A TIME-PHASED PRODUCTION SCHEDULING SYSTEM FOR MULTIPLE ORDER WITH SINGLE ITEM

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ABSTRACT

A mathematical model which minimizes the production cost of an automatic and network-type multiple-order-production (MOP) system with restriction of time-phased order completion is presented. A step-by-step algorithm is described to find the minimal cost of production. In order to solve the combinatorial problems of the production rate, the machine cost, and numerous machines in parallel at each workstation of the production line, a feedback approach is also applied in the algorithm. An example of a large number of orders which is subdivided into different deadlines is shown to demonstrate the application of the algorithm that results in decreased production cost, enhanced production rate, improved efficiency of the production schedule, and on-time completion of production.

UN SYSTÈME DE CALENDRIER DE PRODUCTION AVEC PHASE EN FONCTION DU TEMPS POUR COMMANDES MULTIPLES D'UN PRODUIT UNIQUE

RÉSUMÉ

On présente un modèle mathématique qui permet de diminuer le coût de production d'un système automatique de type réseau de production de commande multiple (PCM) avec restriction d'ordonnancement en fonction du temps. Un algorithme pas à pas est développé pour trouver le coût minimal de production. Dans le but de trouver une solution aux problèmes combinatoires du taux de production, du coût de la machinerie, et des nombreuses machines en parallèle à chaque poste de travail de la ligne de production, nous avons aussi appliqué une approche de rétroaction dans l'algorithme. Un exemple de plusieurs commandes subdivisées en différents délais d'accomplissement est présenté pour démontrer que l'application de l'algorithme a pour résultat de diminuer les coûts de production, d'en augmenter le taux, d'améliorer l'efficacité du calendrier de production, et du travail exécuté dans les délais fixés.

1. INTRODUCTION

With limited production capabilities, it is always a challenge to efficiently and effectively design and plan the production lines. Many researchers [4,5,10,12–16,19,23] presented their approaches in production design and plan through the optimization of production cost; however, these approaches were only restricted to the case of a single product and order. Another approach proposed by Kalir and Arzi [12,13] and Lan and Lan [15] optimized production profitability through the dynamic selection of machine tools for each workstation. Kogan and Levner [14] also developed a mathematical model to control production processes. Kogan and Levner presented their approach to the production of large quantity of orders, but the issue of the order of due dates for production, which is the subject of this paper, was not considered.

Chen and Lan [16] further presented a network-type production system approach to manufacturing same-type products. They developed a step-by-step algorithm to obtain maximum profits of the production system, but they assumed the products to be shipped out immediately after production without consideration of the deadline of the order. To handle the long-term production, factors of order quantity and product finishing time were included to make optimal profit [17,18].

All of the proposed solutions focused on the case of single product, single order, and unlimited delivery of production design, planning, and choices of machines; they did not apply to the realistic production problems of multiple orders and the time constraint of on-time completion of orders. But, to deal with unsynchronized scheduling of arriving and finishing time of multiple orders, Kops and Natarajan [27] proposed an advanced method to decompose the schedule flow of orders into stages to which linear programming can be applied for maximizing machine utilization by optimizing the quantity of handled orders at each constant job-mix stage (CMS). The main objective of this paper is to propose a method, by employing a progressive mathematical model, to minimize the cost of the production of a single product, and of multiple orders with time-phased delivery deadlines.

Production flexibility [9,24] is the ability of a production system to cope with uncertain environmental changes. The higher production flexibility is, the better change management and profitability are [25,1,6]. Nevertheless, as observed by De Toni and Tochia [8], higher production flexibility may also increase the investment cost for the production facility and the complexity of the production system, which then results in more production cost.

Routing flexibility is a theme derived from production flexibility. The routing flexibility of a production system increases the production quantity through dynamic selection of production routes [25,20]. Koste and Malhotra [11] considered the routing flexibility as the capability of each production to engage many machine tools. Therefore, routing flexibility can balance the machine load and execute the production schedule more efficiently. To deal with the arrival of urgent orders in the production system, Tsubone and Horikawa [22] and Zhang et al. [26] treated routing flexibility as the effective solution. The enhancement of the routing flexibility of products can expand the manageable scope of the production resources, reduce the chances of production delay and product inventory, and improve the production control and execution of manufacturing, according to the research of Sheikhzadeh et al. [21]. The mathematical analysis by Benjaafar [2] and Benjaafar and Gupta [3] also discovered that the enhancement of routing flexibility may reduce the batch production cycle, thereby reducing the level of product inventory and the cost of inventory.

When the production capacity cannot fulfill the large number of orders, the production line needs to be more flexible to digest many orders dynamically to deliver products on time and optimize the inventory costs. Routing flexibility provides the solution to the problem of a large quantity of orders for production. Thus, this paper depicts the theory of routing flexibility to explore issues of the most optimum production system under complicated circumstances of multiple orders with multiple product delivery deadlines. To meet order quality and delivery deadline, this paper also introduces the effective way which the production should examine limitations of production and factory equipments to plan the production and stock strategically. Furthermore, a network-type production system is also illustrated for designing production and planning the mathematical model.

To meet delivery deadlines in production, the factory should strategically plan production in advance with consideration of inventory. In fact, this study not only focuses on the layout of the production system but also provides the strategy for the production of a single product for multiple orders, including consideration of production cost and completed stocking cost.

An example showed that the implementation of the modules and production design described in this study can resolve the issues of planning for the production of the single product for multiple orders with multiple delivery deadlines. Moreover, it is developed by the progressive approach assuming that the solution of single product, multiple orders and deadlines can be decided quickly by tuning the parameters (PR_{ij}, t_k, Q_k), the production rate, the production time length, and the demand quantity.

2. ASSUMPTIONS AND NOTATIONS

2.1. Assumptions

Before formulating this study, several assumptions are described below:

1. The whole multiple-production-line system only makes the same type of product during the manufacturing process and a series of production stages of this product are given. The workstations performing the same production stages from different production lines can be linked together as one workstation in this system.
2. The proposed production system consists of automated production lines; every workstation in a production line has its specific sequence of production stages which are performed by the same type of machines.
3. In every machine, there is only one part to be processed at one time.
4. The idle or broken machines do not charge for operation.
5. The proposed production line has enough buffers to avoid blocking and starvation.
6. The cost of buffer space can be ignored because it is far less than the operation cost and maintenance cost of the machine; i.e. the buffer space can be large.
7. The order must be fulfilled before the delivery deadline.
8. Despite full production, the current order demand cannot be fulfilled after accomplishing the previous one.
9. The holding cost of a product per unit time is unchangeable, that is, it is a constant.
10. Once the production system is deployed, the layout of the system will not change during the process. In addition, restarting, or terminating the production system are the only possible problems.
11. The estimation of the profitability after accepting the orders is not discussed in this study.
12. The plant receives different orders of the same product with different delivery time while orders with the same delivery time are considered as the same order.
13. This study only discusses the condition under which the production rate must be larger than the total amount of orders.

2.2. Notation

The following notations are used in this study.

n: the number of production stages for a given product.

N: $N = \{ \text{node } b | b = 0, 1, 2, 3, \ldots, n \}$, where node b represents that all preceding production stages and stage b have been completed and the production stage $b + 1$ is ready to manufacture. In addition, the source node (node 0) only represents that the production stage 1 is ready to process and the sink node (node n) means that the entire production stages (from 1 to n) have been finished.

MS: the set of all available machine types.

ij: $0 \leq i < j \leq n$, *indicates the workstation identification number with production processes from* stage i+1 to j in sequence. Thus, ij is called a feasible workstation if there is a machine $M_{ij} \in MS$; where M_{ij} means the type of the machine tool to perform a sequence of production stages (from i+1 to j).

F:$F = \{ij | 0 \leq i < j \leq n$; ij is a feasible workstation$\}$

t_{ij}: means the processing time per unit product performed by a single machine M_{ij} if $ij \in F$; otherwise, t_{ij} is defined to be a very large number.

l_{ij}: the maximum available number of machine type M_{ij} in the firm; l_{ij} is defined to be zero if ij is infeasible.

r_{ij}: the reliability of machine M_{ij} which is defined by $r_{ij} = \varepsilon_{ij}/(\varepsilon_{ij}+\delta_{ij})$; where ε_{ij} and δ_{ij} are the mean time between failures and mean time to fix a single machine type M_{ij} respectively.

c_{ij}: the operation cost (dollar(s) per unit time) of the single machine type M_{ij}.

c^f_{ij}: the maintenance cost (dollar(s) per unit time) of a single machine type M_{ij}. That is, $c^f_{ij}\delta_{ij}$ is the mean maintenance fees of a single machine type M_{ij}.

c^h_{ij}: the holding cost of unit product per unit time in workstation ij.

pl: $pl = \{i_0 i_1, i_1 i_2, i_2 i_3, \ldots, i_{r-1} i_r\}$ is a production line which indicates a sequence of feasible workstations, $i_0 i_1, i_1 i_2, i_2 i_3, \ldots, i_{r-1} i_r$,where $0 = i_0 < i_1 < i_2 < i_3 < \ldots < i_r = n$.

k_{ij}: $k = c_{ij} t_{ij} + c^f_{ij}\left(\frac{t_{ij}}{r_{ij}} - t_{ij}\right)$ the production cost of a product from the workstation ij, where $c_{ij} t_{ij}$ means the operation cost (dollar(s) per unit time) and $c^f_{ij}\left(\frac{t_{ij}}{r_{ij}} - t_{ij}\right)$ means the maintenance cost (dollar(s) per unit time) in workstation ij.

c_s: the setup cost of a production line.

$pl(i)$: the i-th shortest path in the network-type production system.

U_i: the operational production rate of production line $pl(i)$.

PR_{01}: the through-production rate of the multiple-production-line system.

Q_k: the demand quantity for the end of the k-*th* time interval.

tk-bar: the time length between the due date of the $(k-1)$-*th* order and k-*th* order.

2.3. Decision Variables

PR_{ij}: the operational production rate of workstation ij in the multiple-production-line system.

t_k: the production time length for the k-*th* order.

3. MODEL DEVELOPMENT

In general, a workstation may perform several production stages, a production line may involve a series of workstations, and a production system may consist of several different production lines to manufacture products of the same type. A series of production stages for manufacturing a given product is known prior to the system design. A network-type production system consisting of multiple production lines is discussed in this article. Automated workstations are the essential elements of an automated production line in the production system, as automatic production has been widely adopted in industry today.

For production design and scheduling, a mathematical model is presented to determine the structure and the production time interval of each production line in the manufacturing system under the restriction of order quantity and production deadline. A network model is outlined for the entire workable structure of workstations to make a given product. Each workable structure can be regarded as the feasible path from source node to sink node. Thereby, this study transforms the problem of designing a multiple-production-line system for a given product into an extensive minimal flow problem called a Multiple-Order-Production (MOP) model.

The objective function (1) of the MOP model presents the total cost of the multiple-production-line system for making the given orders. In addition, $c_{ij}t_{ij}$ and $c_{ij}^{f}\left(\frac{t_{ij}}{r_{ij}}-t_{ij}\right)$ are the mean operation cost and the mean maintenance cost per unit in the workstation ij, respectively.

$$\min_{PR_{ij},t_K}\left\{\left[\sum_{ij\in L}k_{ij}PR_{ij}\right].T+\sum_{k=1}\left\{\frac{1}{2}Q_k t_k+\sum_{k=1}\{(1-d_k)[PRt_k-Q_k](t_k-t_{k-1})\}\right\}C_h+C_s\sum_{k=1}^{m}d_k\right\} \tag{1}$$

$$\text{s.t}\ \begin{matrix} d_k=0\ if\ [PRt_k-Q_k]>0 \\ d_k=1\ if\ [PRt_k-Q_k]=0 \end{matrix}\quad Note: Q_0=0,\ t_0=0 \tag{2}$$

$$t_k\leq\sum_{k=1}^{\text{m}}\bar{t}_k\ \forall\text{k}=1,2,\ldots.,\text{m} \tag{3}$$

$$T=(t_1\cup t_2\cup\ldots..\cup t_k) \tag{4}$$

$$\sum_{k=1}^{m}Q_k\leq PR\left(\sum_{k=1}^{k}\bar{t}_k\right) \tag{5}$$

$$PR=\sum_j PR_{oj}=\sum_i PR_{in} \tag{6}$$

$$\sum_i PR_{ij}=\sum_k PR_{jk} \tag{7}$$

$$0\leq PR_{ij}\leq\frac{l_{ij}r_{ij}}{t_{ij}} \tag{8}$$

$$PR_{ij},\ t_k\geq 0 \tag{9}$$

Eq. (2) notes that the production setup cost of the first order should be considered. When the value of the binary variable d_k equals one this means that the production setup cost of the k-th order should be calculated, otherwise d_k is zero. Eq. (3) means that the sum of the available production time interval of the k-th order should be greater than or equal to tk (the production time interval of the k-th order). In eq. (4), T, the total production length of the given multiple-order-production problem, or the total of different delivery time, should be equal to the union of all t_k. Eq. (5) shows that the accumulation of production quantity at the due date of the k-th order should be equal to or greater than the accumulation of the preceding order demands including the k-th order demand. Eq. (6) shows that $\sum_i PR_{in}$ and $\sum_j PR_{oj}$ are both equal to PR (the total production rate of the complete system). Eq. (7) specifies that the total input operational production rate of node j is equal to its total output rate. Eq. (8) shows that the operational production rate of workstation ij is less than or equal to its upper boundary $\frac{l_{ij}r_{ij}}{t_{ij}}$.

4. STEP-BY-STEP ALGORITHM

After introducing the objective function and its constraints, the following step-by-step algorithm to reach the optimal solution is proposed.

This study uses a recursive algorithm for production design and planning to manufacture the given products of the same type by considering multiple orders, different delivery deadlines, and a number of parallel machines in workstations for production to achieve cost minimization. Initially, this proposed solving procedure checks whether the highest production rate provided by the manufacturer can complete all of the order quantities. Then, all feasible production lines are ranked by the unit production cost of each production line; the smaller unit production cost ranked with a lower number. Based on a study (Lan, 2001), the optimal production rate of each production line can be determined. Secondly, the proposed solving procedure examines if the accumulation of order quantities by their delivery deadlines can be done under the maximum production rate. The forward procedure completes the above-mentioned steps.

Subsequently the solving procedure moves to the backward procedure. In this stage, the solving procedure ensures that the quantity of the last delivery order can be completed within its specified production time interval (i.e. the time between the last delivery order and its next prior delivery order) and under the maximum production rate. If yes, the procedure stops and goes to track the next prior delivery order; otherwise, it checks whether the accumulation quantity of the last delivery order and its prior order can be completed within their accumulation time intervals under the maximum production rate. This procedure repeats till the stop situation is attained. Next, if the quantity of the last order can be done within its associated time interval under the maximum production rate of the production system, the step is stopped and the procedure determines how many production lines should be included in the system to complete the entire order quantity. The criterion of selecting the production lines is to choose the production line with smaller number and the higher priority. On the other hand, if the quantity of the last order cannot be completed within the specified time interval under the maximum production rate of the production system, we should add the prior order quantity and then check whether the accumulation quantity can be done within the accumulation time intervals. This step will check again and again until the procedure is stopped. Then find the newer last delivery order and return to the second step until all orders are completed. In addition, a FORTRAN application of the algorithm has been developed to solve such a complicate problem easily. The detailed solving procedure is shown as a flow chart listed in the appendix.

5. A CASE STUDY

Suppose a factory receives an order of 20000 units of merchandise in five deliveries, the delivery times are 120 hours, 300 hours, 900 hours, 1300 hours, and 1800 hours, with quantities of 500, 500, 10000, 500, and 8500, respectively. The factory's production facilities and their pertinent information are shown in Table 1. Then calculate the unit production cost of different production lines and the results are shown in Table 2. With a sequence of cost $pl(1) < pl(2) < pl(3) < pl(4) < pl(5)$, five different production lines are $pl(1)$=01-13-36-67, $pl(2)$=01-12-25-56-67,…etc. Figure 1 depicts the network of these five production lines.

Calculate the mean of the lowest production quantity per hour as the standard quantity of the proposed production plan.

$$PR_{\min} = \frac{\sum_{k=1}^{m} Q_k}{\sum_{k=1}^{m} \bar{t}_k}$$

$$\sum_{k=1}^{m} Q_k = 20000$$

$$\sum_{k=1}^{m} \bar{t}_k = 1800$$

$$PR_{\min} = 20000/1800 = 11.11$$

The production quantity per hour cannot be below 11.11; otherwise, it will not be delivered in time. The algorithm determines if the production capability can satisfy such a large number of orders in time Next, $PR_{ij} = l_{ij}\frac{r_{ij}}{t_{ij}}$ indicates the bottle-neck production rate of each production line, according to the cost of the production line listed in Table 3. The maximum production rate at full capacity is provided by the combination of workstations with the lowest cost. From Table 3, the production rate of line $pl(1)$ is constrained by the production rate of 4.5 (i.e. U_1) of workstation 36 (ij = 36). So are $pl(2)$ and $pl(4)$ which are limited by workstation 25 and 13 respectively. The actual production lines can be determined as $pl(1)$, $pl(2)$, and $pl(4)$, and the actual production rate of each workstation is PR^*_{ij} in Table 3.

The operational production rate of $pl(1)$, $pl(2)$, and $pl(4)$ are $U_1 = 4.5$, $U_2 = 5.7$, and $U_4 = 5.88$ respectively. The full capacity production (i.e. total production rate per hour) is 16.08 (the sum of U_1, U_2, and U_4) which should fulfill these orders.

From the above analysis, different plans can be decided according to three possible combinations of $pl(i)$ in Table 4. The possible combination S_i depends on their mean production rates which are lower than or close to $PR_{\min}$, the lowest production rate per unit time.

In order for the production plan to meet the need of a cross period production, we conduct a checkup of the production quantity, which includes the forward examination and repeated calculations. The results are shown in Table 5. The delivery times are 120, 300, 900, 1300, and 1800 hours, with the maximum production at full capacity production of 1930, 2894, 9648, 6432, and 8040, while the actual production for delivery is 500, 500, 10000, 500, and 8500,

Table 1 The relevant information of the case example

Indices ij	Machine types M_{ij}	Processing Time t_{ij} (h)	Reliability r_{ij}	Maximum available number of machines l_{ij}	Operation cost C_{ij} (\$/h)	Maintenance cost C_{ij}^{f} (\$/h)	Cost of a production line k_{ij} (\$)
01	M_{01}	0.44	0.90	8	20.0	8.5	9.22
12	M_{12}	1.10	0.85	8	14.3	8.0	17.28
23	M_{23}	0.30	0.90	8	15.0	8.0	16.24
34	M_{34}	0.90	0.90	8	21.7	8.6	4.77
45	M_{45}	0.50	0.90	8	20.0	8.5	26.53
56	M_{56}	0.18	0. 95	8	18.3	8.0	20.39
67	M_{67}	0.15	0.90	8	18.3	8.0	26.45
13	M_{13}	0.52	0.90	6	30.0	11.0	10.47
25	M_{25}	1.00	0.95	6	26.0	10.0	14.28
36	M_{36}	1.20	0.90	6	21.0	9.4	3.37
46	M_{46}	0.60	0.95	6	23.3	9.6	2.88
Holding Cost C_{ij}^{h} (\$/h)			0.01				

Table 2: The unit production cost of different production lines

Production Line *pl(i)*	possible combinations of all production	Production cost per unit time	Order
pl(1)	01-13-36-67	54.783	1
pl(2)	01-12-25-56-67	59.273	2
pl(3)	01-12-23-36-67	60.597	3
pl(4)	01-13-34-45-56-67	62.562	4
pl(5)	01-13-34-46-67	63.003	5

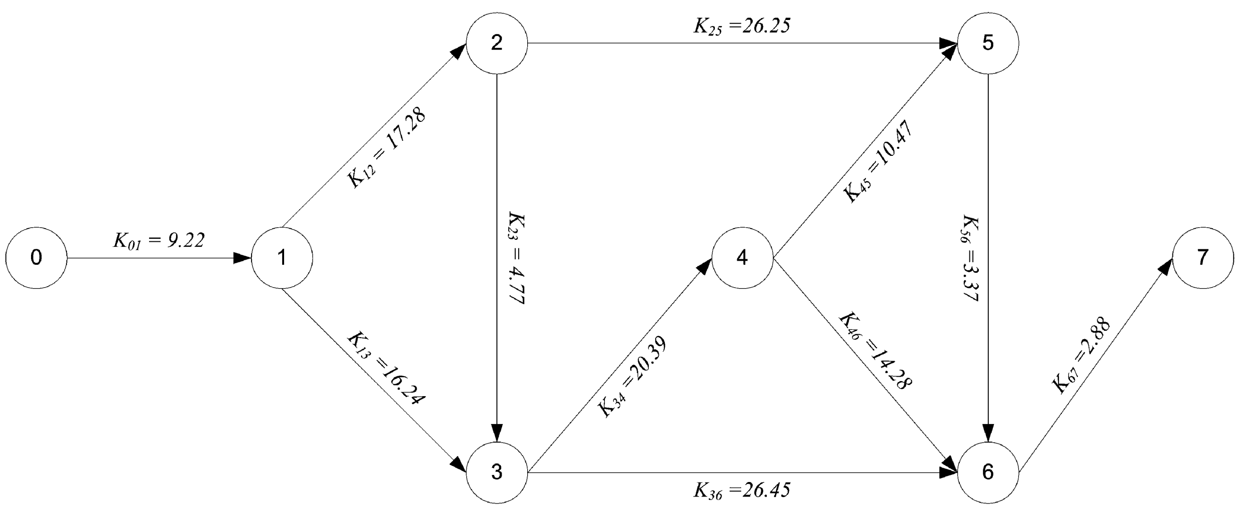

Fig. 1. The network of the five production lines.

Table 3 The maximum production rate

ij	Theoretical production rate PR_{ij}	Actual production rate *pl(1)* $U_1 = 4.5$	Actual production rate *pl(2)* $U_2 = 5.7$	Actual production rate *pl(4)* $U_4 = 5.88$	Actual production rate PR^*_{ij}
01	16.36	11.86	6.16	0.28	16.08
12	6.18	6.18	0.48	0.48	5.7
23	24.00	24.00	24.00	24.00	0.00
34	8.00	8.00	8.00	2.12	5.88
45	14.40	14.40	14.40	8.52	5.88
56	42.22	42.22	36.52	30.64	11.58
67	48.00	43.50	37.80	31.92	16.08
13	10.38	5.88	5.88	0.00	10.38
25	5.70	5.70	0.00	0.00	5.70
36	4.50	0.00	0.00	0.00	4.50
46	9.50	9.50	9.50	9.50	0.00

Table 4 The possible combinations for production plans

Production plan	All possible production combinations	Production rate
1	$S1 = pl(1)$	4.5
2	$S2 = pl(1) + pl(2)$	4.5 + 5.7 = 10.2
3	$S3 = pl(1) + pl(2) + pl(4)$	4.5 + 5.7 + 5.88 = 16.08

respectively. The comparison is the following, 1930 > 500, 2894 > 500, 9648 < 10000, 6432 > 500, and 8040 < 8500.

The order delivery quantity can be done in a full capacity production. Hence it can meet the need of order quantity in accumulation, shown as the following example:

1930 > 500
4824 > 1000
14472 > 11000
20904 > 11500
28944 > 20000

From the order number Q5 of table 6, we can plan and design the production line recursively. For example, Q5's order is 8500, but the actual production rate is 8040, and the insufficiency in production rate can be delivered by Q4. Similarly for Q3, the insufficient production rate can be solved by Q2 and so on. Therefore, the total order quantity is 20000 and can be delivered on time by such a flexible arrangement in the production line in advance. The lowest costs of each production plan are \$27968, \$49463, \$600599, \$55912, and \$496829 respectively. The total lowest cost with an order of 20000 is \$1230772. Table 6 calculates the results from the proposed step-by-step algorithm.

6. CONCLUSIONS

This study also focuses on issues of possible impending practices of a plant. The example described by the mathematical model in this paper can minimize the production cost of the plant. With the comprehensive consideration of multiple orders, multiple delivery deadlines, different costs, and flexible selections of machine tools, the model can be applied to the above mentioned complicated issues.

Table 5 Forward Checkup

Order number	Q Order quantity	Q* Full production quantity	Order quantity forward Checkup	Full production quantity forward Checkup	Checkup result	Order quantity repeated calculation	Full production quantity repeated calculation	Repeated calculation result
1	500	1930	500	1930	✓	500	1930	✓
2	500	2894	1000	4824	✓	10500	12542	✓
3	10000	9648	11000	14472	✓	10000	9648	×
4	500	6432	11500	20904	✓	9000	14472	✓
5	8500	8040	20000	28944	✓	8500	8040	×

Table 6: Calculation of Product Cost under an Actual Production Plan

Order No.	Delivery qty	Actual Production rate	Accumulated Delivery time ***t***	Actual Production time	Average production hour Q / S_i	Product Cost of Each Order $k_{S_i} = \left[\sum_{ij \in L} k_{ij} PR_{ij}\right].T + \sum_{k=1} \left\{\frac{1}{2} Q_k t_k + \sum_{k=1} \{(1-d_k)[PR\, t_k - Q_k](t_k - t_{k-1})\}\right\} C_h + C_s \sum_{k=1}^{m} d_k$
Q_1	500	500	120	111.11	500 / 4.5	$k_{S_1} = (54.78 \times 4.5 \times 111.11) + (1/2 \times 500 \times 0.01 \times 111.11)$ $+ 300 \times 1$ $= 27969$
Q_2	500	500 + 352 = 852	300	83.52	852 / 10.2	$k_{S_2} = (54.783 \times 4.5 \times 83.52) + (59.273 \times 5.7 \times 83.52)$ $+ (1/2 \times 852 \times 0.01 \times 83.52)$ $+ (352 \times 0.01 \times 600) + 300 \times 1$ $= 49463$
Q_3	10000	10000 – 352 = 9648	900	600	9648 / 16.08	$k_{S_3} = (54.783 \times 4.5 \times 600) + (59.273 \times 5.7 \times 600)$ $+ (62.5614 \times 5.88 \times 600)$ $+ (1/2 \times 9648 \times 0.01 \times 600) + 300 \times 1$ $= 600599$
Q_4	500	500 + 460 = 960	1300	213.33	960 / 4.5	$k_{S_4} = (54.783 \times 4.5 \times 213.33)$ $+ (1/2 \times 960 \times 0.01 \times 213.33) + (460 \times 0.01 \times 500)$ $= 55912$
Q_5	8500	8500 – 460 = 8040	1800	500	8040 / 16.08	$k_{S_5} = (54.783 \times 4.5 \times 500) + (59.273 \times 5.7 \times 500)$ $+ (62.5614 \times 5.88 \times 500)$ $+ (1/2 \times 8040 \times 0.01 \times 500) + 300 \times 2$ $= 496829$

Instead of only promoting the production rate, a systematic design is proposed to minimize the entire production costs, which includes operation, maintenance, fixed, setup, and holding costs. With appropriate settings of some determinative parameters, such as different order quantities, different delivery deadlines, different types and amounts of machines, holding and setup costs of the production, the most optimum production plan can be obtained from the model and the solving procedure. Because of the highly repetitive characteristic of the model and the solving procedure, a practical and effective solution to the production management can also be provided.

To extend the scope of the problem resolution, future research can focus on production design and planning for multiple products, multiple orders, and multiple due dates. Additionally, to control production cost more precisely, such research cannot ignore the effects of the product inventories of various plans and policies. The model will become more complex for a more sensible production plan for the realistic situation which deals with small quantities of orders of diversified and customized products. As for the matters of numerous parameters and time consuming computation, advanced study along with the rapid development of computational capacity can resolve more complicated problems of optimum production planning.

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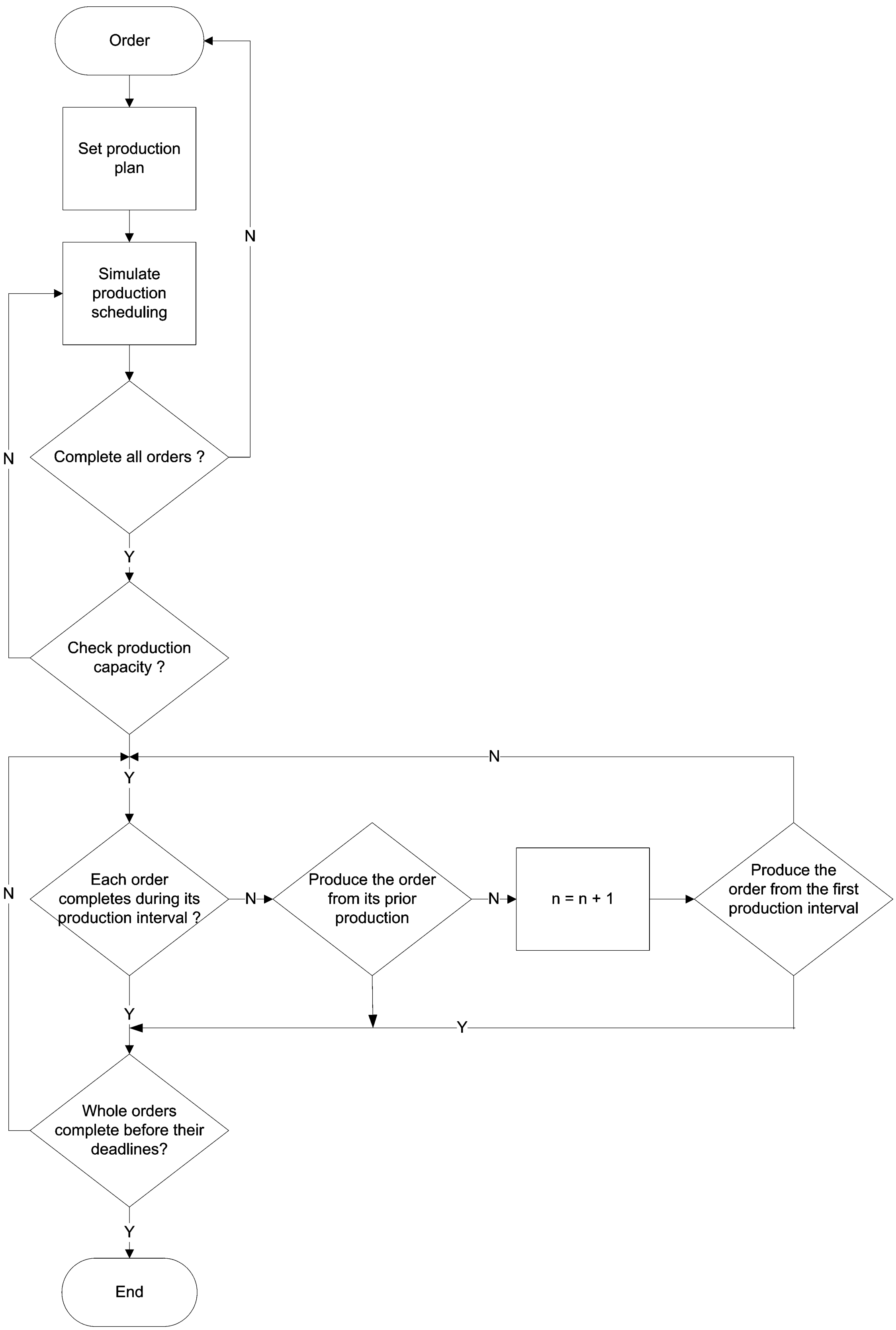
APPENDIX

A Time-Phased Production Scheduling System for Multiple Order with Single Item Flow Chart


Order
Set production plan
Simulate production scheduling
Complete all orders ?
N
N
Y
Check production capacity ?
N
Y
N
Each order completes during its production interval ?
N
Produce the order from its prior production
N
n = n + 1
Produce the order from the first production interval
Y
Y
Whole orders complete before their deadlines?
Y
End