows a peak higher than the passive suspension around sprung mass resonance. The third order controller gives a smoother response with the lowest transmissibility. The improvement for the higher frequencies observable in Figure 8 may be too optimistic, since the improvement is visible for frequencies as high as 100Hz, which is far above the limit of currently available actuators.

As shown on Figure 9, the vertical acceleration has been noticeably reduced, mainly in the range of 4 to 8Hz specified by the HRF filter defined previously. The use of this filter explains the absence of improvements at lower frequencies. As explained previously, improvement at very high frequencies is not realistic. The vertical acceleration is a good example of the effect of performance criteria filtering to ensure the active system is used where it is most important. Since the two comfort criteria have been improved, the active suspension is definitely more comfortable than the passive one.

As stated previously, the amplitude of the road inputs is higher for frequencies below 1Hz making the amplitude of suspension displacement, and risk of suspension bottoming, higher for these low frequencies. The suspension deflection of the active system is clearly improved for frequencies below 2Hz, making this improvement more efficient than a similar improvement in a different frequency range as shown on Figure 10. The reduction of the maximum magnitude reduces the probability of suspension bottoming for a given suspension working space. Since road input amplitude is expected to be smaller around 10Hz the higher response of the active system at unsprung resonance will hardly result in bottoming. For the range where humans are the most sensitive to vibration, the active suspension deflection is close to the passive one. This tends to confirm that acceleration reduction is with difficulty done with a reduction in suspension displacement. These results show that the active system can be used with a realistic suspension travel.

As presented on Figure 11, the tire deflection is also improved for the majority of the frequency of interest with little deterioration compared to the passive suspension. The reduction around sprung mass resonance greatly improves the global stability of the vehicle over a wide range. The higher resonance of the active system at unsprung mass resonance in case of road perturbation matches the higher suspension deflection. Reduced passive damping is a most probable hypothesis to explain this phenomenon. The increase in suspension deflection is not considered as a problem but the increased tire deflection could compromise stability. Further simulations should be done to investigate the phenomenon. Of all the parameters examined, tire deflection is the only significant disadvantage of the active system. Considering the global performance of the active system, its advantages far outweigh its disadvantages.

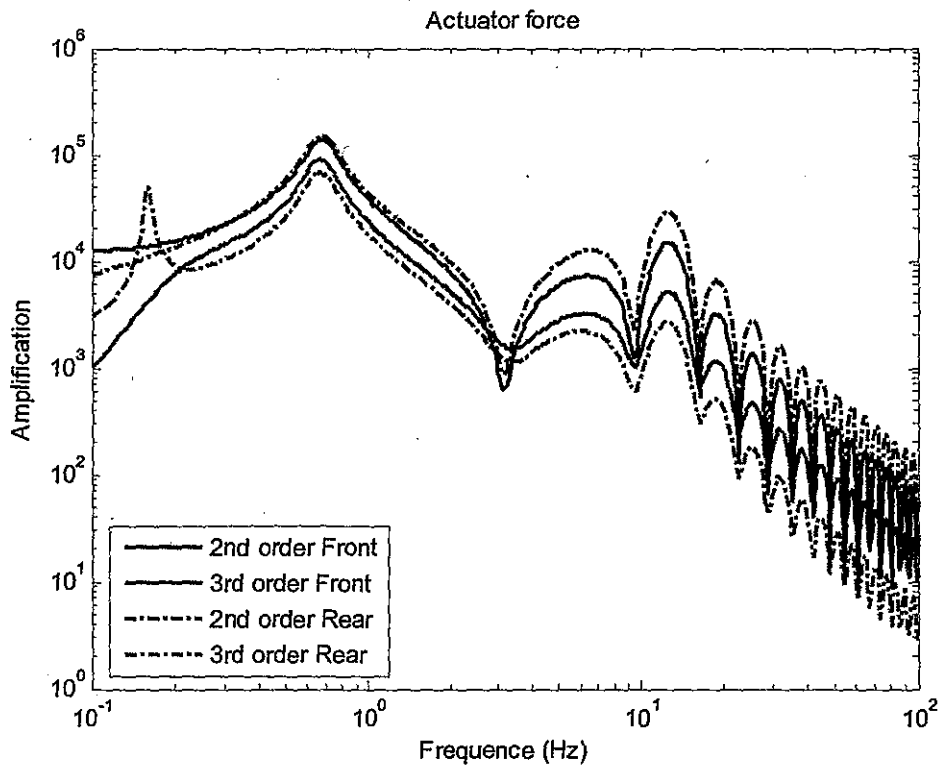


Figure 7: Front and rear actuator force

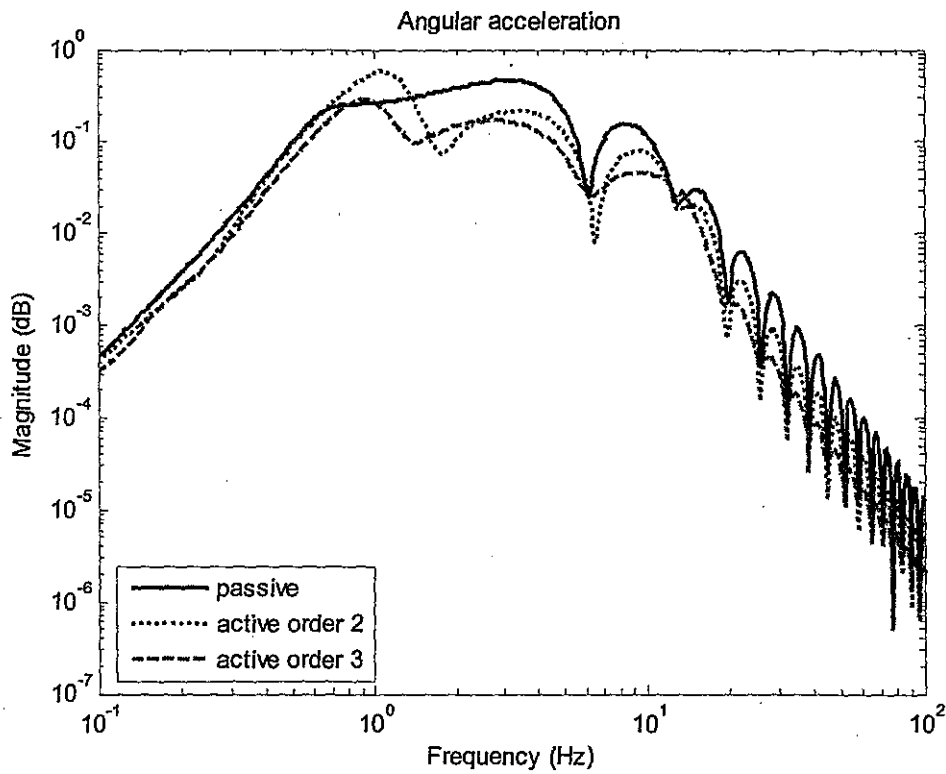


Figure 8: Angular acceleration for road input

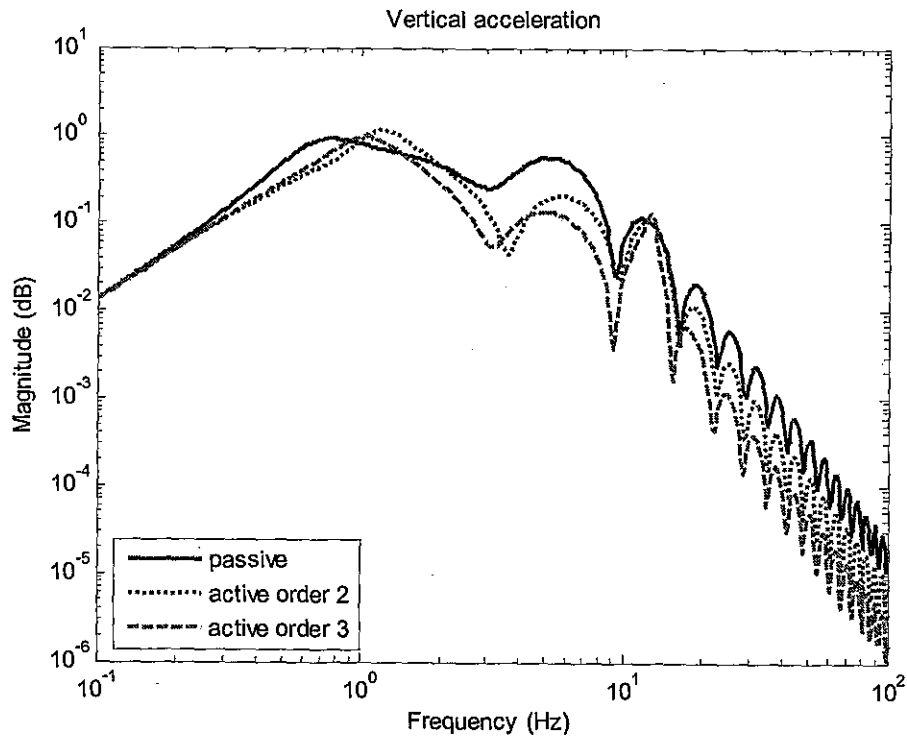


Figure 9: Vertical acceleration for road input

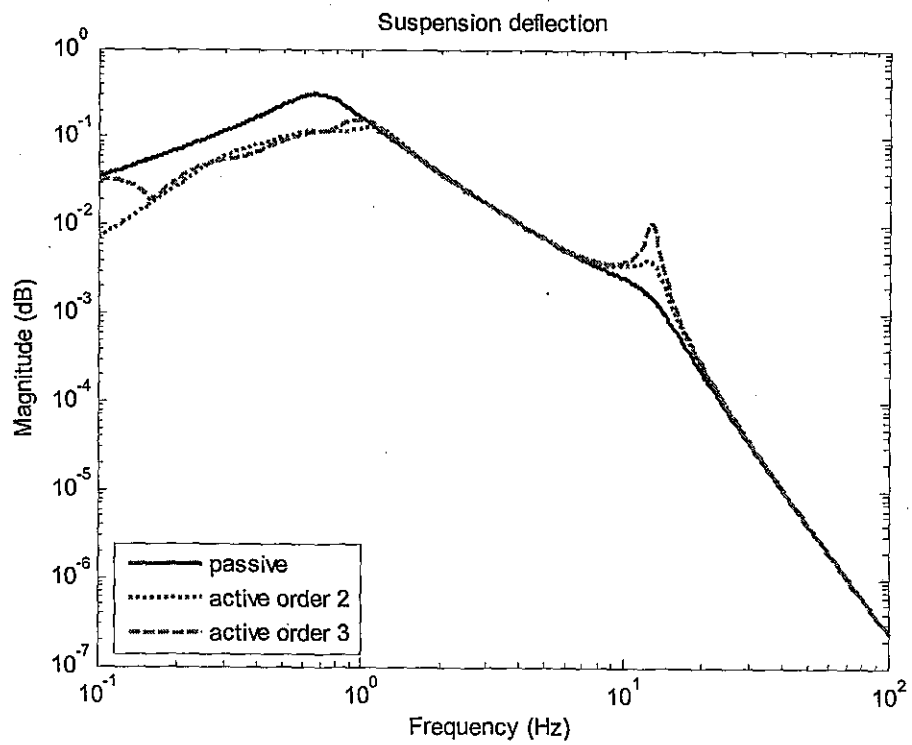


Figure 10: Suspension deflection for road input

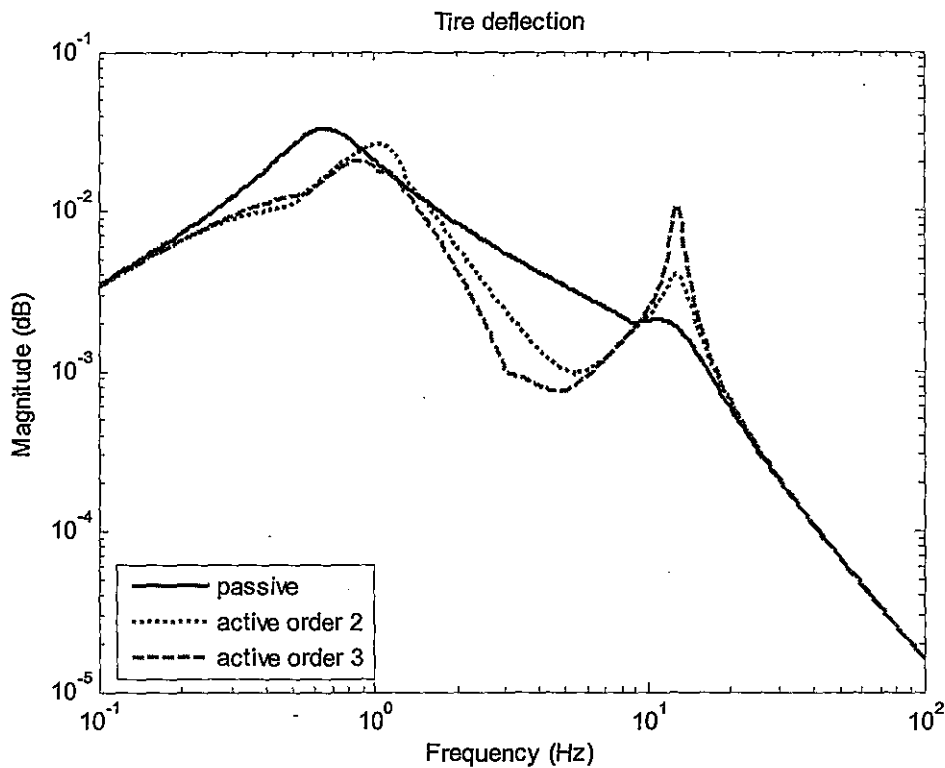


Figure 11: Tire deflection for road input

## CONCLUSION

The controller designed in this study provides a performance improvement of 38%. It was shown that a controller using the measurement of the suspension deflection added to a low order filter can significantly improve suspension performance. Filters have been used to insure that the different parameters are optimized in the desired frequency range. The results show that the 0.1 to 2Hz range is characterized by a reduction in suspension deflection, matching the higher amplitude of perturbation inputs. Frequencies around 4 to 8Hz are characterized by a reduced acceleration to match the range where humans are the most sensitive to acceleration. Tire deflection is reduced for a wide range of frequencies. An increase in tire deflection at unsprung mass resonance is the only disadvantage of the active system.

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