AN APPROACH FOR COUNTING THE NUMBER OF SPECIALIZED MECHANISMS SUBJECT TO NON-ADJACENCY CONSTRAINTS

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ABSTRACT

This paper presents an improved approach to count the number of specialized mechanisms subject to non-adjacency constraints from a candidate kinematic chain. First, the permutation group of the candidate kinematic chain is found. Next, an inventory polynomial named kinematic king polynomial (KKP) to count the specialized mechanisms is modified from the traditional king polynomial related to the count of moves of a king on a chess. Then, an algorithm to calculate the KKP is presented by operations on labeled joint adjacency matrix (LJAM). Finally, two examples are illustrated to verify the approach.

Keywords: non-adjacency constraint; specialized mechanism; permutation group; adjacency matrix.

UNE NOUVELLE APPROCHE POUR ÉNUMÉRER LES MÉCANISMES SPÉCIALISÉS SOUMIS À DES CONTRAINTES NON ADJACENTES

RÉSUMÉ

Cet article présente une amélioration de la méthode de numération de mécanismes spécialisés soumis à des contraintes non adjacentes à partir d'une chaine cinématique donnée. Tout d'abord, tous les groupes de permutation issus de cette chaine cinématique sont décrites. Ensuite, la méthode de calcul des polynômes cinématiques du roi (KKP) permettant d'énumérer les mécanismes spécialisés est modifiée à partir du polynôme du roi traditionnel lié à la numération des mouvements d'un roi sur un jeu d'échecs. Puis, un algorithme de calcul du KKP est présenté par des opérations sur la matrice d'adjacence des articulations nommées (LJAM). Enfin, deux exemples sont illustrés afin de vérifier l'approche.

Mots-clés : contraintes non adjacentes ; mécanisme spécialisé ; groupe de permutation ; matrice d'adjacence.

1. INTRODUCTION

A systematized method of creative mechanism design in type synthesis, the separation of function and structure allows derivation of the necessary topological structures between links and joints and thus deduction of possible topological structures by combination.

In general, mechanisms are type synthesized using the following three-step procedure: identifying the appropriate mechanism type (e.g. the numbers or link and joint types and necessary design constraints), enumerating the basic kinematic chains and their required numbers of links and joints, and the specialization of mechanisms [1]. In this latter, each kinematic chain is specialized through the assignation of link and joint types to obtain all possible mechanism configurations.

Mechanism specialization has been the subject of many studies, which have variously based the structural synthesis of kinematic chains on graph theory [2–9], matrices [10–12] and combinatorial theory [13]. In some such investigations, generation of mechanisms during the synthesis and specialization process requires complex procedures for detection (or computer-aided assignation) and deletion of isomorphic mechanisms [14–16]. For checking the correction of the results of specialized mechanism without isomorphism, the Polya's theory is applied to count the number of mechanisms.

The basic concepts of Polya's theory of enumeration with application to the structural classification of mechanism were proposed at the first by Freudenstein in 1967 [17]. Then Bushsbaun and Freudenstein synthesized the kinematic structure of geared kinematic chains and other mechanisms by using previous work and combinatorial mathematics [18]. In 1991, Yan and Hwang [19] proposed a methodology for enumerating nonisomorphic specialized mechanisms from a specified kinematic chain using the cycle index of permutation groups to calculate the number of the synthesized mechanisms. This method has also been successfully applied in the number synthesis of general simple joint and multiple joints kinematic chains [20–21]. Hwang and Lin [22] applied the concept of generating function and permutation group to generate general specialized mechanisms. Yan and Hung [23–25] then provided a procedure for generating non-isomorphic specialized mechanisms to identify and count the number of mechanism from kinematic chains subject to design constraints. They applied Polya's theory and Burnside's theorem to count the number of mechanisms with a pair of non-adjacent specialized joints. However, the method can not be applied to count the number of mechanisms with arbitrary number of non-adjacent specialized joints.

For instance, the Example 3 in the paper [23] is to count the number of nonisomorphic identified mechanisms, with two nonadjacent prismatic joints, from the Stephenson-III mechanism. The set of the non-adjacency joints of mechanism is listed as $S'' = \{af, ae, ag, bc, be, bf, cd, ce, cg, df, dg\}$, where $a, b, [...], g$ are the joints of Stephenson mechanism, and $af, ae, ..., dg$ are pairs of non-adjacent joints. The permutations are expressed as P''_{J1} and P''_{J2}. The non-adjacency joint group of the kinematic chain, G''_J, is $G''_J = \{P''_{J1}, P''_{J2}\}$, where $P''_{J1} = [ae][af][ag][bc][be][bf][cd][ce][cg][df][dg]$ and $P''_{J2} = [af][ag\,ae][bc\,cd][be\,dg][bf\,df][ce\,cg]$. The cycle index of the non-adjacency joint group P''_G is:

$$P''_G = \frac{1}{4}\left[y_1^{11} + y_1 y_2^5\right], \tag{1}$$

where dummy variable y indicates the types of the non-adjacency joints.

Substituting $y_1 = (B+1)$ and $y_2 = (B^2+1)$ into the Eq. (1), the pattern inventory I can be expressed as:

$$\begin{aligned} I &= \frac{1}{4}\left[(B+1)^{11} + (B+1)(B^2+1)^5\right] = 1 + 6B + 30B^2 + 85B^3 \\ &\quad +170B^4 + 236B^5 + 236B^6 + 170B^7 + 85B^8 + 30B^9 + 6B^{10} + B^{11}. \end{aligned} \tag{2}$$

The coefficient of term B is 6 in Eq. (2). It means that there are six nonisomorphic mechanisms with two non-adjacency prismatic joints as the required in the Example 3. However, the other terms in the equation can not explain anything for the subject. If we want to count the numbers for the constraints with three or more nonadjacent joints, all the tasks have to be re-calculated. The drawback in this paper is that the approach proposed exists the difficult to do with the candidate kinematic chains with a large number of links or joints for the numbers of the elements in the set S'' will be increased with geometric growth. The order of the cycle index will be much higher and the calculation will be complicated.

The purpose of this paper is to propose an approach to count the number of specialized mechanism with any number of non-adjacent specialized joints. Mechanisms with non-adjacency specialized joints mean that the assignment of specific joint types on a kinematic chain can not be adjacent. This is like to find the ways to press non-adjacent kings on a chessboard. In 1977, Motoyama and Hosoya [26] defined the king and domino polynomials on a chessboard or square lattice of arbitrary size and shape. By using the principle of inclusion and exclusion the authors showed that the king polynomial of a chessboard G, $K_G(x)$ is recursively related to subgraph. The similar paper in 1985, Balasubramanian and Ramaraj [27] proposed the computer generation of king and the color polynomial of graphs and lattices.

In this paper we propose a modified king polynomial for kinematic chains in the first time called kinematic king polynomial or KKP in short to develop inventory polynomials for type synthesizing mechanisms subject to non-adjacency constraints.

2. TERMINOLOGY

For the purpose to atlases of kinematic chains, we will present several terminology and definitions according to some basic concepts from the graph and the combinatorial theory.

2.1. Labeled link and joint adjacency matrix

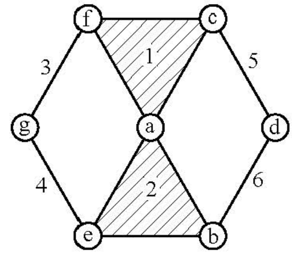
Traditionally, for a kinematic chain with n links, the link adjacency matrix (LAM) $[a_{ij}]$ is an $n \times n$ matrix; if links i and j are adjacent, then $a_{ij} = 1$, otherwise $a_{ij} = 0$. Likewise, for a kinematic chain with m joints, the joint adjacency matrix (JAM) $[b_{ij}\ ]$ is an $m \times m$ matrix, and if joints i and j are adjacent, then $b_{ij} = 1$, otherwise $b_{ij} = 0$. For a labeled kinematic chain with its links being labeled by integers $\{1,2,3,...,i,...\}$ and joints by alphabets $\{a,b,c,...,k,...\}$, we define its corresponding labeled link adjacency matrix (LLAM) as a matrix with its elements $a_{ii} = i$ for the ith link, and $a_{ij} = 1$ for links i and j being adjacent, otherwise $a_{ij} = 0$. Similarly, the labeled joint adjacency matrix (LJAM) is defined as a matrix with its elements $b_{ii} = k$, where k is the label of the ith joint, and $b_{ij} = 1$ for joints i and j being adjacent, otherwise $b_{ij} = 0$. To illustrate, the Watt kinematic chain is in Fig. 1(a) whose labeled link and joint adjacency matrices are given in Fig. 1(b) [1].

2.2. Permutation groups

A permutation p is a bijection (one-to-one and onto) of a finite set S into itself. For example, the sequence (a_2,a_3,a_1,a_4) is a permutation of the set $s = (a_1,a_2,a_3,a_4)$ in which $a_1 \rightarrow$ (is transformed into) a_2, $a_2 \rightarrow a_3$, $a_3 \rightarrow a_1$ and $a_4 \rightarrow a_4$. In this permutation, $a_1 \rightarrow a_2 \rightarrow a_3 \rightarrow a_1$ forms a cycle, denoted by $[a_1,a_2,a_3]$, with a length of three, while $a_4 \rightarrow a_4$ forms another cycle $[a_4]$ with a length of one. The cyclic representation of this permutation p is denoted by $[a_1\, a_2\, a_3][a_4]$.

2.3. Link groups and joint groups

If numbers are assigned to the links of a kinematic chain, then, assuming that the chain has n links, assignation of the number $1\sim n$ is arbitrary and the possible ways of numbering the links is $n!$, with each serial number being a permutation. However, because of the links adjacent relationship, the corresponding link number of some permutations will have the same adjacent relationship. The class of these permutations will thus form a permutation group (G), termed a link group (G_L).



(a)

$$LLAM = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 2 & 0 & 1 & 0 & 1 \\ 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 1 & 1 & 4 & 0 & 0 \\ 1 & 0 & 0 & 0 & 5 & 1 \\ 0 & 1 & 0 & 0 & 1 & 6 \end{bmatrix} \quad LJAM = \begin{bmatrix} a & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & b & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & c & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & d & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & e & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & f & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & g \end{bmatrix}$$

(b)

Fig. 1. Watt kinematic chain and its LLAM and LJAM

To exemplify, in the Watt kinematic chain given in Fig. 1(a), the link is assigned numbers 1 to 6 to give a total possible number sequence of $6! = 720$. If the link number in Fig. 1(a) is assumed to be standard, then among all 720 possible permutations, there are four serial numbers of permutations having the same adjacent relationships. These four permutations then become the link permutation group of the kinematic chain as shown in Eq. (3):

$$G_L = \{p_1, p_2, p_3, p_4\}, \tag{3}$$

where $p_1 = [1][2][3][4][5][6]$, $p_2 = [1][2][3\,5][4\,6]$, $p_3 = [1\,2][3\,4][5\,6]$ and $p_4 = [1\,2][3\,6][4\,5]$. Likewise, the joint group of the kinematic chain is formed as shown in Eq. (4):

$$G_J = \{p'_1, p'_2, p'_3, p'_4\}, \tag{4}$$

where $p'_1 = [a][b][c][d][e][f][g]$, $p'_2 = [a][b\,c][e\,f][d][g]$, $p'_3 = [a][b\,e][c\,f][d\,g]$ and $p'_4 = [a][b, f][c\,e][d\,g]$.

2.4. Similar classes

If $s = (a_1, a_2, a_3, \ldots)$ is the set of the elemental labels (links or joints) of a kinematic chain, then, given a permutation p of G_L (or p' of G_L) that transforms a_i into a_k, the two elements a_i and a_k are similar subject to this permutation [17] [28]. The set s can also be partitioned into similar classes by putting like elements into the same class. For instance, in the kinematic chain shown in Fig. 1(a), $\{1,2\}$ and $\{3,4,5,6\}$ are two similar classes of links, and $\{a\}$, $\{b,c,e,f\}$ and $\{d,g\}$ are three similar classes of joints.

2.5. Cycle index

If G is a permutation group of set s, then, because each permutation p in G can be written uniquely as a product of disjoint cycles, the cycle structure representation [13] of a permutation is $x_1^{b1} x_2^{b2} \ldots x_k^{bk} \ldots$, where

x_k is a dummy variable for cycles with a length of k and b_k is their number. For example, the permutation $p = [1][2\ 6][3\ 5][4]$ has the cycle structure representation $x_1^2 x_2^2$.

The cycle index [13] of a permutation group (see Eq. 5), denoted by C_i, is the summation of the cycle structure representations of all the permutations that make up the group's elements divided by the number of permutations (n):

$$C_i(x_1, x_2, x_3, \dots) = \frac{1}{n} \sum_{p \in PG} x_1^{b1} x_2^{b2} x_3^{b3} \dots \tag{5}$$

Thus, for example, the cycle index of the link group of the kinematic chain shown in Fig. 1(a) is as shown in Eq. (6). As shown in Eq. (3), there are four permutations: the first consists of six cycles with a length of one; the second, of two cycles with a length of one and two cycles with a length of two; and the third and fourth, of three cycles with a length of two.

$$C_i(x_1, x_2) = \frac{1}{4}(x_1^6 + x_1^2 + x_2^2 + 2x_2^3)\,. \tag{6}$$

2.6. Polya's theory

In the specialization process, once the cycle index has been calculated as shown in Eq. (3), Polya's theory can be used to calculate the number of results [19] as shown in Eq. (7), where T is the allotting type of kinematic pair. For example, the kinematic chain shown in Fig. 1(a), if fixed link (F) and link (L) are assigned to the kinematic chain, then $x_1 = F + L$ and $x_2 = F^2 + L^2$. Substituting x_1 and x_2 in Eq. (6) produces the results given in Eq. (8): two possible allotments for a fixed link $2L^5F$, six for two fixed links $6L^4F^2$, and six for three fixed links $6L^3F^3$.

$$Ip = C_i(T, T^2, T^3)\,, \tag{7}$$

$$\begin{aligned} Ip &= \frac{1}{4}\left((F+L)^6 + (F+L)^2(F^2+L^2)^2 + 2(F^2+L^2)^3\right) \\ &= L^6 + 2L^5F + 6L^4F^2 + 6L^3F^3 + 6L^2F^4 + 2LF^5 + F^6\,. \end{aligned} \tag{8}$$

2.7. King polynomial

King polynomial is used to express the number of ways to press non-adjacent kings on a chessboard. For example, a chessboard is as shown in Fig. 2. There are 9 ways to press a king on the board, 16 ways to press two kings on the board as shown in Fig. 3, etc. Hence, the results can be expressed by a polynomial called King polynomial as Eq. (9):

$$K_G(x) = 1 + 9x + 16x^2 + 8x^3 + x^4\,. \tag{9}$$

In 1977, Motoyama and Hosoya [26] proposed a recursive formula to calculate the king polynomial as Eq. (10):

$$K_G(x) = K_{G-C}(x) + x \cdot K_{G\Theta C}(x)\,, \tag{10}$$

where $G - C$ is the chessboard obtained by removed a cell "C" from G and $G\Theta C$ is the chessboard resulting from the removal of the cell C and all the neighboring cells of C.

3. INVENTORY POLYNOMIAL

To develop inventory polynomials for type synthesizing mechanisms subject to non-adjacency constraints, we propose a new algorithm, the kinematic king polynomial (KKP), which is applied to specialize kinematic

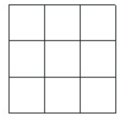


Fig. 2. A 3x3 chessboard.

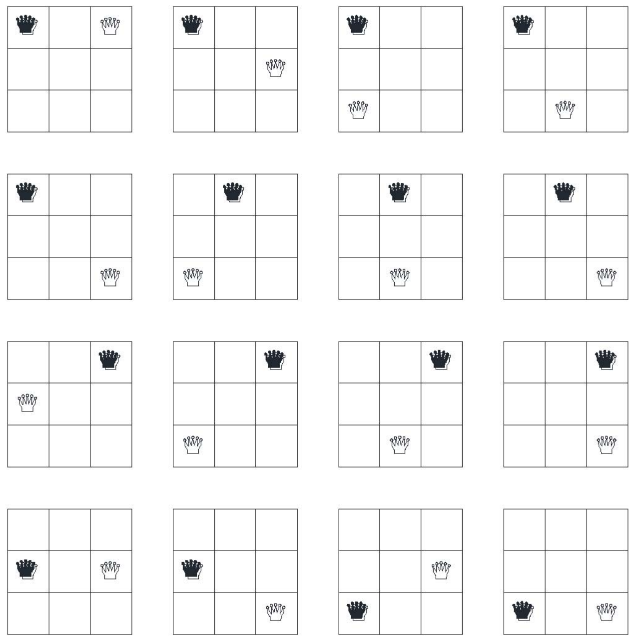


Fig. 3. The results of two kings on the board.

chains or mechanical types constrained by nonadjacent relationships. We define the kinematic king polynomial (KKP) as a representation of type polynomials that have nonadjacent joint mechanisms and express it by Eq. (11):

$$KK(x,C) = \frac{\sum\limits_{p_i \in G} K_i(x,C)}{|G|}\,, \tag{11}$$

where $KK(x,C)$ is the kinematic king polynomial of C, x is a dummy variable, C is a candidate kinematic chain, G is the permutation group of the original kinematic chain C, p_i is each permutation of the permutation group and $K_i(x,C)$ is the king polynomial of C subjected to permutation p_i.

To calculate the KKP of the kinematic chain, we must first derive K_i under each permutation p_i. A permutation can be denoted by cyclic representation as:

$$p_i = [a_1\,a_2\,\ldots\,a_m][b_1\,b_2\ldots]\ldots[c_1\,c_2\ldots]\,, \tag{12}$$

where $[a_1\,a_2\,\ldots\,a_m]$ denotes the first cycle of p_i and $[b_1\,b_2\ldots]\ldots[c_1\,c_2\ldots]$ denote the other cycles. Using the recursive formula given in Eq. (13), the king polynomial K_i for p_i can be done by assigning the elements in the first cycle.

$$\mathrm{K}_i(x,C) = \begin{cases} x^m K_i(x,C_r) + K_i(x,C_s), & \text{if } a_1, a_2, \ldots, a_m \text{ are nonadjacent to each other} \\ K_i(x,C_s), & \text{otherwise}\,. \end{cases} \tag{13}$$

x^m represents the number of m elements substituted as specialized subjects if $a_1, a_2, \ldots, a_m$ are nonadjacent to each other. In permutation p_i, elements within the same cycle are either simultaneously chosen or not. Once these elements are chosen, no adjacent relationship can be selected. C_r is the subchain of C given by deleting special elements and their adjacent elements. $K_i(x, C_r)$ is the K_i polynomial of the subchain C_r. C_s is the subchain of C given by deleting special elements and $K_i(x, C_s)$ is the K_i polynomial of the subchain C_s.

4. PROCEDURE AND EXAMPLES

To identify mechanisms, we make a list of polynomials for the basic forms of kinematic chains shown in Table 1 which joint types expressed as labeled joint adjacency matrix (LJAM). We explain some of the kinematic king polynomials (KKP) based on their joint types as the following:

The joint type is none, meaning there is no joint and its king polynomial is 1. The joint type of no.2 indicates the joint ⓐ, meaning its permutation is $[a]$ and its king polynomial is $x+1$. Here, x means one option for joint selection (joint a). The joint type of no.8 indicates the joints ⓑ–ⓒ–ⓐ. If its permutation is $[ab][c]$, then a and b have the same attributes and must be selected simultaneously or not. Here, the king polynomial is x^2+x+1 , where x^2 means one option for selecting two adjacent joints (joints a and b) and x means one option for c (joint c). Because joints a and b cannot be divided, they offer the option of choosing both or neither.

For another example in the Table 1, the joint type of no.10 indicates the joints ⓓ ⓑ–ⓒ–ⓐ. If its permutation is $[ab][c][d]$, its king polynomial is $(x^2+x+1)(x+1)$. The polynomial x^2+x+1 is derived from the joints ⓑ–ⓒ–ⓐ and its permutation is $[ab][c]$. However, because there is another joint d whose permutation is $[d]$ and whose king polynomial is $x+1$, x^2+x+1 must be multiplied by $(x+1)$, which gives x^3+2x^2+2x+1. Here, x^3 means one option for selecting three nonadjacent joints (joints a, b and d), $2x^2$ means two options for selecting two nonadjacent joints (joint a, b or joint c, d), and $2x$ means two options for selecting one joint (joint c or joint d). The following two examples illustrate the workings of the kinematic king polynomial (KKP) for the counting of specialized mechanisms with nonadjacency constraints.

4.1. Example 1

Count the number and specify prismatic joints to the kinematic chain (8, 10) as shown in Fig. 4. with the condition of the constraint on any of the two prismatic joints is nonadjacent.

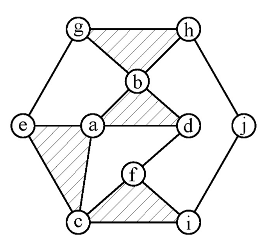


Fig. 4. Labeled (8, 10) kinematic chain.

Once the joints of the chain are designated a, b, c, d, e, f, g, h and i, the joint permutation group of the kinematic chain is as given in Eq. (14):

$$G_J = \{p'_1, p'_2\}\,, \tag{14}$$

where $p'_1 = [a][b][c][d][e][f][g][h][i][j]$ and $p'_2 = [a][bc][de][fg][hi][j]$.

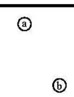
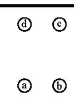
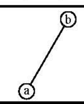
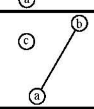
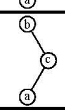
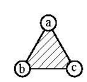
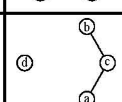
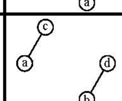
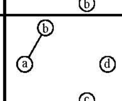
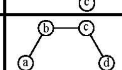
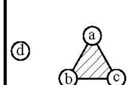
	Joint types	LJAM	Types of Permutations and their King Polynomial				
1	none	none	none				
			1				
2		$[a]$	[a]				
			$x+1$				
3		$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$	[a][b]	[ab]			
			$(x+1)^2$	x^2+1			
4		$\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$	[a][b][c]	[ab][c]	[abc]		
			$(x+1)^3$	$(x^2+1)(x+1)$	x^3+1		
5		$\begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{bmatrix}$	[a][b][c][d]	[ab][cd]	[abc][d]	[abcd]	
			$(x+1)^4$	$(x^2+1)(x^2+1)$	$(x^3+1)(x+1)$	x^4+1	
6		$\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix}$	[a][b]	[ab]			
			$2x+1$	1			
7		$\begin{bmatrix} a & 1 & 0 \\ 1 & b & 0 \\ 0 & 0 & c \end{bmatrix}$	[a][b][c]	[ab][c]	[ac][b]	[abc]	
			$(2x+1)(x+1)$	$x+1$	x^2+x+1	1	
8		$\begin{bmatrix} a & 0 & 1 \\ 0 & b & 1 \\ 1 & 1 & c \end{bmatrix}$	[a][b][c]	[ab][c]	[ac][b]	[abc]	
			x^2+3x+1	x^2+x+1	$x+1$	1	
9		$\begin{bmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{bmatrix}$	[a][b][c]	[ab][c]	[abc]		
			$3x+1$	$x+1$	1		
10		$\begin{bmatrix} a & 0 & 1 & 0 \\ 0 & b & 1 & 0 \\ 1 & 1 & c & 0 \\ 0 & 0 & 0 & d \end{bmatrix}$	[a][b][c][d]	[ab][cd]	[ac][bd]	[ab][c][d]	[ac][b][d]
			$(x^2+3x+1)(x+1)$	$2x^2+1$	x^2+1	$(x^2+x+1)(x+1)$	$(x+1)(x+1)$
11		$\begin{bmatrix} a & 0 & 1 & 0 \\ 0 & b & 0 & 1 \\ 1 & 0 & c & 0 \\ 0 & 1 & 0 & d \end{bmatrix}$	[a][b][c][d]	[ab][cd]	[ac][bd]	[abcd]	
			$(2x+1)(2x+1)$	$2x^2+1$	1	1	
12		$\begin{bmatrix} a & 1 & 0 & 0 \\ 1 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{bmatrix}$	[a][b][c][d]	[a][b][cd]	[ab][c][d]	[ab][cd]	
			$(2x+1)(x+1)^2$	$(2x+1)(x^2+1)$	$(x+1)^2$	x^2+1	
13		$\begin{bmatrix} a & 1 & 0 & 0 \\ 1 & b & 1 & 0 \\ 0 & 1 & c & 1 \\ 0 & 0 & 1 & d \end{bmatrix}$	[a][b][c][d]	[ab][c][d]	[ad][bc]	[ab][cd]	[abcd]
			$3x^2+4x+1$	$2x+1$	x^2+1	1	1
14		$\begin{bmatrix} a & 1 & 1 & 0 \\ 1 & b & 1 & 0 \\ 1 & 1 & c & 0 \\ 0 & 0 & 0 & d \end{bmatrix}$	[a][b][c][d]	[ab][c][d]	[ab][cd]		
			$(3x+1)(x+1)$	$(x+1)(x+1)$	x^2+1		

Table 1. The polynomials for the basic forms of kinematic chains.

Because joint group G_J has two permutations, we must take each permutation p_i separately to obtain the K_i polynomial:

$p'_1 = [a][b][c][d][e][f][g][h][i][j]$:

$$K_1 = (\begin{bmatrix} a & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & b & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & c & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & d & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & e & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & f & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & g & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & h & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & i & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & j \end{bmatrix})$$

$$= x(\begin{bmatrix} f & 0 & 0 & 1 & 0 \\ 0 & g & 1 & 0 & 0 \\ 0 & 1 & h & 0 & 1 \\ 1 & 0 & 0 & i & 1 \\ 0 & 0 & 1 & 1 & j \end{bmatrix}) + \begin{bmatrix} b & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & c & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & d & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & e & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & f & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & g & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & h & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & i & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & j \end{bmatrix}$$

$$= x(x(\begin{bmatrix} f & 1 \\ 1 & i \end{bmatrix}) + \begin{bmatrix} f & 0 & 1 & 0 \\ 0 & g & 0 & 0 \\ 1 & 0 & i & 1 \\ 0 & 0 & 1 & j \end{bmatrix}) + x(\begin{bmatrix} c & 1 & 1 & 1 & 0 \\ 1 & e & 0 & 0 & 0 \\ 1 & 0 & f & 1 & 0 \\ 1 & 0 & 1 & i & 1 \\ 0 & 0 & 0 & 0 & j \end{bmatrix}) + \begin{bmatrix} c & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & d & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & e & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & f & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & g & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & h & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & i & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & j \end{bmatrix}$$

$$= x^4 + 6x^3 + 5x^2 + x + x(x(\begin{bmatrix} e & 0 \\ 0 & j \end{bmatrix}) + \begin{bmatrix} c & 1 & 0 & 0 \\ 1 & e & 1 & 0 \\ 0 & 1 & i & 1 \\ 0 & 0 & 1 & j \end{bmatrix}) + x(\begin{bmatrix} e & 1 & 0 & 0 \\ 1 & g & 1 & 0 \\ 0 & 1 & h & 1 \\ 0 & 0 & 1 & j \end{bmatrix}) + \begin{bmatrix} c & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & d & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & e & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & g & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & h & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & i & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & j \end{bmatrix}$$

$$= 3x^4 + 18x^3 + 18x^2 + 4x + x(x^3 + 3x^2 + 3x + 1) + x(\begin{bmatrix} c & 0 & 1 \\ 0 & d & 0 \\ 1 & 0 & i \end{bmatrix}) + \begin{bmatrix} c & 0 & 0 & 1 \\ 0 & d & 0 & 0 \\ 0 & 0 & g & 0 \\ 1 & 0 & 0 & i \end{bmatrix}$$

$$= 4x^4 + 25x^3 + 29x^2 + 10x + 1 \; .$$

$p'_2 = [a][bc][de][fg][hi][j]$:

$$K_2 = x(\begin{bmatrix} f & 0 & 0 & 1 & 0 \\ 0 & g & 1 & 0 & 0 \\ 0 & 1 & h & 0 & 1 \\ 1 & 0 & 0 & i & 1 \\ 0 & 0 & 1 & 1 & j \end{bmatrix}) + \begin{bmatrix} b & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & c & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & d & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & e & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & f & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & g & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & h & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & i & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & j \end{bmatrix}$$

$$= x(x(\begin{bmatrix} f & 0 \\ 0 & g \end{bmatrix}) + \begin{bmatrix} f & 0 & 0 & 1 \\ 0 & g & 1 & 0 \\ 0 & 1 & h & 0 \\ 1 & 0 & 0 & i \end{bmatrix}) + x^2([j]) + \begin{bmatrix} d & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & e & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & f & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & g & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & h & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & i & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & j \end{bmatrix}$$

$$= 2x^4 + 5x^3 + 5x^2 + 2x + 1 \; .$$

Once each K_i polynomial is derived, the KK polynomial of the whole permutation can be obtained as shown in Eq. (15):

$$KK(x,C) = \frac{(K_1 + K_2)}{2} = 3x^4 + 15x^3 + 17x^2 + 6x + 1 \; . \tag{15}$$

This calculation again represents a kinematic chain that has been allocated specific nonadjacent prismatic joints: $3x^4$ describes three types of four nonadjacent prismatic joints, $15x^3$ describes 15 types of three nonadjacent prismatic joints, $17x^2$ describes 17 types of two nonadjacent prismatic joints, $6x$ describes six types of one nonadjacent prismatic joint and 1 describes one type of zero (0) nonadjacent prismatic joints. These results outlined in Table 2, in which the selected joints were specified as the prismatic joints.

King polynomial	Selected joints as prismatic joints
$3x^4$	ajfg, dehi, jbef
$15x^3$	dhi, bef, bei, deh, dgi, afg, afh, ahi, jfg, jde, jbc, jdg, jbe, jbf, ajf
$17x^2$	fg, hi, de, bc, fh, di, dh, dg, be, bi, bf, af, ah, jf, jb, jd, aj
$6x$	a, b, d, f, h, j
1	none

Table 2. Atlas of the (8, 10) kinematic chain.

4.2. Example 2

Count the number and specify prismatic joints to the kinematic chain (8, 9) as shown in Fig. 5 with the condition of the constraint on any of the two prismatic joints is non-adjacent.

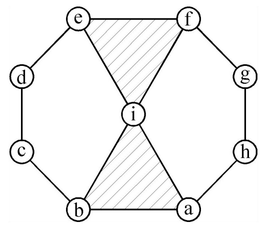


Fig. 5. Labeled (8, 9) kinematic chain.

First, we assign numbers to the joints of (8, 9) kinematic chain as a, b, c, d, e, f, g, h and i. The joint permutation group of the kinematic chain is:

$$G_j = \{p'_1, p'_2, p'_3, p'_4\}\,, \tag{16}$$

where $p'_1 = [a][b][c][d][e][f][g][h][i]$, $p'_2 = [ab][ch][dg][ef][i]$, $p'_3 = [ae][bf][cg][dh][i]$ and $p'_4 = [af][be][cd][gh][i]$.

There are four permutations in joint permutation group G_j. So, we have to get each permutation p_i respectively to obtain K_i polynomial. The steps are as followed:

$p'_1 = [a][b][c][d][e][f][g][h][i]$:

$$K_1\left(\begin{bmatrix} a & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & b & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & c & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & d & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & e & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & f & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & g & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & h & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & i \end{bmatrix}\right)$$

$$= x\left(\begin{bmatrix} c & 1 & 0 & 0 \\ 1 & d & 0 & 0 \\ 0 & 0 & g & 1 \\ 0 & 0 & 1 & h \end{bmatrix}\right) + \begin{bmatrix} a & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & b & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & c & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & d & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & e & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & f & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & g & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & h \end{bmatrix}$$

$$= 2x^4 + 20x^3 + 24x^2 + 9x + 1\,.$$

In the same way we can get the results as below:

$p_2' = [ab][ch][dg][ef][i] : K_2 = 2x^3 + 2x^2 + x + 1;$

$p_3' = [ae][bf][cg][dh][i] : K_3 = 2x^4 + 2x^3 + 4x^2 + x + 1;$

$p_4' = [af][be][cd][gh][i] \; K_4 = 2x^2 + x + 1.$

After finishing each K_i polynomial, the KK polynomial of the whole permutation can be derived. The result is as below:

$$KK(x,C) = \frac{(K_1 + K_2 + K_3 + K_4)}{4} = x^4 + 6x^3 + 8x^2 + 3x + 1 \,. \tag{17}$$

This result is a representation showing that the kinematic chain was allocated to non-adjacent prismatic joints. Among which, x^4 describes one kind of the four non-adjacent prismatic joints; $6x^3$ describes six kinds of the three non-adjacent prismatic joints; $8x^2$ describes eight kinds of the two non-adjacent prismatic joints; $3x$ describes three kinds of the one allotted non-adjacent prismatic joint and 1 describes one of the zero (0) allotted prismatic joints. The allotted results are given in Table 3 in which the solid circle joints indicate the prismatic joints, and the hollow circle joints indicate the rotation joints.

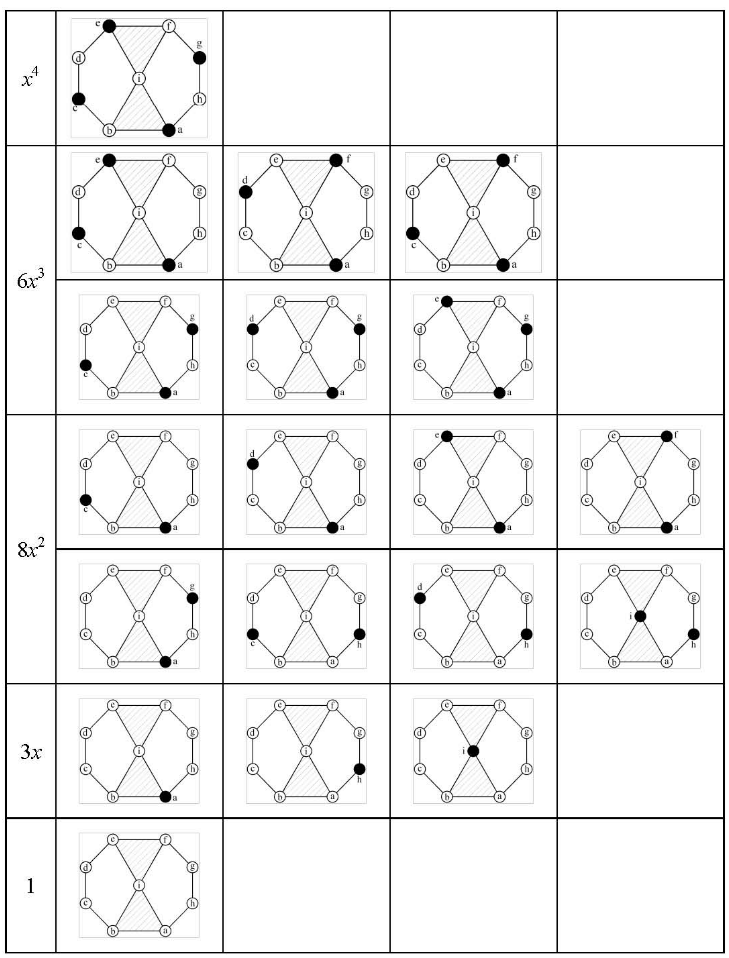


Table 3. Atlas of the (8, 9) kinematic chain.

5. CONCLUSION

In conclusion, based on combinatorial theory and king polynomial, we define a new polynomial named the kinematic king polynomial (KKP) for the counting of specialized mechanisms with nonadjacency constraints. The number of non-isomorphic mechanisms can be counted exactly by mathematic formulas. Furthermore, a recursive approach for calculating the KKP is derived from the labeled adjacency matrices. Two examples are used to illustrate the validity of the approach. The results of this work can be computerized, and it is beneficial to the automation of the creative design of mechanisms.

REFERENCES

1. Yan, H.S., *Creative Design of Mechanical Devices*, Springer, Singapore, 1998.
2. Freudenstein, F. and Dobrjanskyj, L., "On a theory of type synthesis of mechanism", *Proceedings of the 11th International Congress of Applied Mechanics*, pp. 420–428, 1964.
3. Crossley, F.R.E., "The permutations of kinematic chains of eight members or less from the graph-theoretic viewpoint", *Developments in Theoretical and Applied Mechanisms*, Ed. by W.A. Shaw, Pergamon Press, Oxford, Vol. 2, pp. 467–486, 1965.
4. Woo, L.S., "Type synthesis of plane linkages", *ASME Transactions, Journal of Engineering for Industry*, Vol. 89b, pp. 159–172, 1967.
5. Yan, H.S. and Hsu, C.H., "Contracted graphs of kinematic chains with multiple joints", *Mathematical and Computer Modelling*, Vol. 10, No. 9, pp. 681–695, 1988.
6. Yan, H.S. and Hwang, Y.W.,"Number synthesis of kinematic chains based on permutation groups", *Mathematical and Computer Modelling*, Vol. 13, No. 8, pp. 29–42, 1990.
7. Kim, J.T. and Kwak, B.M., "An algorithm of topological ordering for unique representation of graphs", *ASME Journal of Mechanical Design*, Vol. 114, No. 1, pp. 103–108, 1992.
8. Hsu, C.H., "Enumeration of basic kinematic chains with simple and multiple joints", *Journal of the Franklin Institute*, Vol. 329, No. 4, pp. 775–789, 1992.
9. Ding, H. and Huang, Z., "Isomorphism identification of graphs: especially for the graphs of kinematic chains", *Mechanism and Machine Theory*, Vol. 44, No. 1, pp. 122–139, 2009.
10. Chang, Z., Zhang, C., Yang, Y. and Wang, Y., "A new method to mechanism kinematic chain isomorphism identification", *Mechanism and Machine Theory*, Vol. 37, No. 4, pp. 411–417, 2002.
11. Cubillo, J.P. and Wan, J., "Comments on mechanism kinematic chain isomorphism identification using adjacent matrices", *Mechanism and Machine Theory*, Vol. 40, No. 1, pp. 131–139, 2005.
12. Srinath, A. and Rao, A.C., "Correlation to detect isomorphism, parallelism and type of freedom", *Mechanism and Machine Theory*, Vol. 41, No. 6, pp. 646–655, 2006.
13. Liu, C. T., "Introduction to Combinatorial Mathematics", McGraw-Hill, New York, 1968.
14. Yadav, J.N., Pratap, C.R. and Agrawal, V.P., "Computer aided detection of isomorphism among binary chains using the link-link multiplicity distance concept", *Mechanism and Machine Theory*, Vol. 31 No. 7, pp. 873–877, 1996.
15. Sunkari, R.P. and Schmidt, L.C., "Structural synthesis of planar kinematic chains by adapting a Mckay-type algorithm", *Mechanism and Machine Theory*, Vol. 41, No. 9, pp. 1021–1030, 2006.
16. Ding, H., Huang, Z. and Mu, D., "Computer-aided structure decomposition theory of kinematic chains and its application", *Mechanism and Machine Theory*, Vol. 43 No. 12, pp. 1596–1609, 2008.
17. Freudenstein, F., "The basic concepts of Polya's theory of enumeration, with application to the structural classification of mechanisms", *Journal of Mechanics*, Vol. 3, pp. 275–290, 1967.
18. Bushsbaum, F. and Freudenstein, F., "Synthesis of the kinematic structure of geared kinematic chains and other mechanisms", *ASME Journal of Mechanisms*, Vol. 5, pp. 357–392, 1970.
19. Yan, H. S. and Hwang, Y. W., "The specialization of mechanisms", *Mechanism and Machine Theory*, Vol. 26, No. 6, pp. 541–551, 1991.
20. Yan, H. S. and Hwang, Y. W., "Number synthesis of kinematic chains based on permutation groups", *Journal of Mathematical and Computer Modelling*, Vol. 13, No. 8, pp. 29–42, 1990.

21. Hwang. Y. H. and Lin, Y. B., "Number synthesis of generalized chains with multiple joints based on permutation groups", *Proceedings of the 10th National Conference of the Chinese Society of Mechanical Engineers*, pp. 269–276, 1993.
22. Hwang, Y. W., Lin, S. H. and Wu, C. C., "Generating functions for mechanism specialization", *Mechanism and Machine Theory*, Vol. 46, No. 4, pp. 492–514, 2011.
23. Yan, H. S. and Hung, C. C., "Identifying and Counting the Number of Mechanisms from Kinematic Chains Subject to Design Constraints", *ASME Journal of Mechanical Design*, Vol. 128, pp. 1177–1182, 2006.
24. Yan, H. S. and Hung, C. C., "An improved approach for counting the number of mechanisms from kinematic chains subject to design constraints", *Journal of the Chinese Society of Mechanical Engineers*, Vol. 28, No. 5, pp. 471–479, 2007.
25. Hung, C. C., Yan, H. S. and Pennock, G. R., "A procedure to count the number of planar mechanisms subject to design constraints from kinematic chains", *Mechanism and Machine Theory*, Vol. 43, No. 6, pp. 676–694, 2008.
26. Motoyama, A. and Hosoya, H., "King and domino polynomials for polyomino graphs", *Journal of Mathematical Physics*, Vol. 18, p. 1485, 1977.
27. Balasubramanian, K. and Ramaraj, R., "Computer generation of king and color polynomials of graphs and lattices and their applications to statistical mechanics", *Journal of Computional Chemistry*, Vol. 6, No. 5, pp. 447–454, 1985.
28. Harary, F., *Graph Theory*, Addison-Wesley Publishing Company, Reading, 1972