FINITE ELEMENT ANALYSIS OF A SEMI-ELLIPTICAL EXTERNAL CRACK IN A BURIED PIPE

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ABSTRACT

In this research, a buried pipe containing an external semi-elliptical crack has been modeled and investigated using finite element analysis. The interaction between the soil and pipe has been considered according to the Burns and Richard model. A few major parameters, namely, the soil height over pipe, the geometries of pipe and crack and the circumferential position of crack on pipe have been changed and their effects on elastic stress intensity factors have been studied at different positions along the crack front. The results showed that the crack experienced mixed mode loading condition and the circumferential crack position on pipe had a significant influence on the stress intensity factors.

Keywords: Buried pipeline; external semi-elliptical crack; mixed mode loading; finite element analysis.

ANALYSE PAR ÉLÉMENT FINI D'UNE FISSURE SEMI-ELLIPTIQUE EXTERNE DANS UN TUYAU ENTÉRRÉ

RÉSUMÉ

Dans notre étude, un tuyau enterré présentant une fissure semi-elliptique externe a été modélisé et étudié utilisant l'analyse par élément fini. L'interaction entre le sol et le tuyau a été considérée selon le modèle de Burns et Richard. Quelques importants paramètres ont été changés, à savoir, le volume de terre au-dessus du tuyau, les dimensions du tuyau et de la fissure, et la circonférence de la fissure dans le tuyau. Leurs effets sur le facteur d'intensité des contraintes élastiques ont été étudiés à différentes positions le long de la face externe de la fissure. Les résultats ont prouvé que la fissure a éprouvé des conditions de chargements mixtes, et la circonférence de la fissure dans le tuyau a eu une influence significative sur le facteur d'intensité des contraintes.

1. INTRODUCTION

The significance of safety in transmitting oil and gas pipelines has led the engineers to seek reliable approaches to minimize the risk of mechanical failure. The use of buried pipes (Fig. 1) is often considered as one of the reliable and common ways of oil and gas transmission. On the other hand, various factors such as corrosion and static or moving loads like soil weight over the pipe or compressive loads due to the passing vehicles can result in the nucleation of cracks or the propagation of pre-existing cracks in pipes. Crack growth in the pipe may eventually give rise to pipeline rupture and waste a huge amount of gas and oil with catastrophic consequences. To avoid such sudden failures, engineers conduct periodic and careful inspections using the techniques of non-destructive evaluations, so that in the case of crack detection, they can analyze the cracked pipe and decide whether to continue the transmission process or to shut down and repair the damaged part. The above procedure becomes much more crucial for pipelines concealed underground in contrast with the open pipelines.

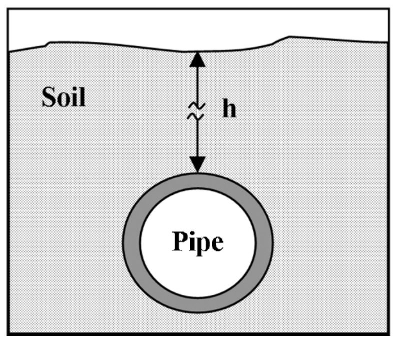


Fig. 1. Schematic of a pipe buried in soil.

In general, a buried pipe is under a wide variety of loading such as internal pressure, bedding and soil weight loads and each of which may cause a specific crack orientation more vulnerable. The finite element analyses showed that the deformation of pipe due to the weight of soil made the axially oriented external cracks more critical than the circumferential cracks. Therefore, this paper aimed to study the effect of soil weight on a buried pipe containing only an axially oriented external crack.

Up to now, extensive researches have been carried out to study axial cracks on pipes and cylinders. Among the first attempts, Underwood [1] and Kobayashi [2] used an engineering estimation to study axial cracks on metallic cylinders under internal pressure. They did not consider wall thickness effect in their investigations. Then, Alturi and Kathiresan [3,4] and McGowan and Raymund [5] could determine the stress intensity factor (SIF) using finite element method for a surface crack on cylindrical part of an internally pressurized vessel only for particular geometric parameters of crack. Later, Newman and Raju [6,7] determined mode-I SIF for a wide range of geometric parameters of semi-elliptical internal and external cracks.

Kirkhop et al. [8,9] calculated SIF for semi-elliptical surface cracks in a pressurized cylinder using a three-dimensional finite element modeling. Zheng et al. [10,11] used the weight functions method to evaluate SIF of longitudinal cracks in thick-walled cylinders and considered a semi-elliptical crack in an open-ended thick-walled cylinder. Lin and Smith [12] analyzed crack growth in pressure vessels using three-dimensional finite element modeling and showed that a crack regardless of its initial shape becomes semi-elliptical after some cycles. Kim et al. [13]

conducted a nonlinear analysis to study internal axial cracks in cylinders under internal pressure. Recently, Nabavi and Shahani [14] calculated the stress intensity factor for an internal semi-elliptical crack in a pressurized finite-length thick-walled cylinder and studied the effects of finite boundary on SIF. A review of papers reveals that none of the researchers have studied SIF for cracks embedded in the underground buried pipes under the soil weight.

In general, axial cracks may be present on either the internal or the external surfaces of a buried pipe. The external cracks can be caused through a host of different reasons such as: an inadvertent collision between the excavator's arm and the pipe or due to corrosion in a dented pipe when the surrounding soil is chemically corrosive. In this research, the stress intensity factors are calculated for buried pipes containing an external crack at different circumferential positions.

2. FINITE ELEMENT MODELLING

A crack in a pipe or a hollow cylinder may be considered and analyzed in the axial, circumferential or inclined orientations. However, the deformation of pipe due to the weight of soil makes the axially oriented external cracks more vulnerable to failure. Hence only axial cracks have been studied in this research (Fig. 2).

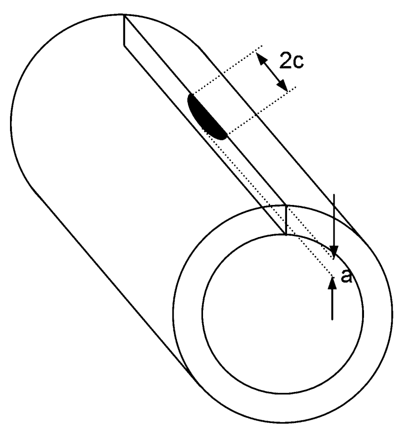


Fig. 2. A 3-dimensional semi-elliptical external crack.

In general, a surface crack can be considered semi-elliptical, since according to Lin and Smith [12], a crack with any initial arbitrary shape becomes semi-elliptical and then propagates in a semi-elliptical shape. In the present research, the crack configuration is described by some non-dimensional parameters, namely, relative wall thickness (t/R), relative crack depth (a/t), crack aspect ratio (a/c), elasticity moduli ratio of pipe and soil (E_p/E_s) and the circumferential position of crack on pipe (θ) (Fig. 3). The 20-node iso-parametric brick elements were employed in the finite element code ANSYS [15] to simulate the cracked pipe. To achieve more accurate results, the quarter-point elements were utilized around the crack front in order to induce a square-root singularity of stress/strain field at the vicinity of crack front. Because of the large number of required analyses and in order to save time, a code in ANSYS Parametric Design Language (APDL) was developed. The proposed code allows the user to obtain automatically the variations of SIF along the crack front for a given set of non-dimensional parameters. The proposed code takes the materials properties and the geometric parameters of pipe and soil and then generates a complicated finite element model of the buried pipe with a 3D semi-elliptical crack.

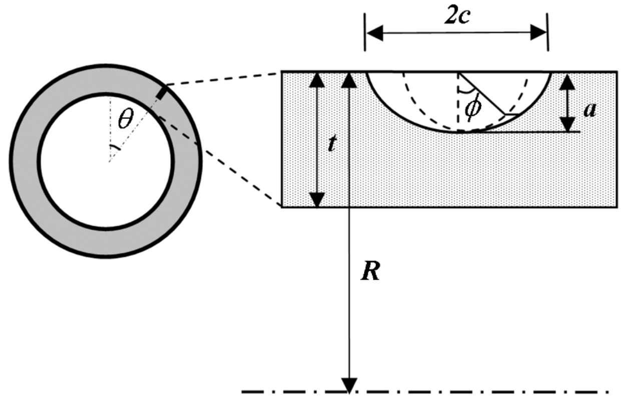


Fig. 3. Geometry of the pipe with an external axial crack of semi-elliptical shape.

The symmetry conditions in the longitudinal direction were exploited to reduce the computational effort. First, a quarter of the cylinder was modeled. The generated quarter was then mirrored to form one half of the pipe and finally only this half of the model was analyzed. As mentioned earlier, the crack front should be surrounded by singular elements, hence a small volume called the crack tunnel comprising the singular elements were created around the crack front. Then other parts of cylinder were meshed (Fig. 4).

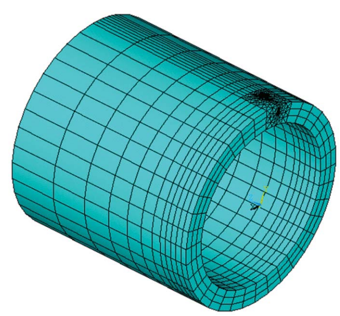


Fig. 4. Finite element model of pipe containing an axial crack.

In order to validate the APDL code developed in ANSYS, the classical problem of a pipe containing a semi-elliptical crack and loaded by a uniform internal pressure was analyzed. The finite element results for the stress intensity factors along the crack front correlated reasonably well with those reported by Raju and Newman [6,7] with a typical error of about 3%.

The Burns and Richard model [16] was employed to simulate the pressure applied on the pipe due to the soil weight. This model provides a varying distributed load on the external pipe surface. The distributed load is symmetric relative to the vertical diameter of the pipe cross section. It is a maximum at the point on top of the pipe and decreases towards the points located along the horizontal diameter of the pipe section. For the sake of brevity, more details of the Burns and Richard model are referred to [16]. As a closed form solution, the Burns and Richard model has been shown to correlate well with the instrumented field studies on pipes

[17]. The specifications of surrounding soil required for simulating this model in the developed APDL code are: height of soil over pipe, ratio of elasticity moduli of pipe and soil, soil specific weight and Poisson's ratio of soil.

3. STRESS INTENSITY FACTORS CALCULATION

The stress intensity factor is one of the most important parameters in fracture mechanics. Brittle fracture in cracked engineering components is usually examined by SIF. After the crack detection in a structure, SIF can be calculated by various experimental or theoretical methods such as finite element analysis. Once SIF is known the risk of brittle fracture can be evaluated by using an appropriate fracture criterion. The mode I and mode II stress intensity factors (K_I and K_{II}) can be calculated from finite element analysis based on the displacement functions (u and v) in a local coordinate system fixed to the crack tip (see Fig. 5).

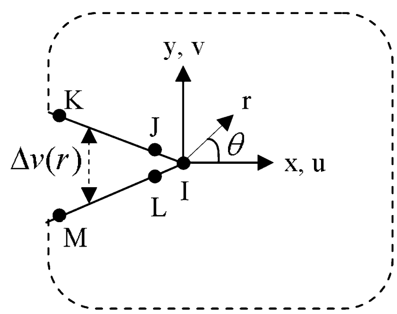


Fig. 5. Local Cartesian coordinates on crack tip.

$$K_I = \sqrt{2\pi}\frac{2G}{(1+k)}\frac{\Delta v(r)}{\sqrt{r}} \tag{1}$$

$$K_{II} = \sqrt{2\pi}\frac{2G}{(1+k)}\frac{\Delta u(r)}{\sqrt{r}} \tag{2}$$

where r is the radial distance from the crack front in the polar coordinates, G is the shear modulus, υ is Poisson's ratio, and k can be defined for plane strain and plane stress problems as follows:

$$k = \begin{cases} 3-4v; & \text{Plane strain} \\ 3v/(1+v); & \text{Plane stress} \end{cases} \tag{3}$$

In Eqs. (1) and (2), Δv and Δu are the relative displacements of near-crack-tip nodes in y and x directions, respectively. By fixing the center of coordinate system at each point along the crack front, the stress intensity factor can be determined for the corresponding point from Eqs (1) and (2). ANSYS finite element code can calculate SIFs with reasonably high accuracy. It obtains displacement functions near the crack tip from finite element analysis and then calculates SIFs according to Eq. (1) and (2). Hereafter the SIFs are considered in the non-dimensional form as:

$$K_{i,non} = \frac{K_i}{K_0}, \quad i = I, II,$$
$$K_0 = \sigma\sqrt{\pi a}, \tag{4}$$
$$\sigma = \gamma h$$

in which a is the crack depth, γ and h are the specific weight and the height of soil over the pipe, respectively.

4. FINITE ELEMENT RESULTS

As mentioned earlier, the axial (or longitudinal) cracks in buried pipelines are more critical than the circumferential cracks, therefore only axial cracks have been studied in this paper. A longitudinal semi-elliptical crack on the outer surface of the buried pipe was investigated by finite element analysis and by taking the advantages of ANSYS-APDL.

A crack located at the outer surface of pipe is called an external crack. The effects of non-dimensional parameters t/R, a/t, a/c and Ep/Es on K_I and K_{II} have been examined. The finite element results showed that the external cracks on horizontal ($\theta = \pm\pi/2$) or vertical directions ($\theta=0$, π) were under pure mode-I loading, otherwise cracks experienced mixed-mode loading. This can be seen for example from Fig. 6 which shows K_{II} is always zero when $\theta=0$ or $\pi/2$. This is because for the axial cracks at $\theta=0$ or $\pi/2$ directions, the only stress component acting due to the soil weight over the pipe, is normal to the crack planes and there is no shear components. Moreover, it can be observed from Fig. 6 that the maximum value of SIF in mode-II is attained when $\theta=45^{\circ}$.

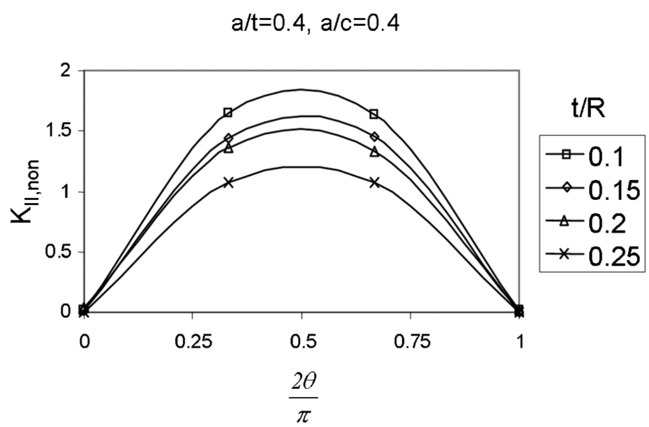


Fig. 6. Effect of crack orientation on K_{II} at deepest point for external semi-elliptical axial crack in buried pipe.

The circumferential crack position in buried pipe had a significant influence on SIFs. As Fig. 7 represents, the crack experienced maximum value of K_I at horizontal orientation (i.e. $2\theta/\pi=1$) and by approaching the vertical orientation, the value of K_I decreased. Therefore, the critical position for maximum crack opening in buried pipelines is horizontal direction.

The finite element results for the external crack in buried pipes revealed that the mode I and mode II SIFs increase as the relative wall thickness (t/R) decreases. This can be seen in Fig. 8

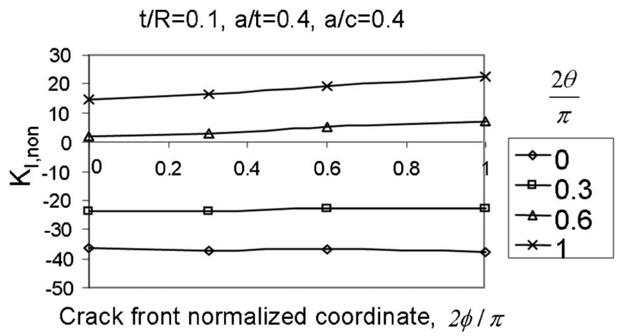


Fig. 7. Variations of $K_{I,non}$ along the crack front for various crack positions θ.

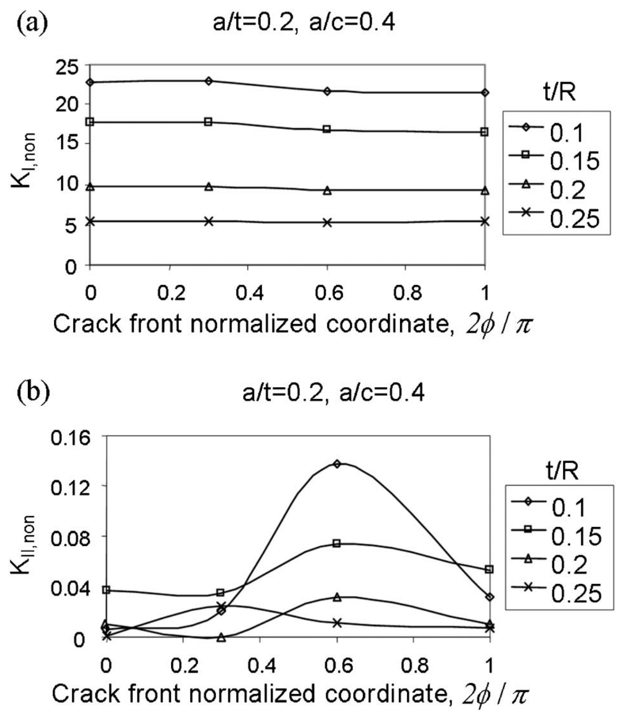
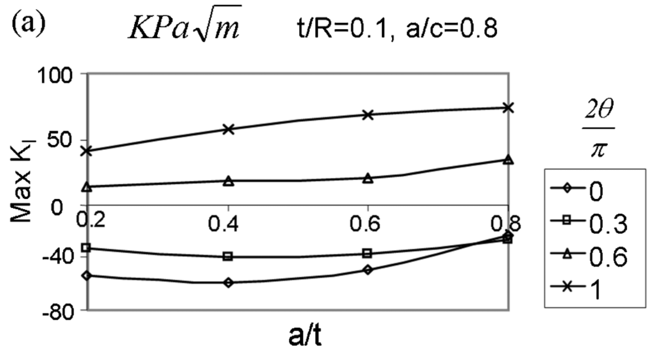


Fig. 8. Effect of relative wall thickness (t/R) on SIFs of an external semi-elliptical crack, a) mode I, b) mode II.

where the variations of $K_{I,non}$ and $K_{II,non}$ are shown along the crack front for various values of wall thickness ratio (t/R).

Fig. 9 shows how the maximum values of K_I and K_{II} along the crack front varied with the crack depth ratio (a/t). It is seen that both K_I and K_{II} increased when a/t became larger. In other words, as the crack penetrated in the pipe wall, it became more critical.

The mode I stress intensity factor at the deepest point of the crack front behaved in a descending manner as the crack aspect ratio (a/c) increased (see Fig. 10(a)). Moreover, Fig. 10(a) indicates the transition of maximum SIF from deepest point ($2\phi/\pi = 0$) to corner point ($2\phi/\pi = 1$), as the aspect ratio varies. Carpinteri [18] analyzed semi-elliptical cracks in metallic round bars under axial loading and observed a similar transition phenomenon. He showed that for all values of relative crack depth, there is an aspect ratio in which the point of



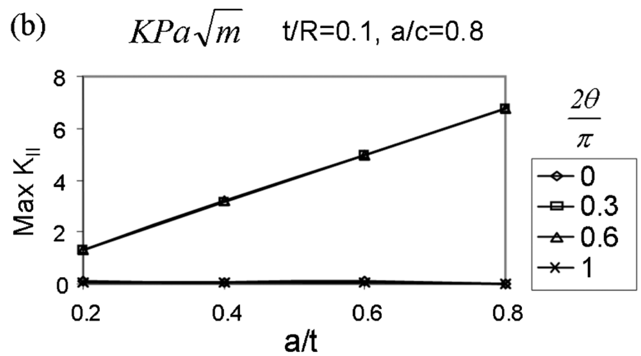


Fig. 9. Effect of relative crack depth (a/t) on SIFs of an external semi-elliptical crack in, a) mode I, b) mode II.

maximum SIF shifts from the deepest point to the corner point. Furthermore he showed that the aspect ratio at which the transition phenomenon takes place is dependent upon the relative crack depth and it decreases as the relative crack depth increases. Similar trend was observed in a cracked buried pipe. As shown in Fig. 10(b), by increasing the a/t ratio, the critical aspect ratio corresponding to the maximum SIF transition reduced.

The results shown in Figs 6 to 10 were obtained from FE analyses in which the ratio of elasticity modulus E_p/E_s was a fixed value of 6×10^3. However, this ratio affects directly the pipe deformation and consequently the extent of crack opening or sliding. As mentioned earlier, the critical position of the external crack is along the horizontal diameter of the pipe section. Hence, the influence of the elasticity moduli of soil and pipe was investigated only for the cracks that exist in this position. As illustrated in Fig. 11, a higher value of E_p/E_s resulted in higher values of mode I and mode II SIFs. In other words, K_I and K_{II} increased as the soil surrounding the pipe became less stiff.

It is noteworthy that, most of the buried pipelines are also subjected to a considerable internal pressure. The stress intensity factors of semi-elliptical cracks in pipes under internal pressure have already been calculated by other researchers e.g. [7]. Therefore, the state of cracks in buried pipeline under internal pressure can be determined by superimposing the values of SIFs for internal pressure and the ones calculated in the current paper due to the soil weight over the pipe, as long as the pipe exhibits a linear elastic behavior.

The negative values of K_I in some Figs, determined by considering the problem in the linear elastic framework, represent a closing tendency in crack deformation. However, when the buried pipe is also under an internal pressure, the resultant SIFs which is the sum of SIFs due to

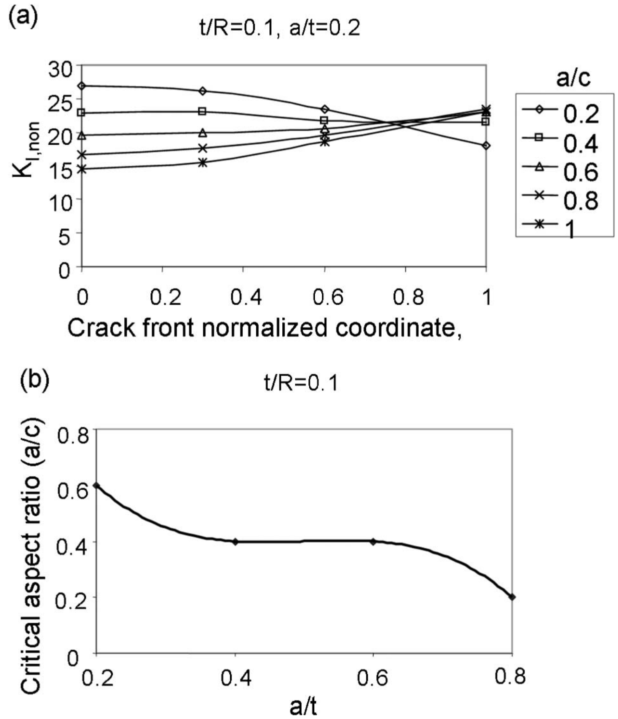


Fig. 10. Effect of crack aspect ratio a/c on mode I SIF for an external semi-elliptical crack in a buried pipe, a) Mode I SIF distribution along the crack front, b) critical (a/c) at which maximum SIF transition takes place as a function of (a/t).

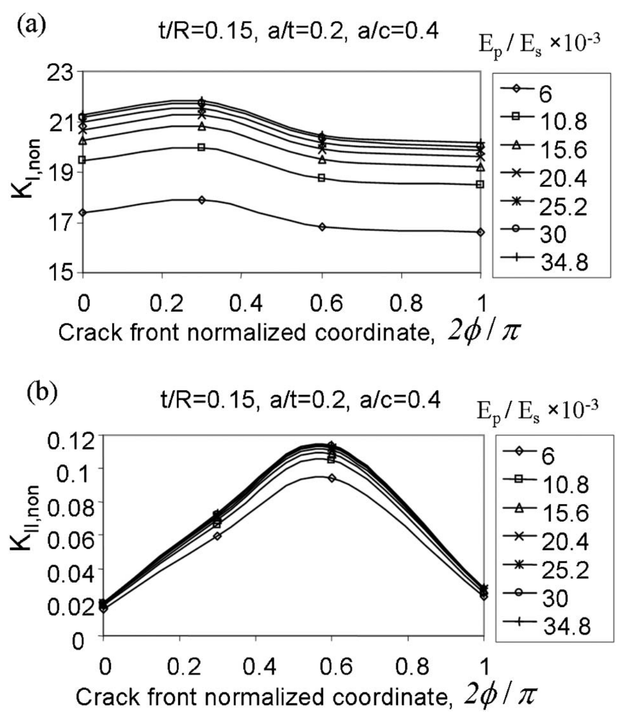


Fig. 11. Effect of E_p/E_s on stress intensity factors for an external semi-elliptical crack in a buried pipe, a) mode I, b) mode II.

the internal pressure and the soil weight may become positive. Thereby, the negative value of SIF presented in this paper can be helpful for analyzing such situations.

5. SUMMARY AND CONCLUSIONS

In this research, the stress intensity factors K_I and K_{II} were determined for an external crack in a pipe buried in the soil. The results can be used for evaluating the load bearing capacity of cracked buried pipes. The results showed that the soil weight over the pipe caused mixed mode conditions for the external crack. Although the cracks at orientations other than the horizontal and vertical directions of the pipe are under mixed mode condition, the value of mode II SIF in comparison with the mode I SIF is much lower. Moreover the cracks oriented at 45^o from the vertical direction, experienced the highest K_{II}. The circumferential crack orientation (θ) in the buried pipe had a significant influence on SIFs. The external cracks at the vicinity of horizontal and vertical directions of pipe behaved completely differently. This is because the weight of soil over the pipe had the most destructive effect on the external cracks located at horizontal direction. Conversely, it led to crack closure (negative SIF) for cracks located at the vertical orientation. The results also revealed that, according to the Burns and Richard model, the relative stiffness of pipe and its surrounding soil affected the stress intensity factors significantly. The increase of relative crack depth (a/t), the decrease of relative wall thickness (t/R) and the decrease of crack aspect ratio (a/c) resulted in an increase of SIFs.

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Nomenclature

r	radial distance from crack tip
R	outer radius of pipe
t	wall thickness of pipe
a	crack depth
c	half of crack length
G	shear modulus
U	displacement in X direction
v	displacement in Y direction
h	height of soil over pipe
E_P	pipe modulus of elasticity
E_S	soil modulus of elasticity
K_I	mode-I stress intensity factor
K_{II}	mode-II stress intensity factor
$K_{I,non}$	non-dimensional Mode-I stress intensity factor
$K_{II,non}$	non-dimensional Mode-II stress intensity factor
γ	soil specific weight
θ	circumferential crack position on pipe
ϕ	angular crack front position