SLIP-LINE MODELING OF MACHINING AND DETERMINE THE INFLUENCE OF RAKE ANGLE ON THE CUTTING FORCE

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ABSTRACT

In this study, the effects of the rake angle on main cutting force (Fc), and thrust forces (Ft) was investigated. A new slip line model approach for modelling the orthogonal cutting process was proposed. This model was applied at negative rake angles from 0° to -60° and consists of three regions. The main forces were measured with a computer aided quick stop device. Variance Analysis (ANOVA) was utilized to analyze the effects of the cutting parameters on cutting and thrust forces accordingly. Multi-variable regression analysis was also employed to determine the correlations between the factors and the cutting forces. The cutting forces could be calculated by equation parameters which are the rake angle and the uncut chip thickness.

Keywords: cutting force; slip line; variance analysis (ANOVA); rake angle.

MODÉLISATION DU GLISSEMENT DE LA LIGNE D'USINAGE ET DÉTERMINATION DE L'INFLUENCE DE L'ANGLE DE COUPE SUR LA FORCE DE COUPE

RÉSUMÉ

Dans cet article, les effets de l'angle de coupe sur la force de coupe et la force de poussée ont été examinés. Pour modéliser la force de coupe orthogonale, une nouvelle approche du modèle de ligne de glissement a été suggérée. Ce modèle a été appliqué aux angles de coupes négatifs de 0° à -60° et consiste en trois régions. Les forces principales ont été mesurées à l'aide d'un appareil à arrêt brusque assisté par ordinateur. En conséquence, l'analyse de variance a été utilisée afin d'analyser les effets des paramètres de coupe sur la force de coupe et la force de poussée. De plus, l'analyse de régression multivariable a été utilisée à seule fin de déterminer la corrélation entre les facteurs et les forces de coupe. Les forces de coupe ont pu être calculées par les paramètres d'équation qui sont l'angle de coupe et l'épaisseur du copeau non coupé.

Mots-clés : force coupe; ligne glissement; analyse de variance; angle de coupe.

NOMENCLATURE

F	resultant force
Fc	cutting force
Ft	thrust force
k	material shear flow stress
P and Q	members of a set of basic matrix operators defined by Dewhurst and Collins [5]
P_A	hydrostatic pressure at point A
t	chip thickness
Vc	cutting speed
w	width of cut

Greek symbols

γ	rake angle
σ_1 and σ_2	radii of curvature of two base slip-lines HN and ML
ω	angular velocity of a machined chip
α_0	relief angle
α_1, α_2	angles of the vertices on each side of the intersection of the slip-line AK with the free surface of work material
η, δ	slip-line angles
ρ	total velocity jump across the slip-lines
τ	shear stress

1. INTRODUCTION

The angle between the tool face and the line perpendicular to the new work surface is known as the rake angle. The rake angle is generally considered the most important element of tool geometry for machining. It is also a significant geometrical factor affecting the mechanics of cutting. Small rake angles have high cutting forces that increase the indentation and compression in the cutting zone at and near the tool edge. When possible, cutter-heads and machining situations should be designed to provide satisfactory rake angles. Cutting forces have been related linearly to rake angle and uncut chip thickness. As the rake angle decrease the thrust force usually increases faster than the cutting force.

Models based on slip-line or analytic methods theoretically were used to simplify understanding of the cutting mechanism in machining [3,6,8,15]. Abebe [3] presented the first model with a dead zone for machining with negative rake angle tool. A slip line model to observe the behavior of the material ahead of a tool face was developed by Kıta et al. [15]. Dundur and Das [6] presented two slip line field models for orthogonal machining with a worn tool with a finite flank wear land. Fang [8] defined a slip line model for machining with a large negative rake angle. This model consisted of three regions, one of them was the stagnation zone. In the stagnation point, the direction of flow of the material depends on the rake angle. Petrky [18] presented that according to the value of τ (shear stress) / k (material shear flow stress) and the negative rake angle, metal flow direction and the dead zone change. Fang [8] used an analytical model of the chip formation for the solution. However, it is too difficult to reach the slip line solution because of the vast number of slip line regions. In this work, the slip line field model was simplified by using a Math Cad software program.

Abdelmoneim et al. [1] studied with negative rake angle tools at low cutting speeds to observe the effect of tool rake angle, cutting speed and dept of cut on the cutting and thrust forces for brass workpiece materials. They denoted that chip thickness and the force components decrease with increasing cutting speed. Abdelmoneim and Scrutton [2] realized that cutting forces rise as negative rake angle becomes more negative. Günay et al. [12] studied the effects of cutting speed and negative rake angle on cutting forces. Cutting forces directly influence cutting parameters such as cutting speed, feed rate, depth of cut, tool rake angle and tool life, so various force measuring methods have been developed. They concluded that cutting forces reduce as negative rake angle increases gradually from negative to positive.

Kopalinsky and Oxley [17] studied the effect of rake angle on Ft (thrust force)/Fc (cutting force) in orthogonal machining by using negative rake angle tools. They denoted that the force ratio Ft/Fc increased considerably with the increase in the negative rake angle. Karpat and Ozel [14] revealed the relationship between edge radius and cutting forces. The effect of edge preparation on cutting forces, especially on thrust forces, becomes more noticeable when uncut chip thickness is increased. Fang and Zhang [7] denoted that the tool edge radius causes an extreme negative rake angle in machining, and that the plastic deformation and plowing become dominant action rather than cutting. Moreover, Wang and Mathew [19] presented a model to predict the cutting forces include the effects of rake angles. Fan and Loftus [10] have shown that the quality of the surface is possible when the cutting forces are controlled through feed rate adjustment. It can be stated that a larger cutting edge radius leads to higher cutting forces since a larger portion of the uncut chip thickness will be machined with a negative rake angle [Fleischer et al. (11)]. Rake angle as the effect of the quality of the chip-consolidated products has been studied in previous publication such as Anilchandra and Surappa [4]. Rake angle effect the cutting force and complex combination of process parameters like chip flow angle as depicted by Yussefian et al. [20].

The main motivation of this paper is to determine the cutting and thrust forces and compose a formula of the cutting forces by the experiment studies. This new formula is much easier to handle with only the rake angle and uncut chip thickness used and no further data needed. A new slip line model for machining with negative rake angle tool was developed. Variation of the dead metal zone and slip line regions with negative rake angle was exhibited. Furthermore, Analysis of variance (ANOVA) was applied to analyze the effectiveness of the two parameters. The understanding of these parameters is important to determine the cutting forces of the machining from the cutting experiment of verification with different rake angles.

2. MATHEMATICAL FORMULATION OF THE NEW MODEL

Dewhurst and Collins's [5] matrix technique for numerically solving the slip-line problems is employed in the present study. The slip-line model for machining with a negative rake angle tool is seen in Fig. 1. In this figure, the dead region in front of the tool and next to the rake face of the tool is DLM; the convex region that makes the chip curl is AK; the region providing contact between slip plane surface and HNM region is KLMN; the region which comes into existence as the deformed chip rubs with the rake face of tool is HNM. Figure 2 shows the hodograph of the new slip-line model. P and Q are the basic matrix operators defined by Dewhurst and Collins [5].

Two vectors defined for modeling purposes are; HN and ML are σ_1 and σ_2 vectors respectively. DL in MLD region is being found as follows

$$DL = \sigma_\delta.\sigma_2 \quad (1)$$

MN in the HNM region is being calculated as follows

$$MN = \sigma_\eta.\sigma_1 \quad (2)$$

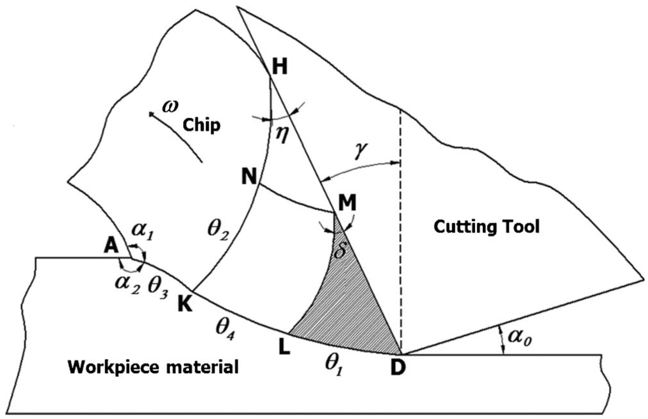

Fig. 1. The new slip line model for machining with negative rake angle tool.

If in the MNKL region,

$$\mathrm{KL} = P_{\eta\theta_2}.MN - Q_{\theta_2\eta}.\sigma_2 \tag{3}$$

$$\mathrm{MN} = P_{\theta_4\theta_2}.KL + Q_{\theta_2\theta_4}.KN \tag{4}$$

$$\mathrm{KN} = P_{\theta_3\eta}.\sigma_2 - Q_{\eta\theta_3}.MN \tag{5}$$

Using the above equations σ_2 is found in terms of σ_1. In this way, the number of vectors decreases to one.

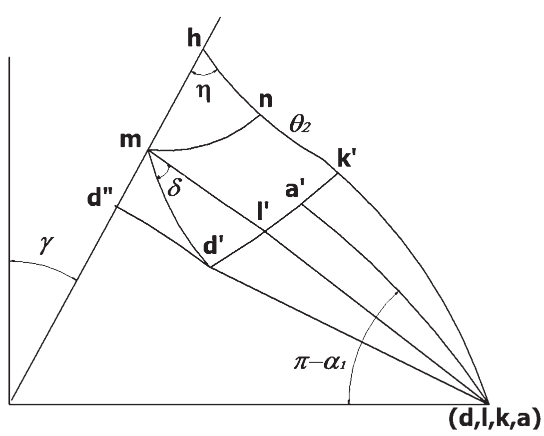

Fig. 2. Hodograph for the new slip line model.

2.1. Relationships in the Hodograph

Equations obtained from the hodograph are:

$$k'n = \omega.KN \tag{6}$$

$$k'l' = \rho.c \tag{7}$$

$$hn = \sigma_\eta.mn \tag{8}$$

$$mn = P_{\eta\theta_2}.\rho.c - Q_{\theta_2\eta}.\omega.KN \tag{9}$$

When these equations are solved, ω is found as

$$\omega = \frac{P_{\eta\theta_2}.\rho.c}{\frac{\sigma_1}{\sigma_\eta} + Q_{\theta_2\eta}.KN} \tag{10}$$

2.2. Determination of the Resultant Force

The force is written as follows when it is transmitted along DL, KL, and AK:

$$\frac{F}{ktw} = \frac{F_{AK}}{ktw} + \frac{F_{KL}}{ktw} + \frac{F_{DL}}{ktw} \tag{11}$$

AK is considered as

$$\mathrm{AK} = \left(\frac{\rho}{\omega}\right).c \tag{12}$$

Friction angle between tool and chip is

$$\delta = \cos^{-1}\left(\frac{\tau}{k}\right) \tag{13}$$

The slip line model can be completed by determining the P_A/k and τ/k values. Fang and Jawahir [9] had presented specified change of P_A/k and τ/k as a table in their study. Table 1 shows the accepted values for the new slip line model with a brass material. Table 2 gives the value of P_A/k that is used for the case study.

It can be seen from the Table 2 that τ/k value decreases and P_A/k value increases, as the rake angle becomes more negative. For the new slip line model, α_1 is determined from the hodograph in Fig. 2 and is calculated as:

Table 1. Accepted values of δ and τ/k.

γ	0°	−10°	−20°	−30°	−40°	−50°	−60°
τ/κ	1	0.95	0.9	0.85	0.80	0.75	0.70
δ	0	9°	13°	16°	18°	20°	21°

$$\alpha_1 = \pi + \theta_3 - \delta - \gamma - \theta_1 - \theta_4 \tag{14}$$

3. EXPERIMENTAL TEST

3.1. Experimental Setup

Quick Stop Device is a research instrument developed for collecting the chip-root samples in shaping based machining. The image of the quick stop device is given in Fig. 3. The cutting process is stopped suddenly by reducing the relative velocity between workpiece and tool to zero with the help of the servo motor and drive system. There are two basic motions in the device designed for this study. One of them is cutting and the other is sensitive depth cut motions. Control of the planning based machining device is provided by a servo motor and driver. Both speed and stopping distance of the single plane cutting motion may be adjusted in advance thanks to the PLC program software. The maximum width and thickness of the chip that can be machined is respectively 1.5 mm and 2 mm. The maximum cutting speed of the quick stop device is 17.5 m/min. A servo drive from Control Techniques is used on a shaping based machining device for controlling the cutting speed and stopping distance of the single axis cutting. The work was performed at various rake angle values. In order to secure this, a special tool holder was designed to rotate the tool. The eDAQ-lite trademarked strain gage indicator has four channels inlet and it communicates with RS 232. Cutting and thrust forces can be measured by single axis strain gages bonded to four facets of the tool.

3.2. Cutting Tools

A set of triangular inserts with no cutting edge radius was used during the experiments with negative rake angle tool. The triangular insert was described as TPGN 160308. The cutting insert mounted on the tool holder by mechanical tightening. Relief angle of cutting tool was 11 degrees. Cutting f1uid was not used during the tests.

3.3. Workpiece Materials

In this work, brass workpiece material was used, chemical composition of the samples was as follows: Cu%=69.809; Zn%=30.138; Sn%=0.0029; Pb%=0.005; Fe%=0.0219; P%=0.0019. Brass produces a continuous-type chip without a built-up edge when it is machined without the use of cutting fluids. The hardness of the workpiece material was 70 HV.

Table 2. Accepted values of P_A/k.

γ	0°	−10°	−20°	−30°	−40°	−50°	−60°
P_A/k	0.85	0.9	0.95	1	1	1	1

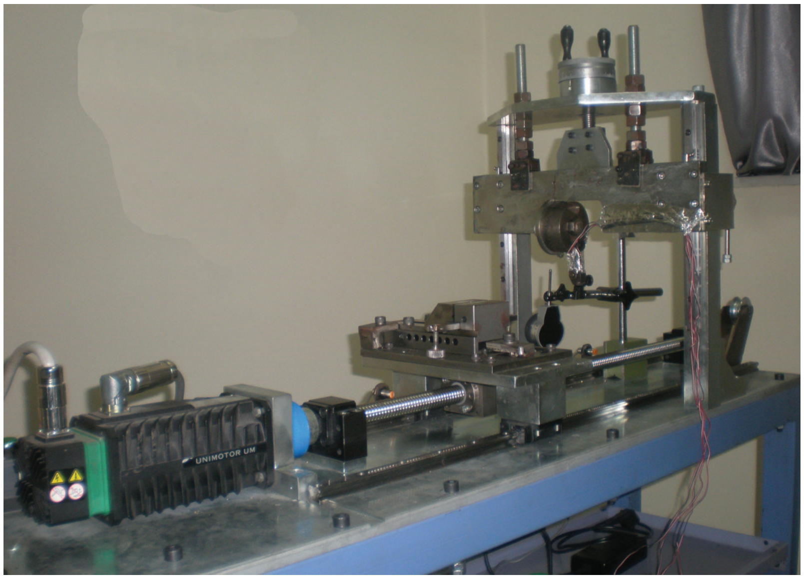

Fig. 3. Quick stop device.

3.4. Process Parameters

Experiments were performed at seven different rake angles and three various cutting depth. Table 3 gave the cutting conditions and other parameters that are used for the case study. All variables were detected by the forces measured in computer controlled quick stop device. Micro strain values converted to force values within the calibration measurements. Cutting speeds used in this work are lower than the speeds normally used in metal cutting. We limited the speeds to prevent wear of the tool. This conjecture is supported by Komanduri [16]. It is revealed that chip formation mechanism in low speed cutting was similar to that in high speed cutting [16].

4. EXPERIMENTAL RESULTS

4.1. Influence on the Forces

Figure 4 shows the variation of Ft and Fc using parameter as uncut chip thickness and rake angle. The lowest cutting force is determined as 309 N at cutting speed 0.5 m/min, 0° rake angle and a constant uncut chip thickness. The highest thrust force is measured as 1270 N at a constant uncut chip thickness, cutting speed 0.5 m/min and −60° rake angle. The increase of the thrust force indicates the material is being compressed and forced under the negative rake angle.

The forces for a −60° rake angle are higher than for a 0° rake angle. From Fig. 4, cutting and thrust forces increase by growing negative rake angle. If the negative rake angle is more negative

Table 3. Experimental cutting conditions.

Uncut chip thickness	50, 100, 150 μm
Cutting speed	0.5 m/min
Rake angle	0°, −10° to −60°
Material	CuZn30
Cutting inserts	TPGN 160308

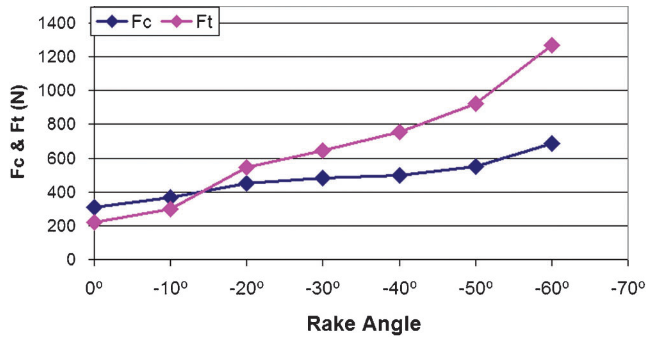

Fig. 4. Thrust (*Ft*) and cutting forces (*Fc*) variation with rake angle (Vc=0.5 m/min, t=100 μm).

than −10°, it is obvious from these graphic that the increase ratio of the thrust force is greater than the increase ratio of the cutting force.

Fc variation with rake angle is shown in Fig. 5. Cutting force increases as the uncut chip thickness increases at the constant cutting speed. In Fig. 5, it can be seen that cutting force increases relatively faster if the rake angle is −50° or less. Thrust force values are given in Fig. 6 versus rake angle for different rake angles. It is obvious from the Fig. 6 that thrust forces increase with increase of uncut chip thickness at constant cutting speed. Along with the enlarged plastic deformation zone the forces increased as the rake angle became more negative. And also, the effect of rake angle on cutting forces, especially on thrust forces, became more noticeable when the negative rake angle increased. Evaluating the effect of the rake angle influencing the value of cutting force is very important for improving the machined product quality.

The experiments are based on design of experiments to obtain equation for machinability values for machining conditions such as uncut chip thickness and rake angle. In the machining, a sufficient rake angle is paramount for good tool life and surface quality, particularly when rake angles negative. The reported results here are a portion of a study including seven rake angles. The thrust forces were also recorded. The thrust and cutting forces were higher for a −60° rake angle than for the other rake angles.

Smaller rake angles have higher cutting forces. The thrust force usually increases faster than the cutting force. The thrust force exceeds the cutting force for −20° rake angle for the first time. As the rake angle changed to more negative, the thrust force showed slow increase up to

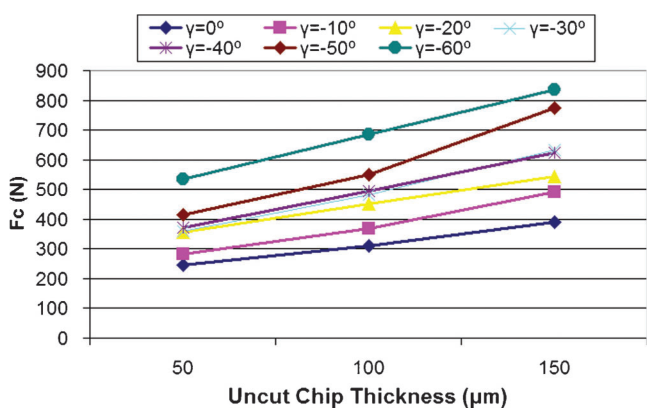

Fig. 5. Fc variation with uncut chip thickness for rake angle tools (Vc=0.50 m/min).

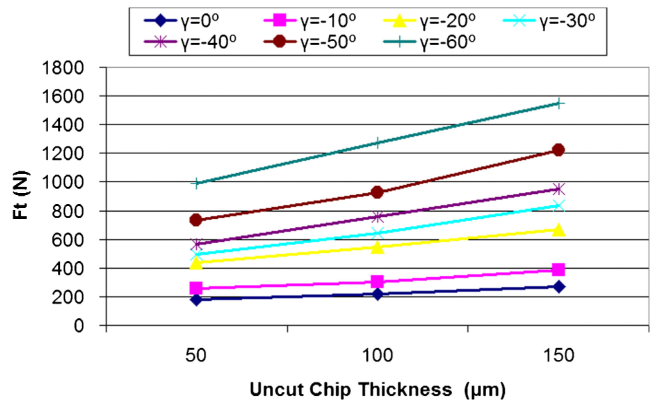

Fig. 6. Ft variation with uncut chip thickness for rake angle tools (Vc=0.50 m/min).

about -40° and then sharp rise up to -60°. These tool force data emphasize that cutting forces can be increased by providing larger negative rake angles.

4.2. Influence on the Slip Line Angles

Based on Hill's theory [13], for every particular value of the tool rake angle, the extension of the stress field into the assumed rigid zones is only possible over a limited range of solutions to the slip line model. If an accepted solution is taken into consideration, machining may eventuate for the specific values of α_1 and α_2 expressed in Fig. 1, for every single surface between AK slip line and free surface. Thus, the conditions expressed in the following equations have to be constituted.

$$\frac{\pi}{2} - 1 - 2\alpha_1 \le \frac{P_A}{k} \le 2\alpha_1 - \frac{3\pi}{2} + 1 \tag{15}$$

$$2.\cos\left(\alpha_2 - \frac{\pi}{4}\right) - 1 \le \frac{P_A}{k} \le 1 + 2\left(\alpha_2 - \frac{\pi}{4}\right) \tag{16}$$

Math Cad software program was used for the solution of the slip-line field formulations. All variables were solved by equating the forces measured in computer aided quick stop device with the F/k.t.w value. Accuracy of the variables depends on the satisfaction of the Eqs. (15) and (16). Variation of slip line angles of the existing model with negative rake angle values is given in Fig. 7.

It can be seen that as negative rake angle increases, dead metal region slip line angle (δ) increases and HNM region slip line angle (η) decreases (Fig. 7). As the negative rake angle becomes more negative, dead metal region (MLD) enlarges.

4.3. Statistical Analysis of the Data

4.3.1. Analysis of variance (ANOVA)

Analysis of Variance provides an elegant way to represent functions that depend on a high dimensional set of parameters. ANOVA is a collection of statistical models, and their associated

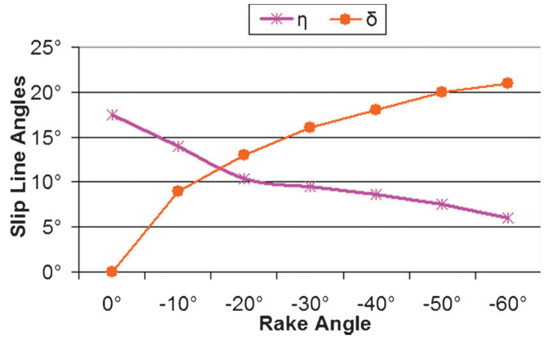

Fig. 7. Relation between slip line angles of the existing model and negative rake angles (k.t.w =24.9).

procedures, in which the observed variance in a particular variable is partitioned into components attributable to different sources of variation. Table 4, which shows ANOVA results, gave evidence to the results mentioned below.

Rake angle was found to be the most significant parameter for *Fc* by analysis of variance (ANOVA) as given in Table 4. It is obvious from the Table 4 that the contribution ratio of the uncut chip thickness is 43.83 %. The analysis of variance table shown here divides the variance of Fc into two components, one for each factor. Each factor after the first is nested in the one above. The goal of such an analysis is usually to estimate the amount of variability contributed by each of the factors, called the variance components. In this case, the factor contributing the most variance is γ. Its contribution represents 56.17% of the total variation in *Fc*.

4.3.2. Multi-Variable regression analysis

Multi-variable regression analysis was employed to determine the relationship between the factors and the cutting force (*Fc*). The factors were rake angle (γ) and uncut chip thickness (t). Table 5 gives the Multi-variable linear regression analysis results. The output shows the results of fitting a multiple regression model to describe the relationship between Fc and two independent variables. The equation of the model is also obtained from the analysis as given in Eq. (17).

$$Fc = 23.7083 + 6.94524.\gamma + 2.67125.t \qquad (17)$$
$$R^2 = 0.\ 9105$$

There is a statistically significant relationship between the variables at the 99% confidence level because the P-value in the ANOVA table is less than 0.01. The R-Squared statistic indicates that the variability of the Fc is 91.0571%. The adjusted R-squared statistic, which is more suitable for comparing models with different numbers of independent variables, is

Table 4. Analysis of Variance for Fc.

Parameter	Sum of square	Degree of Freedom	Mean square	F-ratio	Percentage contribution (%)
γ	666486	7	95212.3	25186.6	56.17
t	314438	16	19652.4	19652.4	43.83
TOTAL	980924	23			

Table 5. Multi-variable linear regression analysis.

Dependent variable: *Fc*				
Parameter	Estimate	Standard Error	T-statistics	p-value
Constant	23.7083	40.3051	0.588222	0.5627
γ	6.94524	0.575787	12.0622	0.0000
t	2.67125	0.32316	8.26604	0.0000

90.2054%. The standard error of the estimate shows the standard deviation of the residuals to be 64.6319. The Durbin-Watson (DW) statistic tests the residuals to determine if there is any significant correlation based on the order in which they occur in your data file. There may be some indication of serial correlation because the DW value is less than 1.4. In determining whether the model can be simplified, notice that the highest P-value on the independent variables is 0.0000, belonging to t. Since the P-value is less than 0.01, the highest order term is statistically significant at the 99% confidence level. No variables can be removed from the model.

5. CONCLUSION

Rake angle had a big effect on cutting and thrust forces. Cutting and thrust forces increase with increasing negative rake angle and uncut chip thickness at constant cutting speeds. This study investigates the impact of rake angle on forces with the help of ANOVA.

The following conclusions and observations can be drawn from these investigations:

- Measuring experiments were completed successfully by quick stop device. Moreover, operation of the device is very simple and it has been found to give reliable performance.
- The new slip line field model was divided into three sub-regions. Change of the sub-regions with rake angle was determined.
- Dead zone enlarged as negative rake angle became more negative.
- Ft/Fc ratio increased as negative rake angle became more negative.
- Fc (cutting force) was calculated by Eq. (17) with only knowing of uncut chip thickness (t) and rake angle (γ). The difficulty in finding a cutting force will be over.
- It was obvious from these experiments that rake angle and uncut chip thickness affects the machining and cutting forces.
- ANOVA gave results in determining the percentage contributions of the key factors that the rake angle is the most important parameter with a contribution ratio of 56.17%.
- Multi-variable regression analysis was also employed to determine the correlations between the factors and the cutting force.

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