SINGULARITY ANALYSIS OF A KINEMATICALLY SIMPLE CLASS OF 7-JOINTED REVOLUTE MANIPULATORS

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Abstract

The singularities for a class of kinematically simple 7-jointed revolute manipulators are found. Within the class, unnecessary lengths and 4-jointed spherical groups of joints are avoided, and successive joints are perpendicular or parallel. Four different joint layouts are required to complete this class. The concept of screw reciprocity is used to find the singular (velocity-degenerate) configurations. Reciprocal screw quantities characterizing the lost instantaneous motions are derived.

ANALYSE DES SINGULARITÉS POUR UNE CLASSE DE MANIPULATEURS AVEC 7 ARTICULATIONS ROTOÏDES

Résumé

Les singularités pour une classe de manipulateurs avec sept articulations rotoïdes sont déterminées. Dans cette classe, toute longueur non nécessaire et des groupes de liaison sphérique à quatre articulations sont évités. De plus, les articulations successives sont perpendiculaires ou parallèles. Le concept de réciprocité des visseurs est utilisé pour déterminer les configurations singulières. Les visseurs réciproques qui caractérisent la perte instantanée de mobilité sont dérivés.

1. INTRODUCTION

1.1. Singularities

Singularities of the screw coordinate (Jacobian) matrix are a problem in the continuous motion control of manipulators. When approaching a singularity, joint rates can become infeasible if motion within the soon-to-be degenerate direction is required. Paul and Stevenson [1] considered the kinematics of spherical robot wrists defining cones of degeneracy (infeasibility) of 60 degrees lying along the direction of the first of the three wrist axes. Waldron, Wang, and Bolin [2] demonstrated the invariance of the dependency of the determinant of the joint screw coordinate matrix on the choice of velocity point and the orientation of the reference frame.

1.2. Techniques for identifying non-redundant manipulator singularities

The solution to the inverse velocity (IV) problem can be expressed as:

$$\dot{\mathbf{q}} = \mathbf{J}^{-1}\mathbf{V} \tag{1}$$

In Eq. (1), $\dot{\mathbf{q}}$ is the vector of joint rates, $\mathbf{J}^{-1}$ is the inverse of the screw coordinate matrix, and $\mathbf{V} = \{\boldsymbol{\omega}^{T}; \mathbf{v}^{T}\}^{T}$ is the angular ($\boldsymbol{\omega}$) and translational ($\mathbf{v}$) velocity vectors of the end effector. The most common method applied to find velocity degeneracies is setting the determinant of the screw-coordinate matrix equal to zero [2–4]. Hunt [4], using the invariance concepts demonstrated in [2], found compact analytical solutions for the IV problem for common industrial manipulators. Within earlier work, Sugimoto, Duffy, and Hunt [5] investigated the special configurations of spatial mechanisms and robot arms. Within [5], it is concluded that a robot arm can produce a desired end-effector instantaneous motion only if the motion is within the system of screw quantities spanned by the robot-arm joints. Furthermore in [5] it is noted that in a velocity-degenerate configuration a screw reciprocal to all of the joint axes will exist. Derivatives of displacement functions were used to identify special configurations in [6–8].

1.3. Singularity reduction via kinematic redundancy

Hollerbach [9] recommended the use of joint-redundant 7-jointed manipulators to reduce singular configurations. Based on the elimination of singularities, kinematic simplicity, and workspace shape, Hollerbach concluded that a spherical-revolute-spherical shoulder-elbow-wrist layout of joints is optimal. Podhorodeski and Pittens [10] defined a kinematically simple class of 3-revolute-jointed non-redundant branches based on avoiding unnecessary link lengths and having successive joints parallel or perpendicular. With a spherical wrist based on three intersecting revolute joints, the defined simple class becomes a non-redundant 6-revolute-jointed class. The manipulator proposed by Hollerbach [9] can be considered as kinematically simple since all the joints are perpendicular and only two link lengths are necessary to define it. This work examines the singularities of kinematically-redundant 7-jointed revelolute arms that can be defined by only two link lengths.

1.4. Identifying singular configurations for redundant manipulators

Whitney [11] proposed using the Moore-Penrose pseudo-inverse of $\mathbf{J}$ to solve the IV problem of kinematically redundant manipulators, i.e.,:

$$\dot{\mathbf{q}} = \mathbf{J}^{+}\mathbf{V} \tag{2}$$

where $\mathbf{J}^{+} = \mathbf{J}^{T}(\mathbf{J}\mathbf{J}^{T})^{-1}$ is the right Moore-Penrose inverse [12] of $\mathbf{J}$. It would be possible to consider singularities of $\mathbf{J}\mathbf{J}^{T}$ to determine velocity degeneracies of redundant manipulators (Luh and Gu [13]). However, while $\mathbf{J}\mathbf{J}^{T}$ is square the elements of $|\mathbf{J}\mathbf{J}^{T}|$, where $| * |$ indicates a determinant, tend to be too complex to allow analytic solutions to be found. An alternative to considering $|\mathbf{J}\mathbf{J}^{T}|$ would be to consider the conditions that would make all normally non-singular 6x6 sub-matrices concurrently singular [14]. This would require finding the determinants of up to seven 6x6 sub-matrices for a seven jointed manipulator, which again is not an attractive method for finding conditions leading to singularities.

Using a modified-Gram-Schmidt-based decomposition of screws, Podhorodeski [15] identified an IV solution for kinematically-redundant manipulators. As well as identifying the null-space basis (self-motions) of the redundant manipulator, the decomposition identified conditions leading to velocity degeneracies of redundant manipulators, including the spherical-revolute-spherical manipulator [16, 17]. A screw-reciprocity-based technique was developed [18, 19] to identify velocity degeneracies of kinematically-redundant manipulators. The screw-reciprocity-based method will be reviewed and applied to a class of 7-jointed revolute manipulators. Kong and Gosselin [20] proposed a dependent-screw suppression approach for the singularity analysis of 7-jointed redundant-serial-manipulators.

2. KINEMATICALLY-REDUNDANT 7-REVOLUTE-JOINTED ARMS

Within the class of simple non-redundant main-arms [10], unnecessary lengths were avoided and joints were chosen to be sequentially perpendicular or parallel to each other. The manipulators considered here are composed of a shoulder joint and an elbow joint joined by a base-proximal link, and a spherical joint consisting of the last three joints. These last three joints are perpendicular to each other and intersect at a common point forming a spherical group of joints (a spherical wrist). This spherical wrist is joined to the elbow joint by a base-distal link.

To consider all the possible arrangements, it is easier to start from the spherical wrist and examine the configurations of the first four joints (see Fig. 1). The joint (joint 4) preceding the wrist must be perpendicular to joint 5 (the first wrist joint) or else the last four joints would

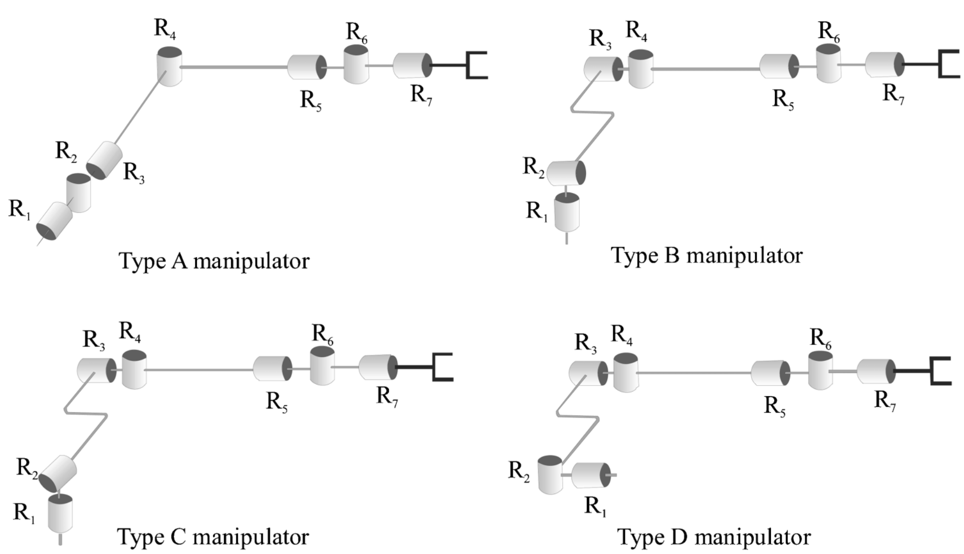


Fig. 1. Four possible configurations for kinematically simple 7-revolute-jointed arms.

intersect at a common point. One length will appear in the Denavit and Hartenberg (D&H) parameters [21] table between joint 4 and the spherical wrist for the base-distal link dimension. In order to have only two lengths in the D&H table, joint 3 must also be perpendicular to joint 4. Joint 3 can be either pointing along or perpendicular to the base-proximal link. If it is pointing along the base-proximal link, the first three joints must be a spherical group since no offsets are allowed. This produces a spherical-revolute-spherical (S-R-S) configuration (manipulator A of Fig. 1.) If joint 3 is perpendicular to the base-proximal link, there are only 3 possibilities for the first two joints, keeping in mind that joints 1 and 2 must be perpendicular to one another. Joint 2 is either parallel to joint 3 (manipulator B of Fig. 1) or perpendicular. There are two perpendicular directions to consider, one with joint 2 pointing towards joint 3, i.e., parallel to the base-proximal link (manipulator C of Fig. 1.) Note that manipulator C has a revolute-spherical-spherical layout of joints. The other layout has joint 2 perpendicular to the base-proximal link (manipulator D of Fig. 1). Manipulators B and D feature active (actuated) universal arrangements of joints at the shoulder and the elbow.

The shoulder-elbow-wrist layouts for manipulators B and D can be termed as universal-universal-spherical layouts. All four manipulators can be characterised by the two lengths of the base-proximal and base-distal links.

The Jacobian (screw-coordinate) matrices will be uniformly referenced to a velocity point on the end effector coincident with the origin of link frame F_4 and the orientation of the corresponding F_4 frame. The above referencing distributes the complexity of the expressions found for the elements of corresponding screw-coordinate (Jacobian) ${}^4\mathbf{J}_{O_4}$ matrices.

3. FINDING SINGULARIIES USING SCREW RECIPROCITY

The screw-coordinate (Jacobian) matrix will be a non-square matrix $\mathbf{J}_{6\times 7}$ for the 7-jointed kinematically-redundant revolute manipulators. The screw coordinates for the ith joint in the Jacobian matrix defined with respect to F_4 can be found with:

$$ {}^4\$_{i_{O_4}} = \left\{ \begin{array}{c} {}^4\hat{\mathbf{z}}_i \\ {}^4\hat{\mathbf{z}}_i \times {}^4\mathbf{r}_{i\rightarrow O_4} \end{array} \right\} \tag{3} $$

where ${}^4\hat{\mathbf{z}}_i$ is the unit vector giving the direction of joint i, and ${}^4\mathbf{r}_{i\rightarrow O_4}$ is the position vector from joint i to O_4, both quantities written with respect to the orientation of F_4.

Applying screw reciprocity [18] to find velocity-degenerate configurations is a two-stage process. Let $\mathbf{J}^*_{6\times 6}$ be a matrix comprised of the screw coordinates of six normally linearly-independent joints. Excluded from this matrix are the screw coordinates of one remaining joint $\$_{\text{red}}$ which is considered to provide redundant (in addition) motion to that provided by the joints forming $\mathbf{J}^*$. Conditions causing velocity-degenerate configurations of the joints forming $\mathbf{J}^*$ are found by setting the determinant of $\mathbf{J}^*$ to zero, i.e.,

$$ |\,\mathbf{J}^*\,| = 0 \tag{4} $$

Assume that $c = 1$ to n_c conditions are identified, where n_c is the total number of unique degenerate configurations.

Screws $\mathbf{W}_{\text{recip}}$ reciprocal to the six joints defining $\mathbf{J}^*$ are found for each of the n_c degenerate conditions identified from $|\,\mathbf{J}^*\,| = 0$ through linear algebra. Enforcing reciprocity with $\$_{\text{red}}$, i.e.,:

$$\$_{red} \circledast \mathbf{W}_{recip} = 0 \tag{5}$$

for each of the reciprocal screws reveals any additional conditions that are required to have the complete 7-jointed kinematically-redundant manipulator velocity degenerate. In Eq. (5), the symbol $\circledast$ indicates the reciprocal product between two screw quantities. For example, if a velocity is expressed as $\mathbf{V} = \left\{ \boldsymbol{\omega}^T;\ \mathbf{v}^T \right\}^T$ where $\boldsymbol{\omega}$ is the angular velocity and $\mathbf{v}$ is the linear velocity, and a wrench is expressed as $\mathbf{W} = \left\{ \mathbf{f}^T;\ \mathbf{m}^T \right\}^T$, where $\mathbf{f}$ is a force and $\mathbf{m}$ is a moment, the reciprocal product of $\mathbf{V}$ and $\mathbf{W}$ is $\mathbf{V} \circledast \mathbf{W} = \boldsymbol{\omega} \cdot \mathbf{m} + \mathbf{v} \cdot \mathbf{f}$. When two screws are reciprocal, their reciprocal product is zero.

In terms of screw coordinates, $\mathbf{W}$ can be expressed as a primary vector, $\mathbf{w}$, and a secondary vector, $\mathbf{w}_o$, and as a wrench intensity, W, multiplying the unit screw coordinates associated with $\mathbf{W}$, i.e.,

$$\mathbf{W} = \left\{ \mathbf{w}^T; \mathbf{w}_0^T \right\}^T = \mathrm{W}\left\{ \$_W{}^T; \$_{W_0}{}^T + p_W \$_W{}^T \right\}^T \tag{6}$$

In Eq. (6), W is the Euclidean norm of $\mathbf{w}$, $\$_W$ is the direction of the reciprocal screw, $\$_{W_0}$ is the moment of $\$_W$ about the reference origin, and p_W is the pitch of $\mathbf{W}$. Taking the dot product of the primary and secondary vectors:

$$\mathbf{w} \cdot \mathbf{w}_0 = \mathrm{W}^2[(\$_W \cdot \$_{W_0}) + p_W(\$_W \cdot \$_W)] = \mathrm{W}^2[(0) + p_W(1)] = \mathrm{W}^2 p_W \tag{7}$$

In Eq. (7) it has been noted $\$_W \cdot \$_{W_0} = 0$, since $\$_W$ and $\$_{W_0}$ are perpendicular to each other, and it has been noted that $\$_W \cdot \$_W = 1$ since $\$_W$ is a unit direction vector. That is, taking the dot product of the primary and secondary vectors of a non-unit screw will yield the pitch of the screw multiplied by the norm of the primary vector squared.

One of the advantages of using screw reciprocity to determine the singularities is that the resulting reciprocal screws allow a physical interpretation of the lost motion degrees of freedom.

4. SINGULARITIES FOR THE SIMPLE CLASS OF 7-JOINTED MANIPULATORS

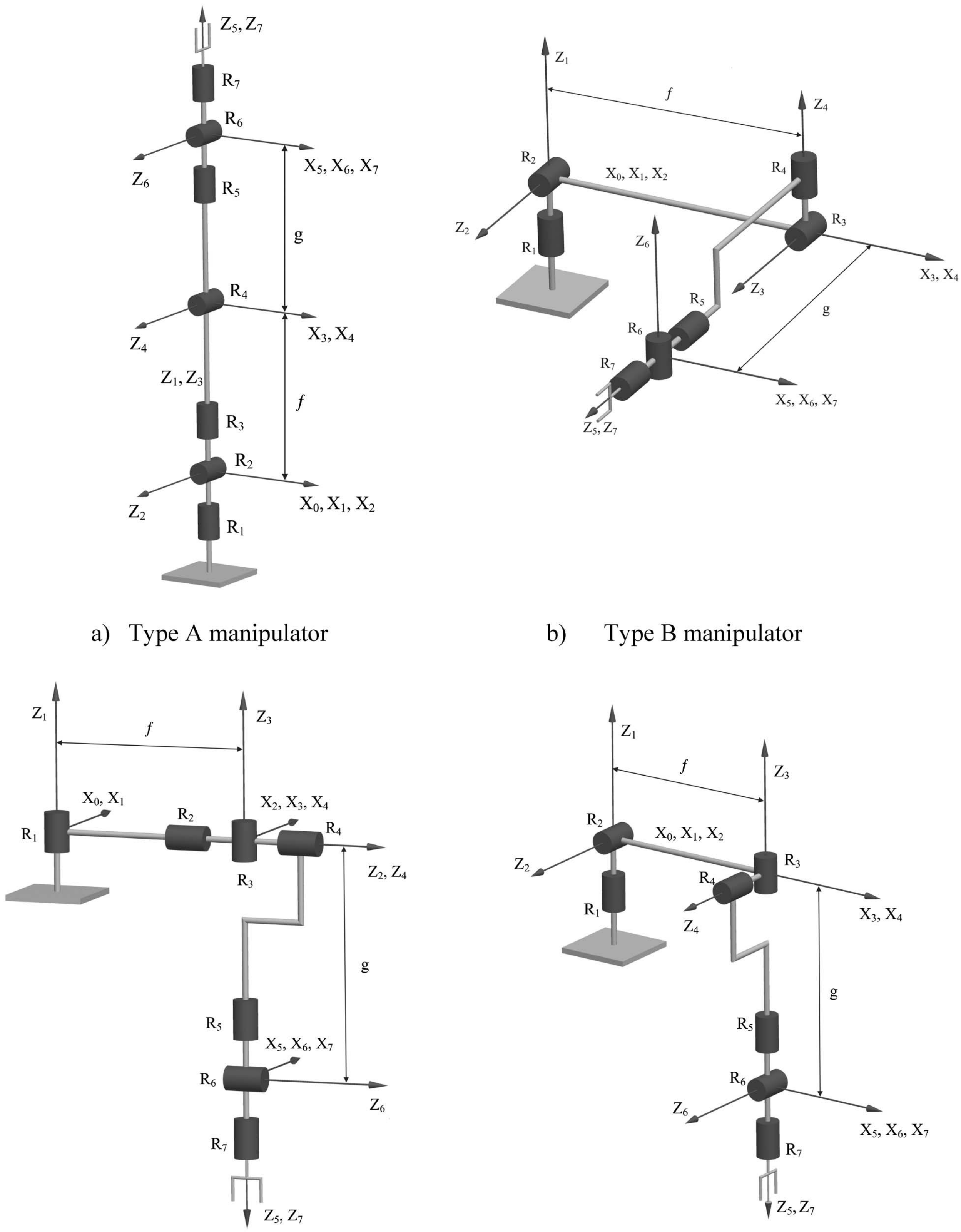
Illustrated in Fig. 2 are the zero-displacement configurations (configurations where $\theta_i = 0$, $i = 1, 2, \ldots, 7$) of each manipulator that are kinematically equivalent to those shown in Fig. 1. Table 1 gives the D&H parameters of each manipulator following the frame assignment of Craig [22].

The singularities for each manipulator were found using the screw reciprocity technique. Manipulator B is used as an example to demonstrate how the method is used.

Choosing a velocity reference point on the end effector coincident with O_4 and using a reference frame oriented as F_4 produces the following screw coordinate Jacobian matrix:

$$^4\mathbf{J}_{O_4} = \begin{bmatrix} ^4\$_{1_{O_4}} & ^4\$_{2_{O_4}} & ^4\$_{6_{O_4}} & ^4\$_{7_{O_4}} & ^4\$_{3_{O_4}} & ^4\$_{4_{O_4}} & ^4\$_{5_{O_4}} \end{bmatrix} \tag{8}$$

where $^4\$_{i_{O_4}}$ is the *i*-th joint screw coordinates with respect to the reference frame 4, where the order of the screws has been chosen to group the joints forming a spherical group of joints (joints 3, 4, and 5) at the end of $^4\mathbf{J}_{0_4}$. The expressions for the screw coordinate matrix for manipulator B are given by:



a) Type A manipulator

b) Type B manipulator

c) Type C manipulator

d) Type D manipulator

Fig. 2. Zero-displacement configurations.

Table 1. D & H parameters for each manipulator.

i-1	$\alpha_{i\text{-}1}$	$a_{i\text{-}1}$	d_i	θ_i
Type A manipulator				
0	0	0	0	θ_1
1	90°	0	0	θ_2
2	−90°	0	f	θ_3
3	90°	0	0	θ_4
4	−90°	0	g	θ_5
5	90°	0	0	θ_6
6	−90°	0	0	θ_7
Type B manipulator				
0	0	0	0	θ_1
1	90°	0	0	θ_2
2	0	f	0	θ_3
3	−90°	0	0	θ_4
4	90°	0	g	θ_5
5	−90°	0	0	θ_6
6	90°	0	0	θ_7
Type C manipulator				
0	0	0	0	θ_1
1	90°	0	f	θ_2
2	−90°	0	0	θ_3
3	90°	0	0	θ_4
4	90°	0	g	θ_5
5	−90°	0	0	θ_6
6	90°	0	0	θ_7
Type D manipulator				
0	0	0	0	θ_1
1	90°	0	0	θ_2
2	−90°	f	0	θ_3
3	90°	0	0	θ_4
4	90°	0	g	θ_5
5	−90°	0	0	θ_6
6	90°	0	0	θ_7

$$
{}^{4}\mathbf{J}_{O_4} = \begin{bmatrix} s_{23}c_4 & -s_4 & -s_5 & c_5 s_6 & -s_4 & 0 & 0 \\ -s_{23}s_4 & -c_4 & 0 & -c_6 & -c_4 & 0 & -1 \\ c_{23} & 0 & c_5 & s_5 s_6 & 0 & 1 & 0 \\ c_2 s_4 f & s_3 c_4 f & -c_5 g & -s_5 s_6 g & 0 & 0 & 0 \\ c_2 c_4 f & -s_3 s_4 f & 0 & 0 & 0 & 0 & 0 \\ 0 & c_3 f & -s_5 g & c_5 s_6 g & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} {}^{4}\$_{1_{O_4}} & \begin{matrix} \mathbf{B}_{3\times 3} & \mathbf{A}_{3\times 3} \\ \mathbf{C}_{3\times 3} & 0_{3\times 3} \end{matrix} \end{bmatrix} \quad (9)
$$

where $c_{23} = \cos(\theta_2 + \theta_3)$ and $s_{23} = \sin(\theta_2 + \theta_3)$. Choosing the first joint as the redundant joint, the determinant of the reduced Jacobian is $\left|{}^{4}\mathbf{J}^{*}_{0_4}\right| = -|\mathbf{A}||\mathbf{C}| = -s_3 s_4^2 s_6 f g^2$. The required conditions for velocity degeneracy of the six screws comprising the reduced Jacobian are thus: *a*) $s_3 = 0$; *b*) $s_4 = 0$; or *c*) $s_6 = 0$.

Condition (a): when $s_3=0$, the reduced Jacobian becomes:

$$
{}^{4}\mathbf{J}^{*}_{0_4}=\begin{bmatrix} -s_4 & -s_5 & c_5s_6 & -s_4 & 0 & 0 \\ -c_4 & 0 & -c_6 & -c_4 & 0 & -1 \\ 0 & c_5 & s_5s_6 & 0 & 1 & 0 \\ 0 & -c_5g & -s_5s_6g & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ c_3f & -s_5g & c_5s_6g & 0 & 0 & 0 \end{bmatrix} \tag{10}
$$

The reciprocal screw for the six joints in the reduced Jacobian can be found by inspection to be: ${}^{4}\mathbf{W}_{s_3=0}=\{0,\quad 1,\quad 0;\quad 0,\quad 0,\quad 0\}^{T}$. Taking the reciprocal product of this screw with $\mathbf{\$}_1$ and setting the result to zero yields: ${}^{4}\$_{1_{O_4}}\circledast{}^{4}\mathrm{W}_{S_3=0}=c_2c_4f=0$. Velocity-degenerate condition-sets are therefore: a_1) $s_3=0$ & $c_2=0$; and a_2) $s_3=0$ & $c_4=0$.

Condition (b): when $s_4=0$, the reduced Jacobian is equal to:

$$
{}^{4}\mathbf{J}^{*}_{0_4}=\begin{bmatrix} 0 & -s_5 & c_5s_6 & 0 & 0 & 0 \\ -c_4 & 0 & -c_6 & -c_4 & 0 & -1 \\ 0 & c_5 & s_5s_6 & 0 & 1 & 0 \\ s_3c_4f & -c_5g & -s_5s_6g & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ c_3f & -s_5g & c_5s_6g & 0 & 0 & 0 \end{bmatrix} \tag{11}
$$

The reciprocal screw is ${}^{4}\mathbf{W}_{s_4=0}=\{0,\quad 1,\quad 0;\quad 0,\quad 0,\quad 0\}^{T}$. Taking the reciprocal product of this screw with $\mathbf{\$}_1$ and setting the result to zero yields: ${}^{4}\$_{1_{O_4}}\circledast{}^{4}\mathbf{W}_{S_4=0}=c_2c_4f=0$. A velocity-degenerate condition-set is therefore: b_1) $s_4=0$ & $c_2=0$.

Condition (c): when $s_6=0$, the reduced Jacobian is equal to:

$$
{}^{4}\mathbf{J}^{*}_{0_4}=\begin{bmatrix} -s_4 & -s_5 & 0 & -s_4 & 0 & 0 \\ -c_4 & 0 & -c_6 & -c_4 & 0 & -1 \\ 0 & c_5 & 0 & 0 & 1 & 0 \\ s_3c_4f & -c_5g & 0 & 0 & 0 & 0 \\ -s_3s_4f & 0 & 0 & 0 & 0 & 0 \\ c_3f & -s_5g & 0 & 0 & 0 & 0 \end{bmatrix} \tag{12}
$$

The following screw is reciprocal to all of the screws (solved by linear algebra) in the reduced Jacobian: ${}^{4}\mathbf{W}_{s_6=0}=\{s_3s_4s_5,\quad -c_3c_5+s_3c_4s_5,\quad -s_3s_4c_5;\quad 0,\quad 0,\quad 0\}^{T}$. The reciprocal product with $\mathbf{\$}_1$ set to zero gives: ${}^{4}\$_{1_{O_4}}\circledast{}^{4}\mathrm{W}_{S_6=0}=c_2(s_3s_5-c_3c_4c_5)=0$. Therefore, two velocity-degenerate condition-sets are found: c_1) $s_6=0$ & $c_2=0$; and c_2) $s_6=0$ & $s_3s_5-c_3c_4c_5=0$.

The velocity degeneracy results are summarized in Table 2 for each of the KS redundant revolute manipulators. Each condition produces a singularity and its associated reciprocal screw is indicated. Note that the coordinates of the reciprocal screws have not been normalized and therefore do not necessarily form a unit screw. These screws allow a physical interpretation of each singularity.

Table 2. Summary of velocity-degeneracy condition-sets.

Type	Velocity Degeneracy Conditions	Associated Reciprocal Screw
A	$a_1)\ c_3=0\ \&\ s_2=0$	${}^4\mathbf{W}_{c_3=0}=\{0,\ \ 1,\ \ 0;\ \ 0,\ \ 0,\ \ 0\}^T$
	$b_1)\ s_4=0$	${}^4\mathbf{W}_{s_4=0}=\{0,\ \ 1,\ \ 0;\ \ 0,\ \ 0,\ \ 0\}^T$
	$c_1)\ s_6=0\ \&\ s_2=0$	${}^4\mathbf{W}_{s_6=0}=\{c_3s_4s_5,\ c_3c_4s_5+s_3c_5,\ c_3s_4c_5;\ 0,\ 0,\ 0\}^T$
	$c_2)\ s_6=0\ \&\ c_5=0$	
B	$a_1)\ s_3=0\ \&\ c_2=0$	${}^4\mathbf{W}_{s_3=0}=\{0,\ \ 1,\ \ 0;\ \ 0,\ \ 0,\ \ 0\}^T$
	$a_2)\ s_3=0\ \&\ c_4=0$	
	$b_1)\ s_4=0\ \&\ c_2=0$	${}^4\mathbf{W}_{s_4=0}=\{0,\ \ 1,\ \ 0;\ \ 0,\ \ 0,\ \ 0\}^T$
	$c_1)\ s_6=0\ \&\ c_2=0$	${}^4\mathbf{W}_{s_6=0}=\{s_3s_4s_5,\ -c_3c_5+s_3c_4s_5,\ -s_3s_4c_5;\ 0,\ 0,\ 0\}^T$
	$c_2)\ s_6=0\ \&\ s_3s_5-c_3c_4c_5=0$	
C	$a_1)\ s_4=0\ \&\ s_2=0$	${}^4\mathrm{W}_{s_4=0}=\left\{0,\ c_2s_3-s_2c_3c_4\dfrac{g}{f},\ s_2c_4;\ -s_2c_4g,\ 0,\ 0\right\}^T$
	$a_2)\ s_4=0\ \&\ s_3=0$	
	$b_1)\ s_6=0$	${}^4\mathbf{W}_{s_6=0}=\{-s_5(s_2c_4+c_2c_3s_4),\ -s_5(-s_2s_4+c_2c_3c_4)$ $+\ c_2s_3c_5,\ c_5(s_2c_4+c_2c_3s_4);\ 0,\ 0,\ 0\}^T$
	$c_1)\ s_2c_4+c_2c_3s_4=0$	${}^4\mathrm{W}_{s_2c_4+c_2c_3s_4=0}=\{0,\ \ 1,\ \ 0;\ \ 0,\ \ 0,\ \ 0\}^T$
D	$a_1)\ c_4=0\ \&\ c_2=0$	${}^4\mathbf{W}_{c_4=0}=\{0,\ \ 1,\ \ 0;\ \ 0,\ \ 0,\ \ 0\}^T$
	$a_2)\ c_4=0\ \&\ s_3=0$	
	$b_1)\ s_4=0\ \&\ c_3=0$	${}^4\mathbf{W}_{s_4=0}=\{0,\ \ s_3,\ \ -f/g;\ \ f,\ \ 0,\ \ 0\}^T$
	$b_2)\ s_4=0\ \&\ c_2f+s_2c_4g=0$	
	$c_1)\ s_6=0\ \&\ c_2=0$	${}^4\mathbf{W}_{s_6=0}=\{-c_4s_5,\ \ s_4s_5,\ \ c_4c_5;\ \ 0,\ \ 0,\ \ 0\}^T$
	$c_2)\ s_6=0\ \&\ s_3s_5+c_3c_4c_5=0$	

Considering the primary and secondary screw coordinate vectors associated with the singular configurations of the kinematically simple 7-jointed revolute redundant manipulator class, tabulated in Table 2, and Eq. (7) for pitch, the pitch of all of the reciprocal screws, $\mathbf{W}$, associated with the degenerate configurations are found to be zero.

Figures 3–6 show examples of singular configurations for each type of manipulator. In each figure, the reciprocal screw is indicated by the solid arrow and the letter W. The axes of rotation of the revolute joints are indicated by dashed lines. In each figure, the reciprocal screw intersects each revolute joint. A solid circle indicates this intersection when it is not on the manipulator. From Table 2, it can be seen that the moment of each reciprocal screw with respect to Frame 4 is zero, except for singularity conditions a_1 and a_2 for manipulator C, and conditions b_1 and b_2 for manipulator D. Figures 5 and 6 illustrate that the reciprocal screws do not pass through the origin for these conditions. This result is expected since the secondary vectors (the moment) $\mathbf{w}_o$ vector of $\mathbf{W}$ are not null, as indicated in Table 2. Recall that each of the reciprocal screws is a zero-pitch screw. Using the analogy that the associated reciprocal screws are forces, the fact that they are of zero-pitch indicates that these forces are pure forces. This implies that the manipulator loses the ability to produce a translational motion that would instantaneously perform work subject to the associated reciprocal screw.

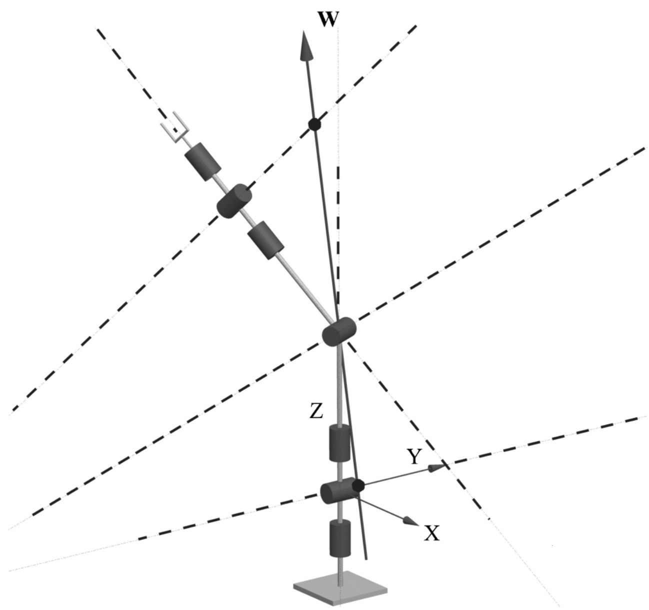


Fig. 3. Type A manipulator: Example of singularity c_1 ($\theta = [0°, 0°, 25°, 45°, 80°, 0°, 0°]^T$).

5. DISCUSSION

The velocity-degeneracy condition-sets of Podhorodeski et al. [15–17] and Nokleby et al. [18, 19] have been confirmed for the Spherical-Revolute-Spherical (S-R-S) manipulator. A more

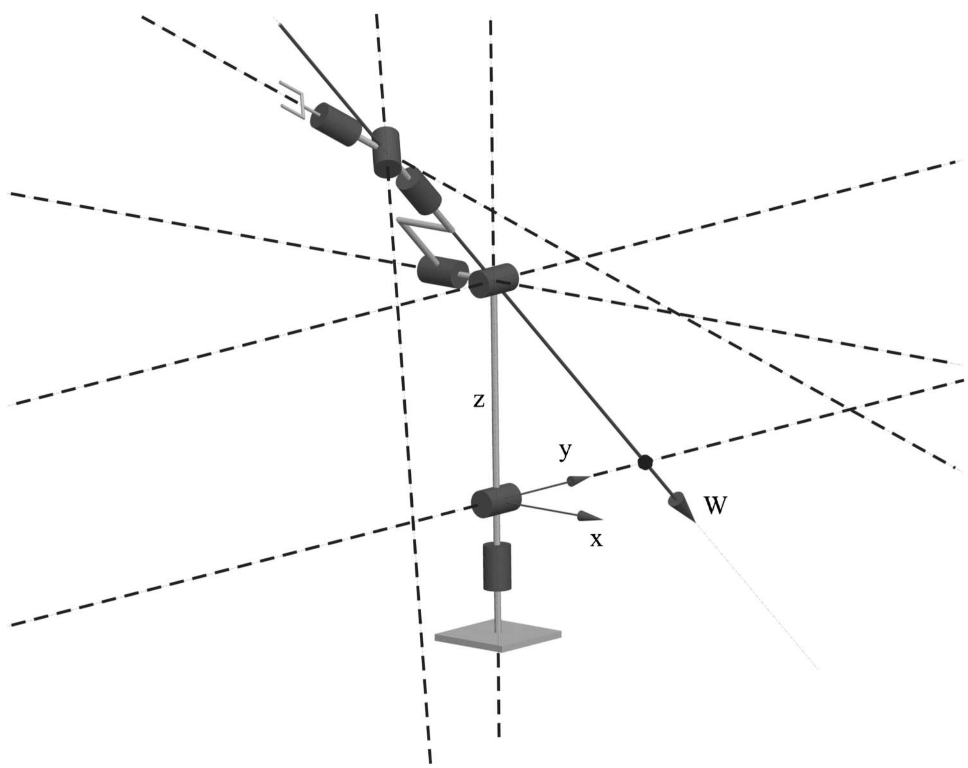


Fig. 4. Type B manipulator: Example of singularity a_1 ($\theta = [0°, 90°, 0°, 45°, 125°, 20°,0°]^T$).

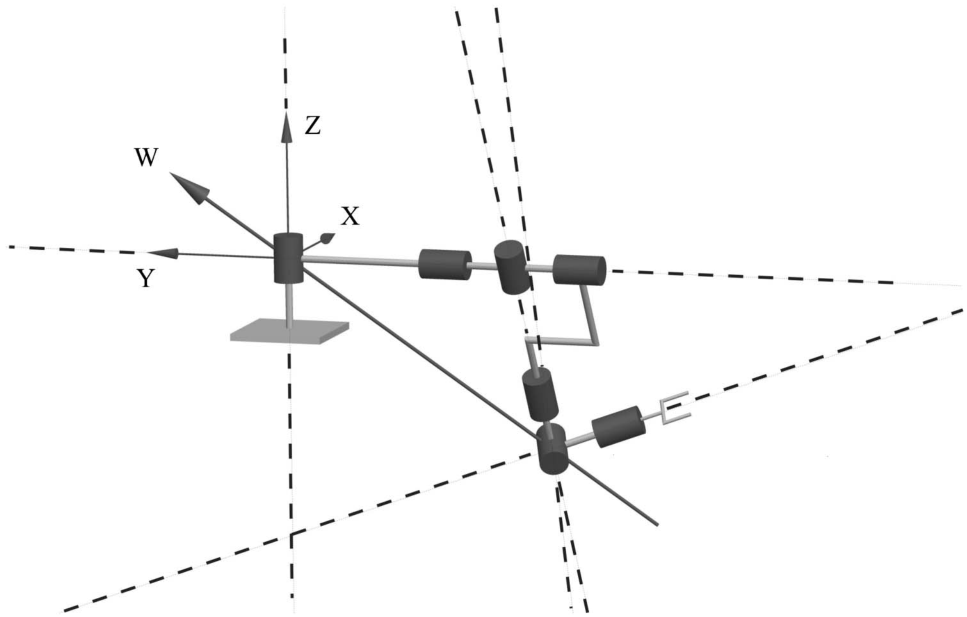


Fig. 5. Type C manipulator: Example of singularity a_2 ($\theta = [0°, 30°, 0°, 0°, 70°, 100°, 0°]^T$).

appropriate choice of translational velocity reference point and orientation reference has been used in this confirmation. The work presented in this paper completes the identification of singularities for a kinematically simple class of seven-revolute-jointed serial manipulators. Other than the above mentioned S-R-S manipulator identified by Hollerbach [9] to be optimal,

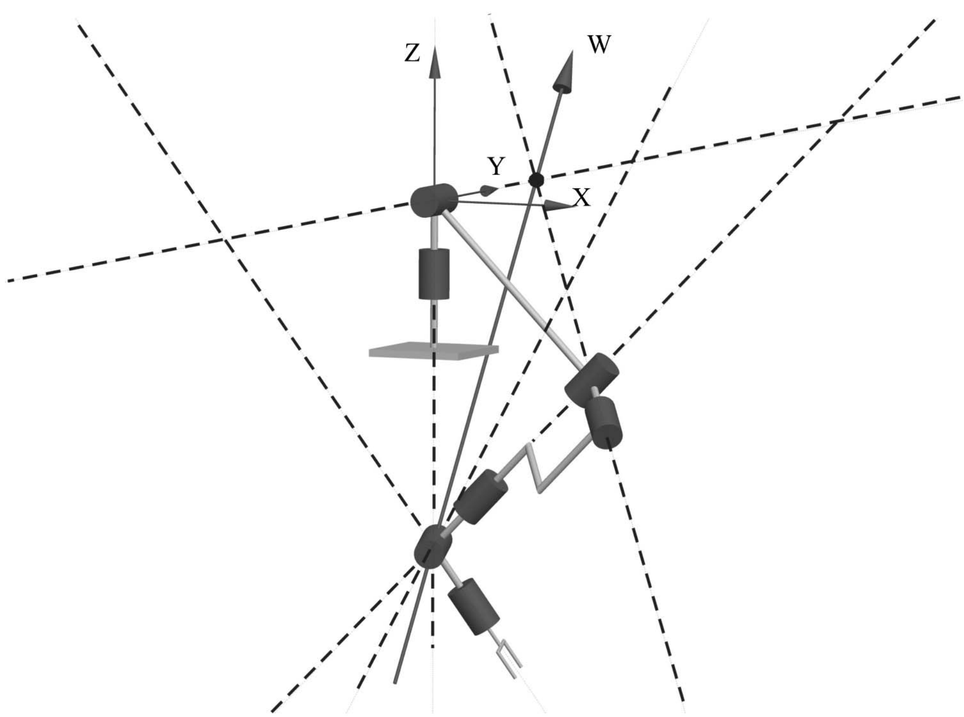


Fig. 6. Type D manipulator: Example of singularity b_2 ($\theta = [0°, 315°, 45°, 0°, 25°, 75°,0°]^T$).

three other manipulators were found to be necessary to complete the simple class. The S-R-S manipulator was identified in [9] as being optimal from the point of view of minimizing the number of velocity degeneracies and from the point of view of having the best work volume. However, manipulator *C*, which can be considered a R-S-S configuration, has the same number of condition-sets for velocity degeneracies. Also, condition c_1 for manipulator *C* is less likely to happen since it requires a specific combination involving three angles. Manipulators *B* and *D* have U-U-S layouts, where U denotes an active universal joint, and have five and six condition-sets leading to velocity degeneracies, respectively. Each of these five or six condition-sets requires the satisfaction of two displacement conditions. The need to satisfy two displacement conditions makes it less likely that a velocity degeneracy will be encountered. The work volumes of manipulators B and D need to be further investigated.

6. CONCLUSIONS

Four joint layouts are required to complete a simple class of seven-jointed revolute redundant manipulators. The four condition-sets leading to velocity degeneracy for the S-R-S layout (manipulator A) have been confirmed. An R-S-S layout (manipulator C) was found to produce the same number of velocity degeneracy condition-sets. For both manipulators A and C one condition-set leading to velocity degeneracy only requires the satisfaction of one joint displacement condition. The other three condition-sets require the satisfaction of two displacement conditions. The other two manipulators can be considered to be U-U-S layouts. Depending on the specific layout, five (manipulator B) or six (manipulator D) joint displacement condition-sets lead to velocity degeneracies. For manipulators B and D all velocity-degeneracy condition sets require the satisfaction of two joint-displacement conditions. One of the advantages of using screw reciprocity is the resulting reciprocal screw that allows a physical interpretation of the velocity degeneracies.

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