REDUNDANCY RESOLUTION OF WIRE-ACTUATED PARALLEL MANIPULATORS

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Abstract

Redundant manipulators have potential advantages of using their degree(s) of redundancy to satisfy additional task(s). To achieve desirable performance criteria, various optimization techniques can be applied to redundant manipulators. In the work presented, the redundancy resolution of planar wire-actuated parallel manipulators is investigated at the kinematic and dynamic levels in order to perform desirable tasks while maintaining positive tensions in the wires. Local optimization routines are used in the simulations in order to minimize the norm of actuator forces/torques or to minimize the norm of the mobile platform velocity, subject to positive tension in the wires. This paper presents techniques to alter wire tensions, mobile platform trajectory, mobile platform velocity, and length rate of wires, in order to maintain positive wire tensions. The effectiveness of the presented approaches is studied through simulations of an example planar wire-actuated manipulator. The presented approaches can be utilized in the design of controllers, trajectory planning, and dynamic workspace analysis.

Keywords: wire-actuated parallel manipulators; redundancy resolution; dynamics; positive wire tension.

RÉSOLUTION DE LA REDONDANCE DES MANIPULATEURS PARALLÈLES ACTIONNÉS PAR CÂBLES

Résumé

Les manipulateurs redondants ont des avantages potentiels à utiliser leurs degrés de redondance pour accomplir des tâches additionnelles. Pour atteindre les critères de performance désirés, une variété de techniques d'optimisation peuvent être appliquées aux manipulateurs redondants. Dans notre travail, nous étudions la résolution de la redondance de manipulateurs parallèles plans actionnés par des câbles, concernant la cinématique et la dynamique, dans le but d'accomplir les tâches voulues tout en maintenant des tensions positives dans les câbles. Des enchaînements d'optimisations locales sont utilisés dans les simulations, dans le but de diminuer la norme des forces des actionneurs ou de diminuer la norme de la vitesse de la plateforme mobile, sous contrainte de maintenir les câbles tendus. Cet article présente des techniques pour modifier les tensions des câbles, la trajectoire de la plateforme mobile, la vitesse de la plateforme mobile, et les vitesses d'enroulement des câbles, dans le but de maintenir les tensions positives des câbles. L'efficacité des approches présentées est étudiée par l'exemple de simulations sur un prototype de manipulateur plan actionné par câbles. Ces approches peuvent être utilisées dans la conception de commandes, de planification de trajectoires, et dans l'analyse de l'espace de travail dynamique.

Mots-clés : manipulateurs actionnés par câbles; résolution de la redondance; dynamique; tension positive des câbles.

1. INTRODUCTION

Employing robot manipulators has drawn a considerable attention to repetitive operations, mass production, high precision, and for hazardous environments to assure the accuracy and reduce the cost involved. Redundant manipulators are an important research subject in the field of robotics. Kinematically redundant manipulators have more degrees of freedom than are necessary to perform a given task, which means that for a given end effector velocity, an infinite number of joint velocities exists. In closed-loop manipulators, redundancy can also be in the form of actuation. If the number of actuated joints is greater than the degrees of freedom of the manipulator, then for a given end effector trajectory and external forces/moments, an infinite number of actuator torques/forces exists. Redundant manipulators can use their degree(s) of redundancy to satisfy additional desirable task(s). Redundancy gives the manipulator great versatility and broad applicability to avoid obstacles, avoid structural limitations (e.g., joint limits), minimize joint forces/torques, and avoid singularities (e.g., configurations at which mobility of the manipulator is reduced and it would not be possible to impose an arbitrary motion to the end effector).

A configuration of a manipulator is a complete specification of the location of every point on the manipulator. A manipulator is said to be parallel if its kinematic structure takes the form of a closed-loop chain of links connected by joints. A parallel manipulator consists of a mobile platform (end effector) connected to a fixed base by several branches/legs/limbs. If all links of a manipulator move in a plane or in parallel planes, then the manipulator is called planar. Wire-actuated parallel manipulators are a special kind of parallel manipulator with multiple wires attached to a mobile platform and with the advantage of having a larger workspace that the mobile platform can reach, being able to be disassembled and reconfigured, increased manoeuvrability, and being lightweight and transportable. The light weight and the long range of wires allow high speed motion, as well. Wires can only apply force in the form of tension (i.e., pulling the mobile platform but not pushing it). Therefore, to design a fully controllable wire-actuated parallel manipulator, the manipulator has to be redundantly actuated. Thus, at least n+1 wires are required for a manipulator with n degrees of freedom (DOF) to keep positive tension in all wires [1–4].

Various optimization techniques have been applied to resolve redundancy of redundant manipulators. In most redundancy resolution schemes, there are dynamic interactions between the end effector motion and the self-motion (or null-motion) of the manipulator. Self-motion refers to those joint velocities that result in zero motion of the end effector. At the torque level, the null term of actuator forces/torques is interpreted as portions of actuator forces/torques that result in zero forces/moments the end effector could apply/resist. The proper use of the null term is of great importance in redundancy resolution. General kinematic, dynamic and stiffness analyses, as well as, the design of wire-actuated manipulators were investigated [1–8]. Choe et al. [2] investigated stiffness analysis of a wire-actuated manipulator and proposed a design to reduce vibration caused by the elasticity of wires. Kawamura et al. [1] investigated the kinematics and dynamics of a high speed wire-actuated parallel manipulator. They analysed the motion stability and investigated the non-linear elasticity of their proposed manipulator. Williams and Gallina [5] introduced translational planar wire-actuated manipulators and presented kinematic, static and dynamic modelling as well as the control architecture. Oh and Agrawal [7] investigated how to design positive tension controllers for wire-actuated manipulators with redundant wires to follow prescribed trajectories. Notash and Kamalzadeh [8] investigated the inverse dynamics of wire-actuated parallel manipulators with a constraining linkage and redundancy in actuation. The workspace of planar wire-actuated parallel manipulators was investigated in [3, 4, 9, 10].

When redundancy in actuation is used, obtaining a unique solution among the infinite inverse dynamic solutions is complicated and needs several considerations such as avoiding negative tensions in wires, reducing the actuator forces/torques, and reducing wire length rates. Therefore, wire actuation and redundancy add more complexity to the inverse dynamics and redundancy resolution in the wire-actuated parallel manipulators. In the presented work, it is attempted to resolve some of the challenges associated with the redundancy resolution of wire-actuated parallel manipulators, benefit from potential advantages of wire-actuated parallel manipulators, and extend and modify existing redundancy resolution techniques proposed by other researchers. The kinematic and dynamic modelling of an example planar wire-actuated parallel manipulator is developed in Section 2. The redundancy resolution schemes at the torque and velocity levels are given in Section 3 and Section 4, respectively. Simulation results at the torque and velocity levels are then developed in Section 5 in order to verify the effectiveness of the redundancy resolution techniques at the torque and velocity levels. The conclusion of the article is in Section 6.

2. MODELLING

The inverse dynamics of a planar wire-actuated parallel manipulator shown in Fig. 1(a) is investigated. According to McColl and Notash [3], comparing the layouts of Fig. 1(a) and Fig. 1(b), the layout shown in Fig. 1(a) offers a larger static workspace and a larger range of orientations without interference problems between the wires and the mobile platform. So, the layout of Fig. 1(a) is selected for the simulations. The dynamic problem is first formulated in the task space and then formulated in terms of the wire lengths and derivatives of wire lengths. Using the kinematic and dynamic analyses, the redundancy of the manipulator is resolved at various levels (as will be explained in Section 3 and Section 4). Figure 1(c) shows the coordinates and parameters used for the analysis of the planar wire-actuated parallel manipulator shown in Fig. 1(a). The fixed coordinate system $\Psi(X, Y)$, located at O, is attached to the base, while the moving coordinate system, $\Gamma(X', Y')$, is attached to the mobile platform at its centre of mass P with coordinates (x, y) in the base frame $\Psi(X, Y)$. The base attachment point of each wire (i.e., anchor) is denoted A_i. The position vector of point P with respect to $\Psi(X, Y)$ is given by $\mathbf{p}$ and $\mathbf{l}_i$ is the vector of the magnitude and direction of each wire length.

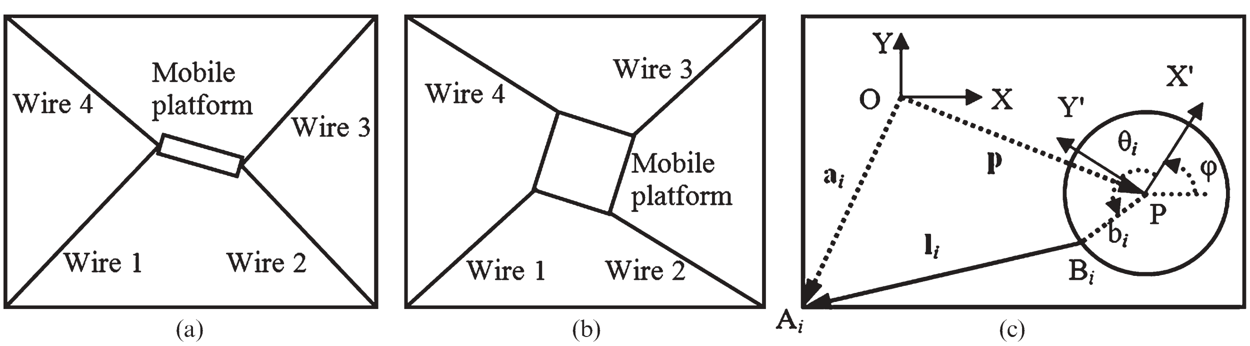


Fig. 1. (a), (b) Example planar 3-DOF wire-actuated parallel manipulators (c) coordinates and variables.

The pose (position and orientation) of the mobile platform can be written as

$$\begin{bmatrix} x \\ y \\ \varphi \end{bmatrix} = \begin{bmatrix} a_{i_x} - l_{i_x} - b_i \cos(\theta_i + \varphi) \\ a_{i_y} - l_{i_y} - b_i \sin(\theta_i + \varphi) \\ \varphi \end{bmatrix}, \; i = 1, \ldots, n \tag{1}$$

where a_{i_x} and a_{i_y} are the coordinates of position vector $\mathbf{a}_i$ of anchor i in $\Psi(\mathrm{X}, \mathrm{Y})$, l_i is the length of wire i, $l_{i_x} = l_i \cos\alpha_i$, $l_{i_y} = l_i \sin\alpha_i$, α_i is the direction for the axis of wire i, b_i is the distance between the mass centre P and the attachment point B_i of wire i on the mobile platform, θ_i is the orientation of line segments $\overline{PB_i}$ with respect to $\Gamma(\mathrm{X}', \mathrm{Y}')$, φ is the orientation of the mobile platform, and n ($n \geq 4$) is the number of wires.

Differentiating Eq. (1) results in

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} -c_i & l_i s_i & b_i s(\theta_i + \varphi) \\ -s_i & -l_i c_i & -b_i c(\theta_i + \varphi) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{l}_i \\ \dot{\alpha}_i \\ \dot{\varphi} \end{bmatrix}, \; i = 1, \ldots, n \tag{2}$$

where $\dot{x}$ and $\dot{y}$ are the components of linear velocities of the mobile platform and $\dot{\varphi}$ is the angular velocity of the mobile platform, c_i, s_i and $s(\theta_i + \varphi)$ stand for $\cos\alpha_i$, $\sin\alpha_i$ and $\sin(\theta_i + \varphi)$, respectively, $\dot{l}_i$ is the length rate of wire i, and $\dot{\alpha}_i$ is the rate of change of wire angle α_i. The solution of $[\dot{l}_i \quad \dot{\alpha}_i \quad \dot{\varphi}]^T$ is given by

$$\begin{bmatrix} \dot{l}_i \\ \dot{\alpha}_i \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} -c_i & -s_i & b_i s(\theta_i + \varphi - \alpha_i) \\ \dfrac{s_i}{l_i} & -\dfrac{c_i}{l_i} & -\dfrac{b_i c(\theta_i + \varphi - \alpha_i)}{l_i} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\varphi} \end{bmatrix}, \; i = 1, \ldots, n \tag{3}$$

To relate wire length rates to the velocity of the mobile platform, the first row of Eq. (3) is used to construct the overall inverse velocity solution as

$$\begin{bmatrix} \dot{l}_1 \\ \vdots \\ \dot{l}_n \end{bmatrix} = - \begin{bmatrix} c_1 & s_1 & -b_1 s(\varphi + \theta_1 - \alpha_1) \\ \vdots & \vdots & \vdots \\ c_n & s_n & -b_n s(\varphi + \theta_n - \alpha_n) \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\varphi} \end{bmatrix} = -\mathbf{J} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\varphi} \end{bmatrix} \tag{4}$$

The Jacobian matrix $\mathbf{J}$ is defined as the negative of the coefficient of the mobile platform velocity in Eq. (4). It should be noted that within the context of serial manipulators the Jacobian matrix is referred to as the coefficient matrix of joint velocities. The alternate forward velocity solution of Eq. (4) is

$$[\dot{x} \quad \dot{y} \quad \dot{\varphi}]^T = -\mathbf{J}^{\#} [\dot{l}_1 \quad \ldots \quad \dot{l}_n]^T \tag{5}$$

where $\mathbf{J}^{\#}$ is the generalized (Moore-Penrose) inverse of $\mathbf{J}$, i.e., $\mathbf{J}^{\#} = (\mathbf{J}^T \, \mathbf{J})^{-1} \, \mathbf{J}^T$. Using techniques based on the generalized inverse of matrices may, in general, lead to non-invariant and inconsistent results (i.e., results that are not invariant with respect to changes in the reference frame and/or changes in the dimensional units used to express the problem [11]). In such cases, the generalized inverse could be weighted using a suitable weighing metric to solve linear physical systems without producing inconsistencies and errors resulting from mixed physical units in the problem formulation. According to Doty et al. [11], when the left-hand side of Eq. (4) is unit consistent, $\mathbf{J}^{\#}$ is invariant to the choice of any weighing metric. So, using a weighing metric will not be required.

For the dynamic modelling, Eq. (5) is differentiated as

$$[\ddot{x} \quad \ddot{y} \quad \ddot{\varphi}]^T = -\frac{d}{dt}\mathbf{J}^{\#}[\dot{l}_1 \quad \ldots \quad \dot{l}_n]^T - \mathbf{J}^{\#}[\ddot{l}_1 \quad \ldots \quad \ddot{l}_n]^T \tag{6}$$

From dynamic force and moment balances

$$\mathbf{J}^T \begin{bmatrix} \tau_1 \\ \vdots \\ \tau_n \end{bmatrix} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I_z \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\varphi} \end{bmatrix} + \begin{bmatrix} 0 \\ -mg \\ 0 \end{bmatrix} + \begin{bmatrix} F_{ext_x} \\ F_{ext_y} \\ M_{ext_z} \end{bmatrix} = \mathbf{M} \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\varphi} \end{bmatrix} + \mathbf{g} + \begin{bmatrix} F_{ext_x} \\ F_{ext_y} \\ M_{ext_z} \end{bmatrix} \tag{7}$$

where m and I_z are the mass and the moment of inertia of the mobile platform respectively, $g = 9.81$ m/s^2 is the gravitational constant, $\ddot{x}$ and $\ddot{y}$ are the components of linear accelerations and $\ddot{\varphi}$ is the angular acceleration of the mobile platform, $[\tau_1 \ldots \tau_n]^T$ is the vector of dynamic wire forces, F_{ext_x} and F_{ext_y} are the components of the external force acting on the mobile platform, M_{ext_z} is the external moment acting on the mobile platform about the z-axis, $\mathbf{M}$ is the inertia matrix, and $\mathbf{g}$ is the vector of gravitational force. Assuming no external forces/moments acting on the mobile platform, the solution to Eq. (7) is given by

$$[\tau_1 \quad \ldots \quad \tau_n]^T = \mathbf{J}^{T\#}\left(\mathbf{M}[\ddot{x} \quad \ddot{y} \quad \ddot{\varphi}]^T + \mathbf{g}\right) + (\mathbf{I} - \mathbf{J}^{T\#}\mathbf{J}^T)\boldsymbol{\lambda}_\tau = \boldsymbol{\tau}_p + \boldsymbol{\tau}_h \tag{8}$$

The first term after the first equal sign of Eq. (8) is denoted by $\boldsymbol{\tau}_p$ which is the minimum norm (particular) solution of Eq. (7) derived from the generalized inverse of matrix $\mathbf{J}^T$ and $\mathbf{J}^{T\#}$ is the generalized inverse of $\mathbf{J}^T$, i.e., $\mathbf{J}^{T\#} = \mathbf{J}\,(\mathbf{J}^T\,\mathbf{J})^{-1}$, while the second term is denoted by $\boldsymbol{\tau}_h$ which is the homogeneous solution that maps the free vector $\boldsymbol{\lambda}_\tau$ to the null space of $\mathbf{J}^T$ [3, 4]. The determination of $\boldsymbol{\lambda}_\tau$ depends on the optimization of a criterion function.

To resolve the redundancy at the torque level, for given trajectories of the mobile platform, a $\boldsymbol{\lambda}_\tau$ is identified (if it exists) at each instant such that minimum norm actuator forces/torques are achieved avoiding negative tension in the wires. It should be noted that throughout this paper, the term "norm" stands for the 2-norm, and all minimizations correspond to the 2-norm of the relevant vector.

The constraint tension function is

$$\boldsymbol{\tau} = \boldsymbol{\tau}_p + (\mathbf{I} - \mathbf{J}^{T\#}\mathbf{J}^T)\boldsymbol{\lambda}_\tau \geq \boldsymbol{\tau}_{\min} \tag{9}$$

where $\boldsymbol{\tau}_{\min}$ is the vector of minimum allowable wire tensions. Substituting Eq. (6) into Eq. (8) results in

$$\begin{bmatrix}\tau_1 & \dots & \tau_n\end{bmatrix}^T = -\mathbf{J}^{T\#}\mathbf{M}\left(\frac{d}{dt}\mathbf{J}^{\#}\right)\begin{bmatrix}\dot{l}_1 & \dots & \dot{l}_n\end{bmatrix}^T - \mathbf{J}^{T\#}\mathbf{M}\mathbf{J}^{\#}\begin{bmatrix}\ddot{l}_1 & \dots & \ddot{l}_n\end{bmatrix}^T + \mathbf{J}^{T\#}\mathbf{g} + (\mathbf{I} - \mathbf{J}^{T\#}\mathbf{J}^T)\boldsymbol{\lambda}_\tau \tag{10}$$

Equation (10) represents the inverse dynamic equation of the wire-actuated parallel manipulator in terms of wire lengths and their derivatives.

In the following sections, the resolution of redundancy considering the minimization of wire tensions will be referred to as the redundancy resolution at the torque level, and the resolution of redundancy considering the minimization of velocity will be referred to as the redundancy resolution at the velocity level. Moreover, the term ''positive tension'' is referred to tensions that are greater than the minimum allowable wire tension.

3. MINIMIZING WIRE TENSIONS FOR A GIVEN TRAJECTORY

To resolve redundancy at the torque level, given the trajectory of the mobile platform, it is required to know the Jacobian matrix at each time instant to construct the constraint function of Eq. (9) as a function of the decision variable $\boldsymbol{\lambda}_\tau$. So, the optimization problem is formulated as follows:

$$\begin{aligned} &\text{minimize } \boldsymbol{\tau}^T\boldsymbol{\tau} = \tau_1^2 + \cdots + \tau_n^2 \\ &\text{subject to } \boldsymbol{\tau} = \boldsymbol{\tau}_p + (\mathbf{I} - \mathbf{J}^{T\#}\mathbf{J}^T)\boldsymbol{\lambda}_\tau \geq \boldsymbol{\tau}_{\min} \end{aligned} \tag{11}$$

and for each pose of the mobile platform, the value of $\boldsymbol{\lambda}_\tau$ is calculated (if it exists) such that minimum positive wire tensions are maintained.

Given the trajectory of the mobile platform, $\mathbf{x}(t) = [x(t)\ y(t)\ \varphi(t)]^T$, and using the inverse velocity analysis of Eq. (4), the Jacobian matrix at each time instant is calculated recursively. Using Eq. (4) and substituting the initial Jacobian matrix, $\mathbf{J}|_{t_0}$, and the mobile platform trajectory at t_1, $\begin{bmatrix}\dot{x} & \dot{y} & \dot{\varphi}\end{bmatrix}_{t_1}^T$, the vector of wire length rates at t_1, $\begin{bmatrix}\dot{l}_1 & \dots & \dot{l}_n\end{bmatrix}_{t_1}^T$, is calculated as

$$-\mathbf{J}|_{t_0}\begin{bmatrix}\dot{x} & \dot{y} & \dot{\varphi}\end{bmatrix}_{t_1}^T = \begin{bmatrix}\dot{l}_1 & \dots & \dot{l}_n\end{bmatrix}_{t_1}^T \tag{12}$$

Following that, the vector of wire lengths $\mathbf{l}$ at t_1, $\begin{bmatrix}l_1 & \dots & l_n\end{bmatrix}_{t_1}^T$ is obtained as

$$\mathbf{l}|_{t_1} = \dot{\mathbf{l}}\big|_{t_1}\Delta t + \mathbf{l}|_{t_0} \tag{13}$$

where $\dot{\mathbf{l}}$ is the vector of wire length rates , and Δt is the time increment. By substituting $\mathbf{l}|_{t_1}$ into Eq. (1), the orientation of wire i, α_i, at t_i is derived for $i = 1,\dots, n$. Therefore, the Jacobian matrix at t_i is obtained.

By repeating the procedure, the Jacobian matrix at each time instant will be derived and substituted into Eq. (9) in order to identify $\boldsymbol{\lambda}_\tau$ such that minimum positive wire tensions are maintained. As explained in Eq. (11), the objective function to be used in the optimization problem is to minimize the norm of tensions in the wires subject to positive wire tensions.

4. RESOLVING REDUNDANCY WHEN MINIMIZING VELOCITY

In order to resolve redundancy at the velocity level considering the minimization of the mobile platform velocity or the norm of wire length rates, Eq. (9) and the mobile platform velocity should be related. In fact, the trajectory of the mobile platform should be modified instantaneously such that either the minimum norm of the wire length rates or the minimum norm of the mobile platform velocity is achieved subject to positive wire tensions. A similar approach was proposed by Oh and Agrawal [7] to minimize the sum of the norms for $\mathbf{x}(t) = [x(t)\ y(t)\ \varphi(t)]^T$ and its derivatives at specific time instants. They used a finite collocation grid in time to form a finite number of inequality constraints of Eq. (9). In the approach proposed in this section, either the norm of the wire length rates or the norm of the mobile platform velocity is minimized at each time instant.

In this approach, for a given mobile platform trajectory, $\mathbf{x}_o(t) = [x_o(t)\ y_o(t)\ \varphi_o(t)]^T$, the trajectory is modified at each time instant such that the objective function and the constraint function are satisfied. For this purpose, the mobile platform trajectory is chosen to have the following form

$$\mathbf{x}(t) = \begin{bmatrix} x \\ y \\ \varphi \end{bmatrix} = \begin{bmatrix} a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^3 (t_f - t) + a_5 t^3 (t_f - t)^2 \\ b_0 + b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^3 (t_f - t) + b_5 t^3 (t_f - t)^2 \\ c_0 + c_1 t + c_2 t^2 + c_3 t^3 + c_4 t^3 (t_f - t) + c_5 t^3 (t_f - t)^2 \end{bmatrix} + \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} t^3 (t_f - t)^3 \tag{14}$$

where t_f is the final time instant, a_i, b_i and c_i are the known constant coefficients, and p_1, p_2 and p_3 are the unknown variable coefficients (the values of p_1, p_2 and p_3 will be calculated at each time instant). The first term on the right-hand side of Eq. (14) is denoted by vector $\mathbf{x}_o(t) = [x_o(t)\ y_o(t)\ \varphi_o(t)]^T$, and the second term by vector $\mathbf{x}_{var}(t, p_1, p_2, p_3) = [x_{var}(t, p_1)\ y_{var}(t, p_2)\ \varphi_{var}(t, p_3)]^T$. Differentiating Eq. (14) with respect to time results in

$$\dot{\mathbf{x}}(t, p_1, p_2, p_3) = \dot{\mathbf{x}}_o(t) + \dot{\mathbf{x}}_{var}(t, p_1, p_2, p_3) \tag{15}$$

where $\dot{\mathbf{x}}$, $\dot{\mathbf{x}}_o$ and $\dot{\mathbf{x}}_{var}$ are the time derivatives of $\mathbf{x}$, $\mathbf{x}_o$ and $\mathbf{x}_{var}$, respectively. $\dot{\mathbf{x}}_{var}$ is referred to as the variable portion of the mobile platform velocity. The arguments of $\mathbf{x}$, $\mathbf{x}_o$ and $\mathbf{x}_{var}$ and their derivative will be omitted in the following paragraphs.

The fifth order x_o, y_o and φ_o trajectories are chosen to satisfy eighteen initial and final boundary conditions of the mobile platform trajectory, $\mathbf{x}$, and its derivatives, i.e., $(x, \dot{x}, \ddot{x}, y, \dot{y}, \ddot{y}, \varphi, \dot{\varphi}, \ddot{\varphi})_0$ and $(x, \dot{x}, \ddot{x}, y, \dot{y}, \ddot{y}, \varphi, \dot{\varphi}, \ddot{\varphi})_f$, respectively. The eighteen coefficients a_i, b_i and c_i are defined from these eighteen boundary conditions. It should be noted that the addition of the second term on the right-hand side of Eq. (14), $\mathbf{x}_{var}$, does not affect the boundary conditions of the mobile platform trajectories and their derivatives because of the chosen form of $t^3(t_f - t)^3$. Using this additional term on the right-hand side, the constraint function represented by Eq. (9) will become a function of p_1, p_2 and p_3, and in the optimization problem p_1, p_2 and p_3 are changed such that the objective function is satisfied as well as achieving positive wire tensions. Three objective functions are defined and either of them could be used in order to resolve redundancy at velocity level. The objective functions are listed as

$$\text{minimizing } \dot{\mathbf{x}}^T \dot{\mathbf{x}} = \dot{x}^2 + \dot{y}^2 + \dot{\varphi}^2 \tag{16}$$

$$\text{minimizing } \dot{\mathbf{l}}^T\dot{\mathbf{l}} = \dot{l}_1^2 + \cdots + \dot{l}_n^2 \tag{17}$$

$$\text{minimizing } \dot{\mathbf{x}}_{\text{var}}^T\dot{\mathbf{x}}_{\text{var}} = \dot{x}_{\text{var}}^2 + \dot{y}_{\text{var}}^2 + \dot{\varphi}_{\text{var}}^2 \tag{18}$$

where $\dot{x}_{\text{var}}$, $\dot{y}_{\text{var}}$ and $\dot{\varphi}_{\text{var}}$ are the components of the variable portion of the mobile platform velocity. The objective function of Eq. (16) minimizes the norm of the mobile platform velocity. Since the first term on the right-hand side of Eq. (14) is constant, the first derivative of the second term (i.e., $\dot{\mathbf{x}}_{\text{var}}$) is used in the optimization problem of Eq. (18). The aim of using the first derivative of the variable term on the right-hand side of Eq. (14) (i.e., $\dot{\mathbf{x}}_{\text{var}}$) to define the third objective function, is to minimize $\dot{\mathbf{x}}_{\text{var}}$ such that $\dot{\mathbf{x}}$ in Eq. (15) traces the given mobile platform velocity (i.e., $\dot{\mathbf{x}}_o$) as close as possible. All three objective functions are subject to

$$\boldsymbol{\tau}(t, p_1, p_2, p_3) = \boldsymbol{\tau}_p(t, p_1, p_2, p_3) + (\mathbf{I} - \mathbf{J}^{T\#}(t, p_1, p_2, p_3)\mathbf{J}^T(t, p_1, p_2, p_3))\boldsymbol{\lambda}_v \geq \boldsymbol{\tau}_{\min} \tag{19}$$

where $\boldsymbol{\lambda}_v$ at the velocity level is an arbitrary vector that is similar to $\boldsymbol{\lambda}_\tau$ at torque level. So, after calculating p_1, p_2 and p_3, at each time instant, the minimum $\boldsymbol{\lambda}_v$ is calculated (if it exists) that maintains positive tension in all wires. Resolving redundancy at the velocity level is only useful for applications in which the specified initial and final poses of the manipulator are of interest, such as pick and place, and spot welding. Since the trajectory is modified instantly, the proposed redundancy resolution scheme may cause jerky motion in addition to discontinuity in wire tensions as the manipulator moves along the trajectory.

Given the trajectory of the mobile platform as a function of p_1, p_2 and p_3, and using the inverse velocity analysis, the Jacobian matrix at each time instant is derived as a function of p_1, p_2 and p_3 using a similar recursive procedure explained in Section 3. In other words, Eq. (12) can be rearranged as

$$-\mathbf{J}(t_j, p_1, p_2, p_3)\dot{\mathbf{x}}(t_{j+1}, p_1, p_2, p_3) = \dot{\mathbf{l}}(t_{j+1}, p_1, p_2, p_3) \tag{20}$$

and Eq. (13) as

$$\mathbf{l}(t_{j+1}, p_1, p_2, p_3) = \dot{\mathbf{l}}(t_{j+1}, p_1, p_2, p_3)\Delta t + \mathbf{l}(t_j, p_1, p_2, p_3) \tag{21}$$

By substituting $\mathbf{l}$ (t_{j+1}, p_1, p_2, p_3) into Eq. (1), cos α_i and sin α_i at t_{j+1} are obtained as functions of p_1, p_2 and p_3 and as a result the Jacobian matrix at t_{j+1} is derived as a function of p_1, p_2 and p_3. $\mathbf{J}$ (t_{j+1}, p_1, p_2, p_3) is then substituted into Eq. (19),

$$\boldsymbol{\tau}(t_{j+1}, p_1, p_2, p_3) = \boldsymbol{\tau}_p(t_{j+1}, p_1, p_2, p_3) + (\mathbf{I} - \mathbf{J}^{T\#}(t, p_1, p_2, p_3)\mathbf{J}^T(t, p_1, p_2, p_3))\boldsymbol{\lambda}_v \geq \boldsymbol{\tau}_{\min} \tag{22}$$

in order to identify p_1, p_2, p_3 and $\boldsymbol{\lambda}_v$ such that the objective function is satisfied maintaining positive tension. In fact, p_1, p_2 and p_3 appearing in the objective function are changed such that the norm of $\dot{\mathbf{l}}^T\dot{\mathbf{l}}$ or $\dot{\mathbf{x}}^T\dot{\mathbf{x}}$ or $\dot{\mathbf{x}}_{\text{var}}^T\dot{\mathbf{x}}_{\text{var}}$ is minimized subject to positive tension in the wires.

5. SIMULATION

In the following subsections, the simulation results of redundancy resolution of the planar wire-actuated manipulator, shown in Fig. 1(a), are presented. The optimization problems are carried out in Matlab using the *fmincon* function to verify the optimization procedures discussed in Section 3 and Section 4. The upper limit on the actuator torques is calculated based on the maximum allowable wire tension of 525 N [4].

5.1. Simulation Results When Minimizing Wire Tensions

As explained in Section 3, the optimization problem is formulated as follows:

$$\begin{aligned} &\text{minimize } \boldsymbol{\tau}^T\boldsymbol{\tau} = \tau_1^2 + \tau_2^2 + \tau_3^2 + \tau_4^2 \\ &\text{subject to } \boldsymbol{\tau} = \boldsymbol{\tau}_p + (\mathbf{I} - \mathbf{J}^{T\#}\mathbf{J}^T)\boldsymbol{\lambda}_\tau \geq \boldsymbol{\tau}_{\min} \end{aligned} \tag{23}$$

and for each pose of the mobile platform, a value $\boldsymbol{\lambda}_\tau$ is calculated (if it exists) such that minimum positive wire tensions are maintained. With four wires, the constraint function of Eq. (9) is reduced to four linear inequalities in terms of $\boldsymbol{\lambda}_\tau$, where $\boldsymbol{\lambda}_\tau$ is reduced to a scalar. The termination tolerances placed on constraint violations were chosen as 10^{-3} N, on the objective function as 10^{-3} N^2, and on the estimated parameter values (i.e., λ_v) as 10^{-4} N.

The following coordinates {(−1, −0.75), (1, −0.75), (1, 0.75), (−1, 0.75)} (units in meters) are used for anchor position vectors $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4\}$ in the fixed frame, and the orientations of line segments $\overline{PB_i}$ with respect to the mobile platform frame, $\{\theta_1, \theta_2, \theta_3, \theta_4\}$, are {180, 0, 0, 180} (units in degree). The mass m and radius b_i of the mobile platform are 2 kg and 0.25 m, respectively, and the minimum allowable tension in the wires, $\boldsymbol{\tau}_{\min}$, is $[2\ 2\ 2\ 2]^T$ (units in Newtons). The mass moment of inertia of the mobile platform, I_z, is 0.0144 kg.m^2. The initial and final boundary conditions $(x, \dot{x}, \ddot{x}, y, \dot{y}, \ddot{y}, \varphi, \dot{\varphi}, \ddot{\varphi})_{0,f}$ are assumed to be $(0, 0, 0, 0, 0, 0, 0, 0, 0)_0$ and $(0.5 \text{ m}, 0, 0, 0.25 \text{ m}, 0, 0, 10 \text{ deg}, 0, 0)_f$, respectively, with $t \in [0, 1]$ s and time step of $\Delta t = 0.001$ s. So, a fifth order polynomial is used for the mobile platform trajectory satisfying eighteen boundary conditions of $\mathbf{x}(t)$, and its first and second derivatives.

Figures 2(a) and 2(b) show the defined motion of the mobile platform, regardless of whether positive tension in the wires is achieved or not. Figure 2(c) represents the wire length rates calculated using Eq. (12). It should be noted that the determination of wire length rates is independent of the optimization of the tension in the wires.

Figure 3(a) shows the minimum λ_τ that guarantees positive tension in the wires. Since the change in the orientation of the mobile platform is small (i.e., 10 deg) the optimization was

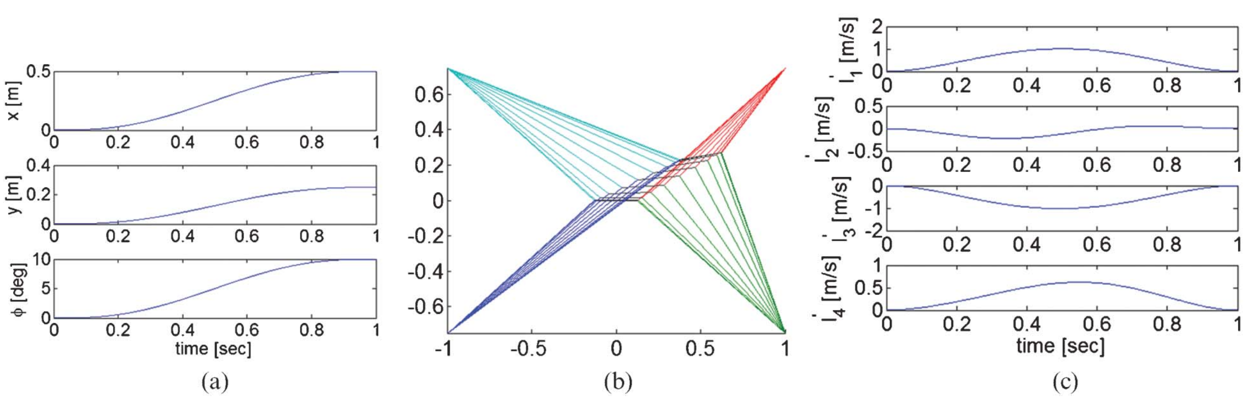


Fig. 2. (a) Pose of the mobile platform, (b) configuration change of the mobile platform, (c) wire length rates.

terminated successfully. If the change in the orientation of the mobile platform is not small enough, the pose of the mobile platform may not lie within the available workspace of the manipulator and the tension constraints may not be met. The plots of the particular solutions $\boldsymbol{\tau}_p$ (i.e., without null space contribution) are given in Fig. 3(b). As it can be seen, tensions in the third and fourth wires are negative. To maintain positive tensions in the third and fourth wires, null space contribution is used. Figure 3(c) illustrates the tension histories resulting from the substitution of minimum λ_τ (shown in Fig. 3a) into Eq. (9). The minimum allowable tension of 2 N in the fourth wire shows that the first three wires are critical for maintaining the configurations shown in Fig. 2(b).

In addition to the 2-norm of the wire tensions used in Eq. (23), higher norms were used, but for the given example, no significant change in the results was observed.

5.2. Simulation Results When Minimizing Velocity

For a given mobile platform trajectory, i.e., $\mathbf{x}_o(t) = [x_o(t)\ y_o(t)\ \varphi_o(t)]^T$ of Eq. (14), the trajectory is modified at each time instant such that:

$$\begin{aligned} &\text{minimize } \dot{\mathbf{x}}_{\text{var}}^T \dot{\mathbf{x}}_{\text{var}} = \dot{x}_{\text{var}}^2 + \dot{y}_{\text{var}}^2 + \dot{\varphi}_{\text{var}}^2 \\ &\text{subject to } \boldsymbol{\tau} = \boldsymbol{\tau}_p + (\mathbf{I} - \mathbf{J}^{T\#}\mathbf{J}^T)\boldsymbol{\lambda}_v \geq \boldsymbol{\tau}_{\min} \end{aligned} \tag{24}$$

and a value is calculated for p_1, p_2, p_3 and $\boldsymbol{\lambda}_v$ (if it exists), at each time instant, that maintains positive tension in the wires. With four wires, the constraint function of Eq. (19) is reduced to four linear inequalities in terms of $\boldsymbol{\lambda}_v$, where $\boldsymbol{\lambda}_v$ is reduced to a scalar.

The simulation parameters in this section are the same as in Section 5.1. For comparison, the given trajectory of the mobile platform, $\dot{\mathbf{x}}_o$, in this section is the same as the desired trajectory of the mobile platform at the torque level (discussed in Section 5.1).

Figures 4(a) and 4(b) show the motion of the mobile platform while maintaining positive tension in the wires. From Fig. 4, it can be seen that the mobile platform moves towards the up-right corner of the base (i.e., towards the third anchor) until it reaches its predefined final position and orientation. The termination tolerances placed on the constraint violations were chosen as 10^{-3} N, on the objective function as 10^{-3}, and on the estimated parameter values as 10^{-4} (m/s^6 for p_1 and p_2, rad/s^6 for p_3, and N for λ_v).

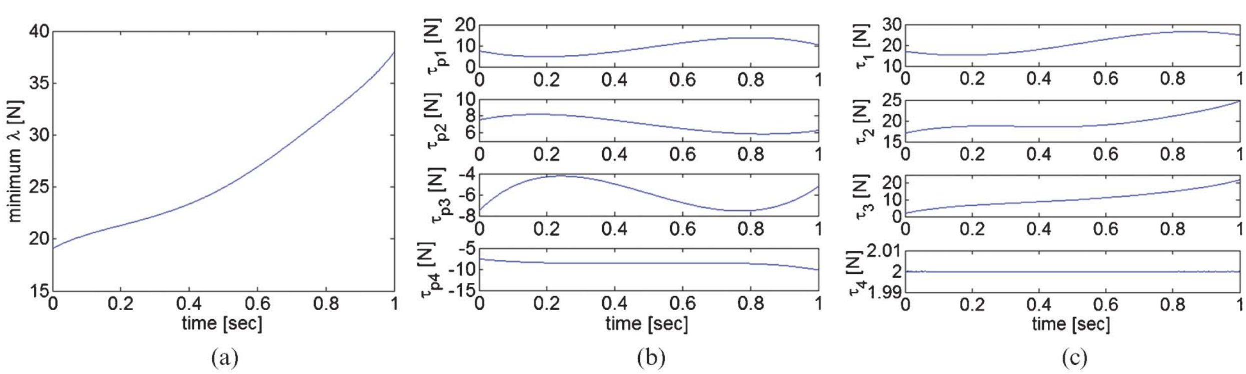


Fig. 3. (a) Minimum λ_τ to maintain positive tension in the wires, (b) tension in the wires without null space contribution, (c) wire tensions with null space contribution.

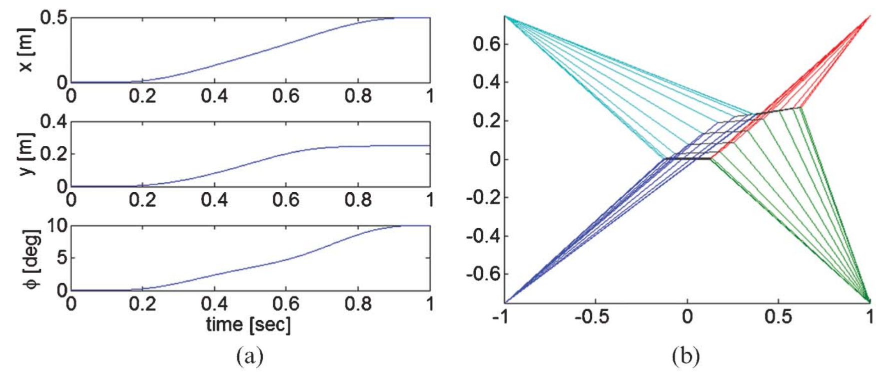


Fig. 4. (a) Pose of the mobile platform, (b) change in the configuration of the mobile platform.

Figure 5 shows the actual velocity of the mobile platform $\dot{\mathbf{x}}$ and the predefined components of mobile platform velocity $\dot{\mathbf{x}}_o$ and the variable portion of the mobile platform velocity $\dot{\mathbf{x}}_{\text{var}} = \dot{\mathbf{x}} - \dot{\mathbf{x}}_o$. The coefficients a_i, b_i and c_i are determined from the eighteen predefined boundary conditions $(x, \dot{x}, \ddot{x}, y, \dot{y}, \ddot{y}, \varphi, \dot{\varphi}, \ddot{\varphi})_{0,f}$ with $t \in [t_0, t_f]$. From the optimization problem represented by Eq. (24), it is desired that the actual velocity of the mobile platform, $\dot{\mathbf{x}}$, be as close to $\dot{\mathbf{x}}_o$ as possible. The variable portion of the mobile platform velocity illustrated in Fig. 5(b) shows how close $\dot{\mathbf{x}}$ is to $\dot{\mathbf{x}}_o$. As it can be seen from Fig. 5(a), for time instants before $t = 0.45$ s, the solid curves of $\dot{x}$, $\dot{y}$ and $\dot{\varphi}$ lie under the dash-dotted curves of $\dot{x}_o$, $\dot{y}_o$ and $\dot{\varphi}_o$, respectively. So, for the first half of the motion of the mobile platform the solution of Eq. (24) has resulted in velocity components (i.e., $\dot{x}$, $\dot{y}$ and $\dot{\varphi}$) which have lower magnitudes compared to the corresponding components of given velocity of the mobile platform $\dot{\mathbf{x}}_o$ (i.e., $\dot{x}_o$, $\dot{y}_o$ and $\dot{\varphi}_o$). Nevertheless, this is not the case for $\dot{x}$ and $\dot{\varphi}$ when compared with $\dot{x}_o$ and $\dot{\varphi}_o$, respectively, for the second half of the motion of the mobile platform. It should be noted that for comparison, $\dot{\mathbf{x}}_o$ was chosen the same as the desired velocity of the mobile platform, $\dot{\mathbf{x}}$, when resolving redundancy at the torque level in Section 5.1. Since at each time instant a new set of p_1, p_2, p_3 and λ_v that satisfies the objective and constraint functions is calculated, there is no guarantee that the solutions of p_1, p_2, p_3 and λ_v are continuous. The optimization tolerances and the initial guess for the optimization parameters can also affect the solutions of p_1, p_2, p_3 and λ_v. An approach to avoid

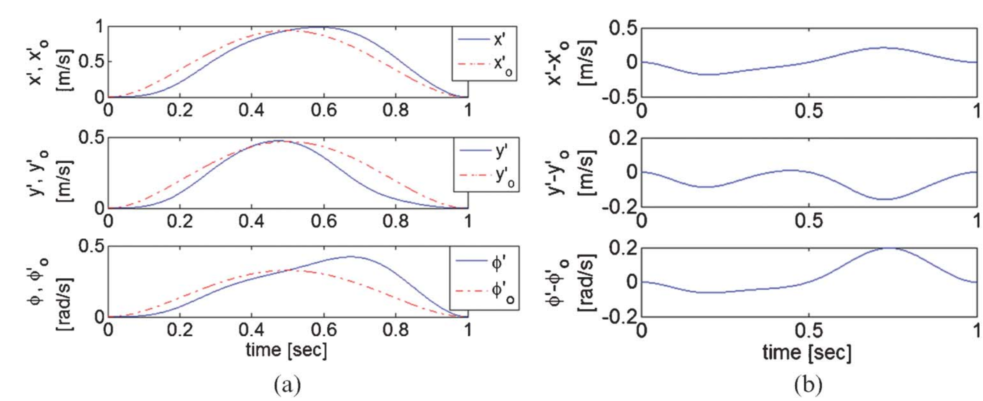


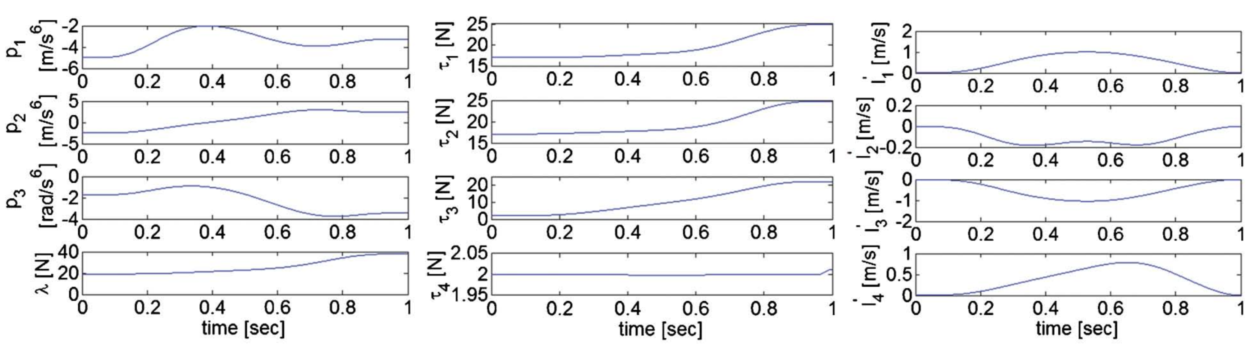
Fig. 5. Components of mobile platform velocity: (a) actual velocity of the mobile platform, $\dot{\mathbf{x}}$, and the predefined component of mobile platform velocity, $\dot{\mathbf{x}}_o$, (b) variable portion of the mobile platform velocity, i.e., $\dot{\mathbf{x}}_{\text{var}} = \dot{\mathbf{x}} - \dot{\mathbf{x}}_o$.

discontinuities is the globally continuous solution proposed by Oh and Agrawal [7] that determines a specific set of p_1, p_2, p_3 and λ_v, for the entire motion of the mobile platform using discrete poses of the mobile platform along the path. They used trajectory parameterization in conjunction with a finite collocation grid in time to ensure smooth tensions during the path. However, it should be noted that their proposed solution minimizes the mobile platform velocity at specific time instants (not at each time instant), i.e., $\dot{\mathbf{x}}$ in Eq. (15) traces $\dot{\mathbf{x}}_o$ as close as possible at specific time instants. In fact, their proposed scheme determines one constant set of p_1, p_2, p_3 and λ_v over $[0, t_f]$ to steer the mobile platform between given boundary conditions satisfying positive tension in the wires.

Figure 6(a) shows the solutions of p_1, p_2, p_3 and λ_v that guarantee positive tension in the wires. Figure 6(b) illustrates the tension histories resulting from the substitution of p_1, p_2, p_3 and λ_v (shown in Fig. 6(a)) into Eq. (22). As it can be seen from Fig. 6(b), the tension in the fourth wire is approximately 2 N. In the optimization problem used in this work (Eq. (24)), the minimum tension in the wires was considered to be 2 N. Similar to Fig. 3(c), Fig. 6(b) shows that the fourth wire is not critical for maintaining the configurations shown in Fig. 4(b). Figure 6(c) represents the wire length rates resulting from the substitution of p_1, p_2, p_3 and λ_v (shown in Fig. 6(a)) into Eq. (20). Considering Fig. 6(c) and the trajectory of the mobile platform that results in the configurations shown in Fig. 4(b), as the mobile platform moves the first and fourth wires are extended whereas the second and third wires are shortened, which is similar to Fig. 2(c). The change in the length of the first, third and fourth wires are more noticeable compared to the second wire.

6. DISCUSSION AND CONCLUSIONS

In this paper, two approaches to resolve actuation redundancy of planar wire-actuated parallel manipulators were investigated. In the first approach, a prescribed trajectory of the mobile platform was followed and the norm of wire tensions was minimized while maintaining positive tension in the wires. In the second approach, the desired mobile platform trajectory was modified at each time instant such that the minimum norm velocity of the mobile platform or minimum norm wire length rates was achieved, subject to positive wire tensions. Simulations of a 3-DOF planar wire-actuated parallel manipulator was developed minimizing either the norm of wire tensions or the norm of the variable portion of the mobile platform velocity. Based on the optimization results, it was observed that using the null space contribution the wire tensions



(a) (b) (c)

Fig. 6. (a) Solutions of p_1, p_2, p_3 and λ_v to maintain positive tensions in the wires, (b) tension in the wires with null space contribution, (c) wire length rates.

can be kept positive successfully. However, it should be noted that this is not always the case, e.g., when the manipulator has a large orientation value or when the change in the orientation of the mobile platform is not small enough. Based on the optimization criteria, the tolerances used in the optimization routine, and the optimization scheme, the torque level and velocity level approaches used continuous mobile platform trajectories and produced continuous wire tensions. At the velocity level, the modified trajectory of the mobile platform and the wire length rates were continuous as well. The choice of a proper optimization scheme, whether to minimize the tension in the wires or to minimize the velocity components, depends on the application of the manipulator. For example, resolving redundancy at the velocity level is, in general, recommended for trajectory planning [7] and/or for applications when the specified initial and final poses of the manipulator and/or initial and final velocities and accelerations of the mobile platform are of interest. Comparing the two described redundancy resolution techniques, the computational procedure at the torque level is faster and less complicated than that at the velocity level.

REFERENCES

1. Kawamura, S., Hitoshi, K. and Choe, W., "High-speed manipulation by using parallel wire-driven robots," *Robotica,* Vol. 18, pp. 13–21, 2000.
2. Choe, W., Hitoshi, K., Katsuta, K. and Kawamura, S., "A design of parallel wire driven robots for ultrahigh speed motion based on stiffness analysis," *Proceedings of the Japan/USA Symposium on Flexible Automation,* pp. 159–166, 1996.
3. McColl, D. and Notash, L., "Configuration and workspace analysis of planar wire-actuated parallel manipulators," *Proceedings of the 17th CISM-IFToMM Symp. Robot Design, Dynamics, and Control (RoManSy),* Japan, 8 pages, 2008.
4. McColl, D. and Notash, L., "Extension of antipodal theorem to workspace analysis of planar wire-actuated manipulators," *Proc. of 5th IFToMM Int. Workshop on Computational Kinematics,* Duisburg, Germany, 8 pages, 2009.
5. Williams, R.L. II and Gallina, P., "Translational Planar Cable-Direct-Driven Robots," *Journal of Intelligent and Robotic Systems,* Vol. 37, pp. 69–96, 2003.
6. Williams, R.L. II, Gallina, P. and Vadia, J., "Planar Translational Cable-Direct-Driven Robots," *Journal of Robotic Systems,* Vol. 20, pp. 107–120, 2003.
7. Oh, S.-R. and Agrawal, S.K., "Cable Suspended Planar Robots with Redundant Cables: Controllers with Positive Tensions," *IEEE Transactions on Robotics,* Vol. 21, pp. 457–465, 2005.
8. Notash, L. and Kamalzadeh, A., "Inverse dynamics of wire-actuated parallel manipulators with a constraining linkage," *Mechanism and Machine Theory,* Vol. 42, pp. 1103–1118, 2007.
9. Gouttefarde, M. and Gosselin, C.M., "Analysis of the Wrench-Closure Workspace of Planar Parallel Cable-Driven Mechanisms," *IEEE Transactions on Robotics,* Vol. 22, pp. 434–445, 2006.
10. Bouchard, S., Gosselin, C.M. and Moore, B., "On the ability of a cable-driven robot to generate a prescribed set of wrenches," *Proceedings of IDETC/CIE 2008,* Brooklyn, New York, USA, 12 pages, 2008.
11. Doty, K.I., Melchiorri, C. and Bonivento, C., "A Theory of Generalized Inverses Applied to Robotics," *International Journal of Robotics Research,* Vol. 12, pp. 1–19, 1995.