THE DOF DEGENERATION CHARACTERISTICS OF CLOSED LOOP OVER-CONSTRAINED MECHANISMS

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ABSTRACT:
A new terminology, "*degenerative degrees of freedom*", to describe mechanisms possessing different degrees of freedom (DOF) while containing the same number of linkages and joints is introduced. A systematic approach is developed for studying this particular type of closed loop mechanism and its degeneration characteristics of DOFs. First, the single closed loop over-constrained mechanism is analyzed and a relationship between the number of over-constraints and the number of joints and DOFs is established. Then, all possible types of independent over-constraints and their combinations are summarized. Further the non-instantaneous condition of the mechanism is analyzed by using an analytical method. The paper delineates three rules that provide the guidelines for the layout of joints, linkages and their assembly. Finally, the degenerative characteristics of all such mechanisms are systematically tabulated along with sketches of some typical ones. To corroborate the literature an example involving two 6R closed loop mechanisms with 1 and 3 DOFs respectively is presented and analyzed, thus validating their degenerative characteristics.

Keywords: over-constrained; degeneration; synthesis; screw theory.

LES CARACTÉRISTIQUES DE DÉGÉNÉRESCENCE DES DDL DES MÉCANISMES SURCONTRAINTS À CHAÎNE FERMÉE

RÉSUMÉ
Nous introduisons une nouvelle terminologie, "degré de liberté dégénéré", pour décrire des mécanismes possédant différents degrés de liberté (DDL) pour un même nombre de liens et d'articulations. Une approche systématique est développée pour étudier ce type particulier de mécanisme à boucle fermée ainsi que les caractéristiques de dégénérescence des DDLs. Dans un premier temps, nous analysons les mécanismes surcontraints à une seule boucle fermée, et nous établissons une relation entre le nombre de surcontraintes et le nombre de joints et de DDL. Par la suite, sont recensés tous les types de sur-contraintes indépendantes, ainsi que toutes leurs combinaisons possibles. Ensuite, la condition non-instantanée du mécanisme est étudiée en utilisant une méthode analytique. L'article définit trois règles donnant les instructions pour l'agencement des joints, des articulations et les différents assemblages. Pour finir, les caractéristiques de dégénérescence de tels mécanismes sont systématiquement croisées avec *des configurations plus typiques*. Afin d'être cohérent avec le reste de la littérature, un exemple impliquant deux mécanismes à chaîne fermée 6R, possédant respectivement 1 et 3 DDLs, est présenté et étudié, dans le but de valider leurs caractéristiques de dégénérescence.

Mots-clés : surcontrainte; dégénérescence; synthèse; théorie des vis.

1. INTRODUCTION

In general, the number of actuators in a closed loop mechanism is identical to its DOFs, and the DOFs have a certain relationship with the number of links and joints. Usually, the Grübler-Kutzbach criterion can be used to synthesize new mechanisms. However, this criterion cannot be applied to an over constrained mechanism, due to the extra constraints. Therefore, the synthesis of over constrained mechanisms is still a long-term problem in the mechanism field.

Most of the earlier one DOF over constrained mechanisms were discovered by ad hoc methods. The single-loop, one DOF, over constrained mechanisms can have four, five, or six links. Waldron [1,2] claimed that almost all four-link, over constrained mechanisms, consist of lower kinematic pairs, have been identified. Wohlhart [3,4] and Baker [5] used the Bennett linkage as a building block to obtain a Bennett-based 6-R over constrained linkage. Deriving the loop-closure displacement equations of spatial five- and six-link mechanisms is another effective method and research field. Pamidi et al. [6] derived the necessary criteria of over constrained five-link mechanisms. Baker [7] and Yan et al. [8,9] also used the loop-closure equation approach to study the conditions for five- or six-link over constrained mechanisms. The loop-closure equation can only be used when the joint arrangement of a mechanism is predetermined due to this equation being structure dependent. Tsai [10] presented a method to enumerate kinematic structures and discussed the characteristics about instantaneous mobility about the mechanisms. Fang and Tsai [11] also presented a new method for the structural synthesis of over constrained mechanisms based on constraint analysis; they also enumerated possible overconstraints about pure force and pure couples. Until now, the synthesis method has become systematic and comprehensive, but the situation for the combination of forces and couples has not been solved.

The synthesis for multi-DOF mechanisms is more difficult than one DOF mechanisms because of the added complexity in their construction and geometric assembling. Fanghella and Galletti [12] analyzed the mobility of single-loop kinematic chains based on displacement groups. Kong [13] analyzed the possible over constrained types and characteristics for multi-DOF, over constrained, closed loop mechanisms. Guo and Fang [14] synthesized possible mechanisms by using an analytical method and the geometric assembling conditions that respond to the different types of overconstraints were studied. Through further research of those synthesized over constrained mechanisms, we noticed that mechanisms with the same number of links and joints sometimes have different DOFs. We call these degeneration characteristics of DOFs.

In this paper, a systematic method is presented to study the degeneration characteristics of DOFs of closed loop, over constrained mechanisms based on screw theory. First, all possible constraints assemblies are enumerated. Then, given certain overconstraints, the geometric rules for synthesis of closed loop mechanisms are developed. Finally, all possible situations of the degeneration characteristics of DOFs are tabulated and several are sketched.

2. ANALYSIS OF THE NUMBER OF OVERCONSTRAINTS

The Grübler-Kutzbach criterion for a spatial mechanism is the traditional DOF formula and can be stated as

$$F=6(n-j-1)+\sum_{i=1}^{j} f_i \tag{1}$$

where F is the number of DOF of a mechanism, n is the number of links, j is the number of joints, and f_i is the degrees of relative motion permitted by joint i.

However, this equation cannot calculate the number of DOFs of over constrained mechanisms. According to this equation, the constraints acting on the entire mechanism will be over calculated. Since the overconstraints can be divided into common constraints and redundant constraints, the general Grübler-Kutzbach criterion can be modified as,

$$F=d(n-j-1)+\sum_{i=1}^{j} f_i+m \tag{2}$$

where d is the order of the spatial mechanism, $d=6-\lambda$, λ denotes the number of common constraints, and m represents the number of redundant constraints.

Note that a spherical joint is equivalent to three intersecting noncoplanar revolute joints, a universal joint is equivalent to two intersecting revolute joints, and so on, and we assume that the revolute and prismatic joints are the basic joint types used for the closed loop mechanism. Each basic joint in a mechanism can be associated with a unit screw.

In this paper, we take the spatial closed loop mechanism without common constraint as an example. For the general spatial closed loop mechanism without common constraint, $\lambda=0$, $d=6$ and the number of links n and the joints j are equal. If $j=n=3$, the mechanism will become a stable triangle and it will no longer be a mechanism. In Eq. (2), $j=n>3$ is required and the redundant constraints m make the whole mechanism lose at least m degrees of freedom. The degree of freedom of the mechanism cannot be more than $6-m$. We have,

$$\begin{cases} j+m=6+F, \ \forall j>3 \\ F\leq 6-m, \ m\in\{1,2,3,4,5\} \end{cases} \tag{3}$$

By solving Eq. (3), we get all possible assemblies with different numbers of F and n, as presented in table 1.

From Table 1, we get an interesting discovery. Mechanisms with the same number of links and joints may have different degrees of freedom. For example, one 6-link closed loop mechanism may have 1, 2, or 3 degrees of freedom. Later, we will present a systematic method to analyze the cause of the degeneration characteristics of DOFs based on screw theory.

3. SCREW THEORY

Any spatial vector can be expressed as a screw

$$\hat{\$}=\begin{bmatrix} \mathbf{s} \\ \mathbf{s}_o+h\mathbf{s} \end{bmatrix} \tag{4}$$

Table1. All different possible DOF with the same number of links.

Links n	DOF 1	DOF 2	DOF 3	DOF 4	DOF 5	DOF 6
4	√	√				
5	√	√				
6	√	√	√			
7		√	√			
8			√	√		
9				√		
10					√	

where, $\mathbf{s}$ is a unit vector pointing in the direction of the joint axis, $\mathbf{s}_o = \mathbf{r} \times \mathbf{s}$ defines the moment of the screw axis about the origin of a reference frame, $\mathbf{r}$ is the position vector of any point on the screw axis with respect to the reference frame, and h is the pitch of the screw. If the pitch equals zero, the screw coordinates reduce to

$$\hat{\$} = \begin{bmatrix} \mathbf{s} \\ \mathbf{s}_o \end{bmatrix} \tag{5}$$

If the pitch of a screw is infinite, the unit screw is defined as

$$\hat{\$} = \begin{bmatrix} \mathbf{0} \\ \mathbf{s} \end{bmatrix} \tag{6}$$

The motion created by a revolute joint is a screw with zero pitch pointing along the joint axis. The unit screw associated with a prismatic joint is a screw of infinite pitch pointing along the direction of the joint axis. All other types of lower pairs such as universal, cylindrical, and spherical joints can be formed by a combination of revolute and prismatic joints.

In the same way, pure force constraints can be expressed as Eq. (5) and pure couple constraints can be expressed as Eq. (6). We call these screws a wrench. If a wrench, $\$_r = \rho \hat{\$}_r$, acts on a rigid body in such a way that it produces no work while the body is undergoing an infinitesimal twist, $\$ = q\hat{\$}$, the two screws are said to be reciprocal screws. The virtual work performed between the wrench and the twist is given as

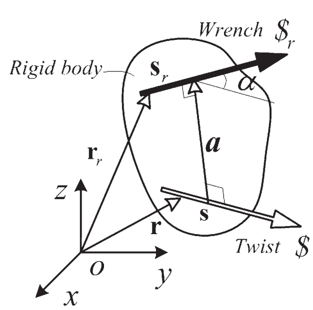


Fig. 1. Twist and wrench in three-dimensional space.

$$\begin{aligned}\delta W &= \rho\dot{q}[\mathbf{s}\cdot(\mathbf{r}_r \times \mathbf{s}_r + h_r\mathbf{s}_r) + \mathbf{s}_r\cdot(\mathbf{r}\times\mathbf{s} + h\mathbf{s})] \\ &= \rho\dot{q}[(h+h_r)(\mathbf{s}\cdot\mathbf{s}_r) + \mathbf{r}_r\cdot(\mathbf{r}\times\mathbf{s}) + \mathbf{s}\cdot(\mathbf{r}_r\times\mathbf{s}_r)]\end{aligned} \tag{7}$$

From the geometry of the lines associated with the two screws shown in Fig. 1, we obtain

$$\mathbf{s}\cdot\mathbf{s}_r = \cos\alpha \tag{8}$$

$$\mathbf{s}_r\cdot(\mathbf{r}\times\mathbf{s}) + \mathbf{s}\cdot(\mathbf{r}_r\times\mathbf{s}_r) = -\boldsymbol{a}\cdot(\mathbf{s}\times\mathbf{s}_r) = -a\sin\alpha \tag{9}$$

where $\boldsymbol{a}$ is a vector along the common perpendicular leading from the screw axis of \$ to $\$_r$, and α is the twist angle between the axes of \$ and $\$_r$ measured from \$ to$\$_r$, about the common perpendicular according to the right-hand rule. By substituting Eqs. (8) and (9) into Eq. (7), we obtain

$$\delta W = \rho\dot{q}[(h+h_r)\cos\alpha - a\sin\alpha] \tag{10}$$

By definition, the virtual work produced by the two reciprocal screws is equal to zero. Since ρ and $\dot{q}$ are generally not equal to zero, the reciprocal condition can be stated as

$$(h+h_r)\cos\alpha - a\sin\alpha = 0 \tag{11}$$

Note that the virtual work is equal to zero by definition when both \$ and $\$_r$ are infinite-pitch screws. In the following sections, all possible overconstraints are discussed for several over constrained closed loops. The above reciprocal condition will be used for identification of the joint-screw system for a given constraint-wrench system.

4. ALL POSSIBLE OVERCONSTRAINTS CONFIGURATIONS

All possible constraints can be considered as pure couples, pure forces or the combination of both. The maximum independent number of couples and forces are three for each. Thus, we give all of the possible constraints a general expression in screw form. By solving the zero work equation to get all possible solutions [11, 14], we can assemble conditions for the joints of these particular mechanisms.

4.1. Pure Couple Overconstraints

According to Eq. (6), any pure couple overconstraints can be written as

$$\hat{\$}_{Ci} = \begin{bmatrix}\mathbf{0}\\ \mathbf{s}_{Ci}\end{bmatrix}, i = 1,2,3 \tag{12}$$

where i is the number of independent overconstraints. For the revolute joints in any of the mechanisms subject to this overconstraint, the virtual work between revolute motion and overconstraints must be equal to zero. Substituting Eqs. (5) and (12) into Eq. (7) yields

$$\delta W_i = \mathbf{s} \cdot \mathbf{s}_{Ci} = \cos \alpha_i = 0, i = 1,2,3 \tag{13}$$

This indicates that all revolute joints should be perpendicular to the pure couple constraints. However, there exists no revolute joint which can be perpendicular to three independent pure couple constraints simultaneously. Therefore, the assembly conditions for a revolute joint under pure constraint couples are

$$\alpha_i = \frac{\pi}{2} + k\pi, \ \ k = 0,1,2,\cdots, \ \ i = 1,2 \tag{14}$$

Hence, all revolute joint axes in pure couple over constrained mechanisms lie on planes that are perpendicular to the direction of the constraint wrench.

For prismatic joints, by definition of virtual work we have

$$\delta W_i \equiv 0, \quad i = 1,2,3 \tag{15}$$

Eq. (15) implies that prismatic joints can be positioned and oriented arbitrarily as long as they are linearly independent. The assembly conditions are shown as Fig. 2(a).

4.2. Pure Force Overconstraints

Any pure force overconstraints can be written as

$$\hat{\$}_{Fi} = \begin{bmatrix} \mathbf{s}_{Fi} \\ \mathbf{r}_{Fi} \times \mathbf{s}_{Fi} \end{bmatrix}, \ \ i = 1,2,3 \tag{16}$$

where i is the number of independent overconstraints. For revolute joints, substituting Eqs. (5) and (16) into Eq. (7), will yield

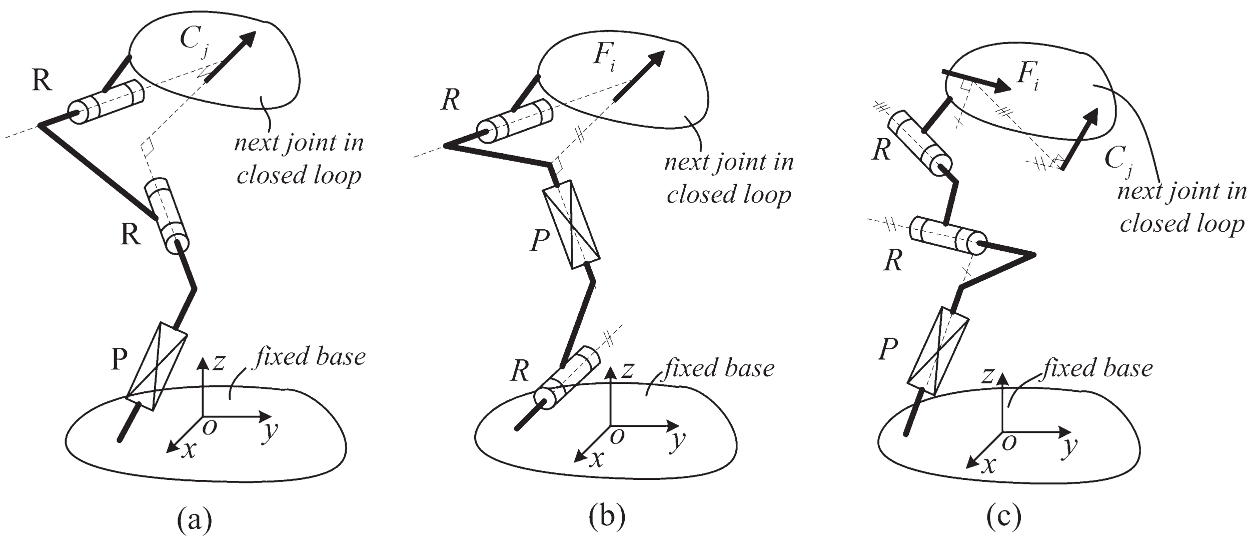


Fig. 2. The assembly conditions under overconstraints.

$$\delta W_i = -a_i \sin \alpha_i = 0, \;\; i = 1,2,3 \tag{17}$$

The solution can be written as

$$a_i = 0, or, \alpha_i = k\pi \quad k = 0, 1, 2, \cdots, \;\; i = 1,2,3 \tag{18}$$

Hence, all revolute joint axes in pure force over constrained mechanisms must either intersect with or be parallel to the constraint force.

By substituting Eqs. (6) and (16) into Eq. (7), we obtain the following for prismatic joints.

$$\delta W_i = \mathbf{s} \cdot \mathbf{s}_{Fi} = \cos \alpha_i = 0, \;\; i = 1,2,3 \tag{19}$$

$$\alpha_i = \frac{\pi}{2} + k\pi, \quad k = 0,1,2, \cdots, \quad i = 1,2,3 \tag{20}$$

Equation (20) implies that all prismatic joint axes must be perpendicular to any constraint forces. The assembly conditions are shown as Fig.2 (b).

4.3. Constraint Combinations

An C*i*F*j* joint-screw system consists of all screws that are reciprocal to the independent constraint couple, C*i*, and the independent force, F*j*. The combination of (*i*+*j*) overconstraints can be expressed as,

$$\begin{cases} \hat{\$}_{Ci} = [0 \; ; \; \mathbf{s}_{Ci}]^T & i = 1,2,3 \\ \hat{\$}_{Fj} = [\mathbf{s}_{Fj} \; ; \; \mathbf{r}_{Fj} \times \mathbf{s}_{Fj}]^T & j = 1,2,3 \\ \quad 2 \leq i + j \leq 5 \end{cases} \tag{21}$$

where the subscripts *i* and *j* represent the number of overconstraints.

From Eq. (21), we obtained the possible combinations of overconstraints, such as C1F1, C1F2, C1F3, C2F1, C2F2, C2F3, C3F1, C3F2.

For revolute joints, the virtual work between the screws and reciprocal screws are given by

$$\begin{cases} \delta W_i = -a_i \sin \alpha_i = 0, & i = 1,2,3 \\ \delta W_j = -a_j \sin \alpha_j = 0, & j = 1,2,3 \end{cases} \tag{22}$$

$$\begin{cases} \alpha_i = \dfrac{\pi}{2} + k\pi, \; k = 0,1,2, \cdots, \;\; i = 1,2,3 \\ a_j = 0, or, \alpha_i = k\pi \quad k = 0, 1, 2, \cdots, j = 1,2,3 \end{cases} \tag{23}$$

Equation (23) implies that a revolute joint axis must be perpendicular to the given couple constraints. At the same time, they can either intersect with constraint forces, or they are parallel to the constraint forces.

For prismatic joints, the virtual work from couple constraints are always zero because $\delta W_i \equiv 0, i=1,2,3,$. Therefore, the virtual work can be written as

$$\delta W_j = \mathbf{s} \cdot \mathbf{s}_{Fj} = \cos \alpha_j = 0, \;\; j=1,2,3 \tag{24}$$

$$\alpha_j = \frac{\pi}{2} + k\pi, \quad k=0,1,2,\cdots, \;\; j=1,2,3 \tag{25}$$

Equation (25) implies that a prismatic joint axis must be perpendicular to the given force constraints and it has no relationship with the couple constraint. The assembly conditions are shown in Fig. 2(c).

In table 2, F represents the force constraints and the number following is the number of independent forces. C denotes the couple constraints and the number following is the independent number of couples. Table 2 shows that mechanisms with the same number of links may have different numbers of degrees of freedom.

5. GEOMETRIC CHARACTERISTICS OF OVER CONSTRAINED MECHANISMS

In closed loop over constrained kinematic chains, relative motion between links may affect the relative orientation of the axes of the revolute joints. This can result in dissatisfaction of the required geometric conditions between overconstraints and joints axes. In such a case, the mechanism will be an instantaneous one. In order to avoid this situation we arrange all revolute joints in groups such that parallel joints are grouped together. Every group should have at least 2 parallel joints in it. Therefore, the geometric relationships between the joints are preserved. The prismatic joints in the mechanism will ensure that the associated links will preserve their orientation. This is not the case with revolute joints where associated links keep changing their orientation during motion. According to this analysis, the summation of the products of the angular velocity of each joint (with respective to the ground link) with its associative unit screw should be zero. That is

Table 2. All possible degree of freedom under different overconstraints.

DOF F	Links n	Joints j	Overconstraint type	Number of overconstraints m
1	6	6	F1,C1	1
	5	5	F2,C2,F1C1	2
	4	4	F3,C3,F2C1,F1C2	3
2	7	7	F1,C1	1
	6	6	F2,C2,F1C1	2
	5	5	F3,C3,F2C1,F1C2	3
	4	4	F3C1,C3F1,F2C2	4
3	8	8	F1,C1	1
	7	7	F2,C2,F1C1	2
	6	6	F3,C3,F2C1,F1C2	3
4	9	9	F1,C1	1
	8	8	F2,C2,F1C1	2
5	10	10	F1,C1	1

$$\sum_{i=1}^{n} \dot{q}_i \hat{\$}_i = 0 \tag{26}$$

where $\dot{q}_i$ is theith joint rate and $\hat{\$}_i$ is the unit screw associated with the ith joint. Assuming that the joints are either revolute or prismatic joints, Eq. (26) can be decomposed into

$$\sum_{i=1}^{k} \dot{q}_i \mathbf{s}_i = 0 \tag{27}$$

$$\sum_{i=1}^{k} \dot{q}_i (\mathbf{r}_i \times \mathbf{s}_i) + \sum_{i=k+1}^{p} \dot{q}_i \mathbf{s}_i = 0 \tag{28}$$

where k. and (p-k) denote the numbers of revolute and prismatic joints respectively. Each term in Eq. (27) denotes the angular velocity of the ith revolute joint relative to the fixed link. Similarly, each term in Eq. (28) represents the linear velocity of the ith revolute or prismatic joint relative to the fixed link. Base on this point and the assembly condition of kinematic joints subject to the overconstraints analyzed in section 4, also with references [11,14], we proposed three rules about synthesis of over constrained mechanisms.

Rule 1:

If the overconstraints are pure C1, all the revolute joint axes in this C1 joint-screw system must lie on parallel planes perpendicular to the direction of the given constraint couple, whereas the prismatic joint axes can be positioned and oriented arbitrarily.

If the overconstraints are pure C2, all the revolute joint axes in this C2 joint-screw system must be parallel to each other and to the common normal of the two given constraint couples, whereas the prismatic joint axes can be positioned and oriented arbitrarily.

If the overconstraints are pure C3, this C3 joint-screw system consists of only prismatic joint screws, and the prismatic joint axes can be positioned and oriented arbitrarily as long as they are linearly independent.

Rule2:

If the overconstraints are pure F1, all the revolute joint axes in this F1 joint-screw system must be either intersecting or parallel to the constraint force, and the prismatic joint axes must lie on planes that are perpendicular to the direction of the constraint force.

If the overconstraints are pure F2, all the revolute joint axes in this F2 joint-screw system must either intersect with the two given constraint forces or must be parallel to one and intersect the other constraint force. The prismatic joint axis must be parallel to the common normal of the two constraint forces.

If the overconstraints are pure F3, all the revolute joint axes of this F3 joint-screw system must intersect with the three given constraint forces at a point, and there are no feasible prismatic joints in this F3 joint-screw system.

Rule3:

If the overconstraints are any of the following: F1C1, F1C2, F1C3, F2C1, F2C2, F2C3, F3C1, F3C2, all the revolute joint axes must be perpendicular to the constraint couples and must either intersect or be parallel to the constraint force. All prismatic joint axes must lie on planes that are perpendicular to the direction of the constraint forces. There are no prismatic joints in F3C1 and F3C2 over constrained mechanisms. There are no revolute joints in F1C3, F2C3 over constrained mechanisms.

6. VARIABLE DOF MECHANISMS WITH SAME NUMBER OF LINKS

6.1. Degenerative DOF Characteristics

From the three synthesis rules discussed before together with Table 1 and Table 2, we obtain Table 3. In Table 3, the first column specifies 7 families of mechanisms with the number of links n (i.e. the number of single DOF joints), ranging from 4 to 10. For each family, all the members belonging to it are classified based on their DOF ranging from 1 to 6 and their architecture is given for every overconstraint type. The symbol $\times$ means there is no possibility for this kind of DOF with corresponding number of links.

From Table 3 we can easily find the DOF degenerative characteristics of single closed loop mechanisms. It depends on the type of overconstraint employed. For example 6R, 5R1P, 4R2P mechanisms can give rise to one DOF, two and three DOF mechanism based on the type of overconstraint chosen. The geometric assembly conditions for these mechanisms are dictated by the three synthesis rules. In single DOF, C1 type of 6R mechanisms, all revolute joint axes are classified into two groups and they lie on the plane which is perpendicular to the given couple constraint. In two DOF, C2 type of 6R mechanisms, all revolute joint axes are parallel to the common normal line of the two given overconstraints. In three DOF F1C2 type of 6R mechanisms, all revolute joint axes are parallel to the common normal line of the two given overconstraints and they intersect or stay parallel to the given force overconstraints. From this we can draw a conclusion that for more DOF characteristics needs more strict geometric assembling requirements.

Note: The spatial mechanisms without common constraints and with 9 links can have only 4 DOF and those with 10 links can have only 5 DOF, so they do not show any degenerative DOF characteristics.

6.2. Design Examples

Figures 3, 4, and 5 illustrate some mechanisms designed using Table 3 and the three synthesis rules. The nomenclature used to name the figures is given as follows: the first number represents the degree of freedom, the following letter and number represent the type of overconstraint and the last letter and number represent the number of joints and their type.

6.3. DOF Validation

In this section, we calculate the DOF of two different 6R closed loop mechanisms and validate their degenerative characteristics. The 6R closed loop mechanism can be considered as a parallel mechanism with two identical 3R chains between a fixed base and a moving platform.

For parallel mechanisms, the motion of the moving platform can be treated as a resultant of coupled movement of all individual limbs. Since these mechanisms posses only revolute joints, the instantaneous twist of the moving platform can be expressed as a linear combination of each revolute screw

$$\$_p = \sum_{i=1}^{2} \theta_{ij} \$_{ij} \qquad j = 1,2,3 \tag{29}$$

Table 3. Possible degrees of freedom for same number of links (n).

n	Constraint Type	Possible degrees of freedom from 1 to 6					
		1	2	3	4	5	6
4	F3	4R	×	×	×	×	×
	F1C2	4R,1P3R 3P1R,4P	×	×	×	×	×
	F2C1	4R,1P3R 3P1R,4P	×	×	×	×	×
	C3	4P	×	×	×	×	×
	F3C1	×	4R	×	×	×	×
	C3F1	×	4P	×	×	×	×
	F2C2	×	2R2P,4P	×	×	×	×
5	C2	3R2P,2R3P	×	×	×	×	×
	F2	3R2P,2R3P	×	×	×	×	×
	F1C1	3R2P,2R3P	×	×	×	×	×
	F3	×	5R	×	×	×	×
	C3	×	5P	×	×	×	×
	F2C1	×	5R,4R1P, 3R2P,2R3P	×	×	×	×
	F1C2	×	5R,4R1P, 3R2P,2R3P	×	×	×	×
6	C1	6R,5R1P,4R2P	×	×	×	×	×
	F1	6R,5R1P,4R2P	×	×	×	×	×
	F2	×	6R,5R1P,4R2P, 3R3P,2R4P	×	×	×	×
	C2	×	6R,5R1P,4R2P, 3R3P,2R4P	×	×	×	×
	F1C1	×	6R,5R1P,4R2P, 3R3P,2R4P	×	×	×	×
	F3	×	×	6R	×	×	×
	C3	×	×	6P	×	×	×
	F2C1	×	×	6R,5R1P,4R2P, 3R3P,2R4P	×	×	×
	F1C2	×	×	6R,5R1P,4R2P, 3R3P,2R4P	×	×	×
7	F1	×	7R,6R1P,5R2P, 4R3P,3R4P,2R5P	×	×	×	×
	C1	×	7R,6R1P,5R2P, 4R3P,3R4P,2R5P	×	×	×	×
	F2	×	×	7R,6R1P,5R2P, 4R3P,3R4P,2R5P	×	×	×
	C2	×	×	7R,6R1P,5R2P, 4R3P,3R4P,2R5P	×	×	×
	F1C1	×	×	7R,6R1P,5R2P, 4R3P,3R4P,2R5P	×	×	×

Table 3. Cont.

n	Constraint Type	Possible degrees of freedom from 1 to 6					
		1	2	3	4	5	6
8	F1	×	×	8R,7R1P,6R2P, 5R3P,4R4P, 3R5P,2R6P	×	×	×
	C1	×	×	8R,7R1P,6R2P, 5R3P,4R4P, 3R5P,2R6P	×	×	×
	F2	×	×	×	8R,7R1P, 6R2P, 5R3P, 4R4P, 3R5P, 2R6P	×	×
	C2	×	×	×	8R,7R1P, 6R2P, 5R3P, 4R4P, 3R5P, 2R6P	×	×
	F1C1	×	×	×	8R,7R1P, 6R2P, 5R3P, 4R4P, 3R5P, 2R6P	×	×
9	F1	×	×	×	√	×	×
	C1	×	×	×	√	×	×
10	F1	×	×	×	×	√	×
	C1	×	×	×	×	√	×

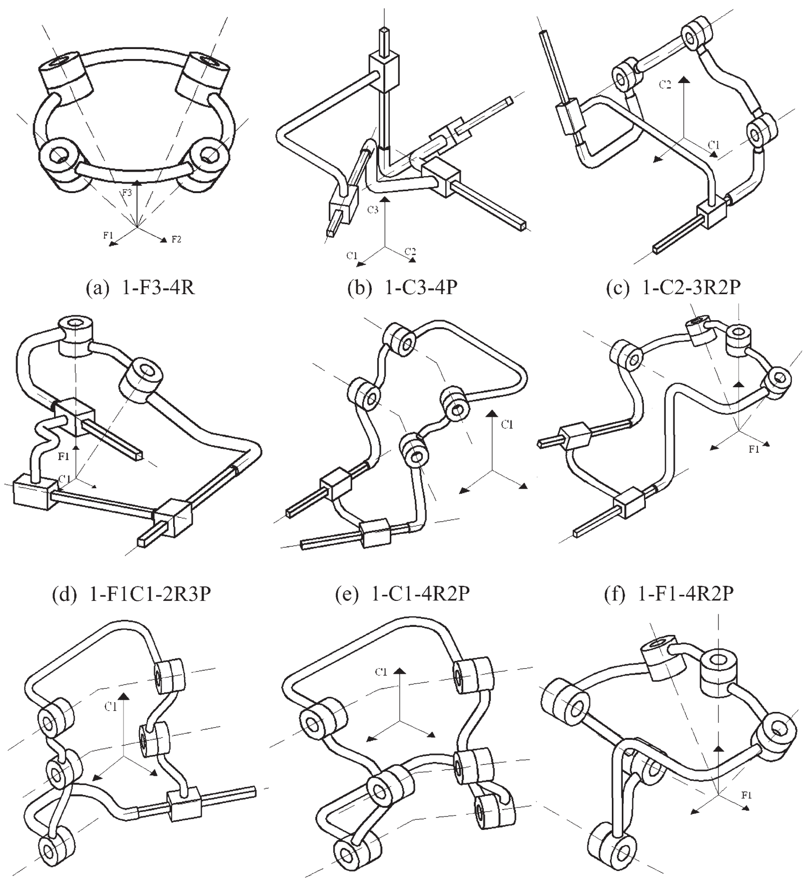
Note: The spatial mechanisms without common constraints and with 9 links can have only 4 DOF and those with 10 links can have only 5 DOF, so they do not show any degenerative DOF characteristics.

where $\dot{\theta}_{ij}$ represents the intensity and $\$_{ij}$ represents the screw associated with the jth revolute joint of the ith limb. We choose 1-C1-6R, 2-F2-6R and 3-F3-6R configurations respectively for the three mechanisms to validate their degeneration characteristics of DOFs.

From Fig.6, we obtain the following screws:

$$\begin{cases} \$_1 = [1\ \ 0\ \ 0\ ;\ 0\ \ 0\ \ 0]^T \\ \$_2 = [1\ \ 0\ \ 0\ ;\ 0\ \ e_2\ \ f_2]^T \\ \$_3 = [1\ \ 0\ \ 0\ ;\ 0\ \ e_3\ \ f_3]^T \end{cases} \quad \begin{cases} \$_4 = [0\ \ 1\ \ 0\ ;\ d_4\ \ 0\ \ f_4]^T \\ \$_5 = [0\ \ 1\ \ 0\ ;\ d_5\ \ 0\ \ f_5]^T \\ \$_6 = [0\ \ 1\ \ 0\ ;\ 0\ \ 0\ \ 0]^T \end{cases} \tag{30}$$

where the vector $[d_i\ \ e_i\ \ f_i]$, defines the moment of the screw axis about the origin of the reference frame chosen.

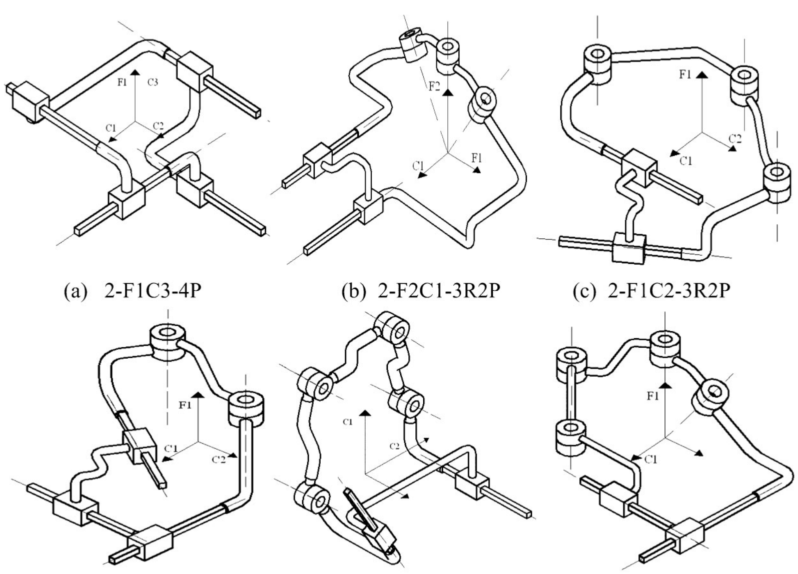


(a) 1-F3-4R (b) 1-C3-4P (c) 1-C2-3R2P

(d) 1-F1C1-2R3P (e) 1-C1-4R2P (f) 1-F1-4R2P

(g) 1-C1-5R1P (h) 1-C1-6R (i) 1-F1-6R

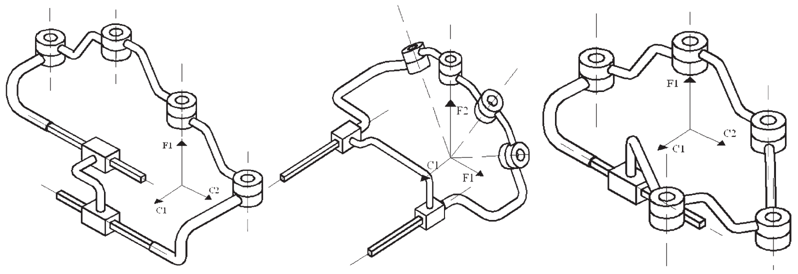
Fig. 3. 1-DOF closed loop over constrained mechanism.



(a) 2-F1C3-4P (b) 2-F2C1-3R2P (c) 2-F1C2-3R2P

(d) 2-F1C2-2R3P (e) 2-C2-4R2P (f) 2-F1C1-4R2P

Fig. 4. 2-DOF closed loop over constrained mechanism.



(a) 3-F1C2-4R2P (b) 3-F2C1-4R2P (c) 3-F1C2-5R1P

(d) 3-F1C2-6R (e) 3-F3-6R

Fig. 5. 3-DOF closed loop over constrained mechanism.

Since the motion of the moving platform can be treated as a resultant of coupled movement of all individual limbs, we use Eq. (30) to obtain,

$$v_z = \frac{\delta_1}{e_2 - e_3}\dot{\theta}_1,\ v_z = \frac{\delta_1}{e_3}\dot{\theta}_2,\ v_z = \frac{-\delta_1}{e_2}\dot{\theta}_3,\ v_z = \frac{\delta_2}{d_5}\dot{\theta}_4,\ v_z = \frac{-\delta_2}{d_4}\dot{\theta}_5,\ v_z = \frac{\delta_2}{d_4 - d_5}\dot{\theta}_6 \tag{31}$$

where v_z is the translation velocity along the z axis, $\delta_1 = e_3 f_2 - e_2 f_3, \delta_2 = d_5 f_4 - d_4 f_5$.

From the above equations, we validate that the above 1-C1-6R closed loop mechanism certainly has one DOF, namely the movement along the direction of the z axis. So we can choose any one of the joints as the actuated joint.

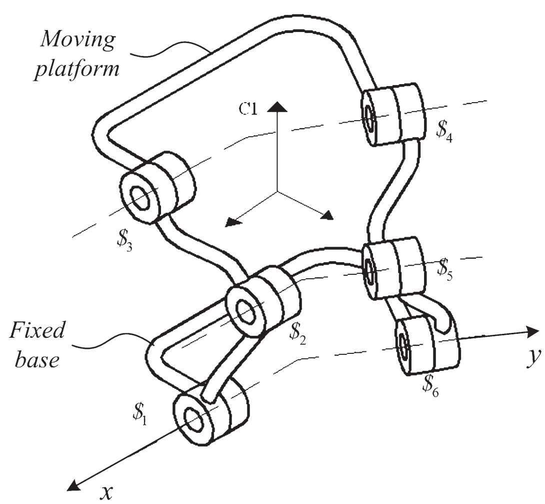


Fig. 6. 1-C1-6R closed loop mechanism.

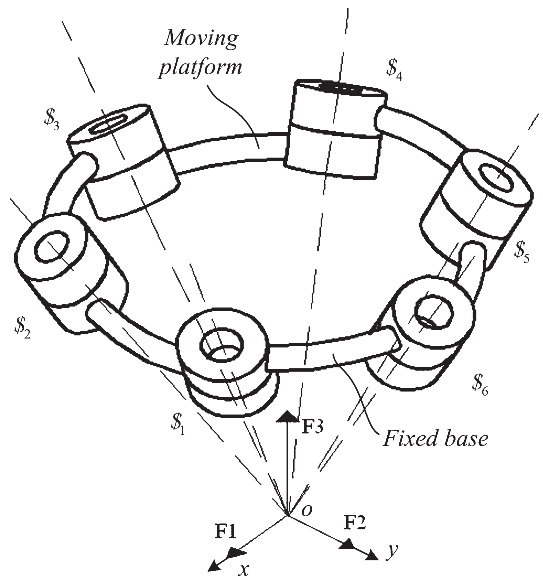


Fig. 7. 3-F3-6R closed loop mechanism.

From Fig.7, we obtain the following screws:

$$\$_i = [a_i \;\; b_i \;\; c_i \,; \; 0 \;\; 0 \;\; 0]^T \quad i = 1,2,\cdots,6 \tag{32}$$

The three independent parameters *a*, *b* and *c* are required to identify the motion. This proves that mechanism is certainly a three DOFs mechanism.

7. CONCLUSIONS

A systematic method is presented to study the degeneration characteristics of DOFs of closed loop, over constrained mechanism. The major works and conclusions of this paper are drawn as follows:

1. The configurations of revolute and prismatic joints in closed loop, over constrained mechanism are obtained. The closed loop mechanisms are classified based on the type of overconstraints.
2. The degeneration characteristics of DOFs of closed loop, over constrained mechanisms are studied with a concept of "*Degenerative DOF*", which explains the existence of different DOF for mechanisms containing the same type and number of links and joints.

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