DIRECT ADAPTIVE FUZZY SLIDING OBSERVATION AND CONTROL

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ABSTRACT

In this paper, a new direct adaptive fuzzy sliding observation and control (AFSOC) method is proposed. The method can be applied to a class of unknown nonlinear systems. The proposed observer estimates the closed-loop state tracking error asymptotically, provided that the output gain matrix includes Hurwitz coefficients. The chattering phenomenon is overcome by using a boundary layer around the sliding surface. The stability of the AFSOC method is proved using the Lyapunov stability theory. Numerical simulation on a benchmark chaotic system depicts the effectiveness of the proposed algorithm.

Keywords: direct adaptive fuzzy sliding; adaptive observation; AFSOC; chaotic systems; unknown systems.

L'OBSERVATION ET LA COMMANDE DIRECTE PAR MODE ADAPTATIVE FLOUE GLISSANTE

RÉSUMÉ

Dans cet article, une nouvelle méthode directe d'observation et de commande basée sur une méthode adaptative floue glissante (AFSOC) est proposée. Cette méthode peut être appliquée à une classe de systèmes non linéaires inconnus. L'observateur proposé estime asymptotiquement l'erreur de suivi des variables d'état de système en boucle fermée, à condition que la matrice de gain de sortie comprenne les coefficients de Hurwitz. Le phénomène de cliquetis (*chattering*) est surmonté par l'utilisation d'une couche limite autour de la surface de glissement. La stabilité de la méthode AFSOC est prouvée en utilisant la théorie de stabilité de Lyapunov. La simulation numérique sur un système de référence chaotique représente l'efficacité de l'algorithme proposé.

Mots-clés : commande adaptative floue glissante ; observation adaptative ; AFSOC ; systèmes chaotiques ; systèmes inconnus.

1. INTRODUCTION

While there are a huge number of model-based control methods, in many practical situations no exact information about the model is available. Therefore, a major trend on the subject of model-free control methodologies has appeared in the literature in recent decades.

One of the most powerful methods which is widely used for control of uncertain nonlinear systems, is the sliding mode control (SMC) method [1,2]. This approach provides good results, when a nominal model of the plant is available, and the bound of uncertainty is small and known [3].

Another approach for control of an unknown dynamical system is the conventional fuzzy logic controllers (FLC) [4]. Despite the initial expectations, the parameters of the FLC are to be tuned using trials and errors on the real plant, which may not be easy or even possible. Stability analysis is another issue with the conventional fuzzy-based methods.

Many newer researches address the idea of combining the FLC approach with various nonlinear control methodologies [5–9]. One such method is the, so called, adaptive fuzzy sliding mode (AFSM) control. This approach exploits the advantages of both SMC and FLC schemes.

The idea of AFSM controllers was first proposed by Sinn-Cheng Lin and Yung-Yaw Chen [10] in 1994. In 1998, an indirect AFSM algorithm was proposed by Yoo and Ham [11], where, the unknown model was initially estimated by an adaptive fuzzy engine and the control signal was then designed based on the sliding mode theory.

Another approach is the direct AFSMC [12]. Direct AFSM algorithms are more effective than the indirect ones, for the lower possibility of singularities, compared with the indirect methods [13]. In addition, the indirect approaches require multiple estimation algorithms, while in the direct approaches the control signal is estimated using a single approximation algorithm, and hence they may converge faster.

More recent articles have improved the AFSM methods. Wang et al. [14] used the indirect AFSM algorithm to approximate the unknown system functions to design an equivalent sliding mode control signal, and also used the direct AFSM algorithm to estimate the switching control signal. Therefore, they managed to design an AFSM controller with less information on the upper bound of the uncertainty. Wai et al. [15] used the direct AFSM method to estimate the bound of the approximation error in the real time. Wai [16] used the direct AFSM algorithm to estimate the hitting part of control signal in such a way that the chattering phenomenon could be eliminated. In [17], Chen et al. employed the AFSM algorithm to estimate the bound of uncertainties. Wai et al. [18] developed a cascade AFSM algorithm, using two direct AFSM algorithms; one for estimating the set point and another for estimating the control signal. Later on, Hwang et al. [19] developed a direct AFSM algorithm by a type-2 fuzzy system for controlling unknown chaotic systems.

AFSM algorithm has also been applied in different practical nonlinear control systems such as depth control of remotely operated underwater vehicles, Bessa et al. [20], synchronization of chaotic systems, Roopaei and Zolghadri [21], control of unknown chaotic systems, by Poursamad and Markazi [22], control of MEMS resonators, by Sohanian and Markazi [23] and various other applications [24–27].

The underlying assumption of the AFSM method is that all the states of the plant are directly measurable, a matter which may not be possible for some practical applications. Therefore, several studies consider an output feedback AFSM algorithm [28–31], where an observer is employed for estimation of the tracking error vector.

In this article, a new observer is proposed, where, the measured output vector is any combination of states, provided that the output gain matrix includes Hurwitz coefficients, and observability property exists. Two AFSM sub-algorithms are used, where one of them provides the control input directly, and the other one improves the estimate of the states observation error. Possible chattering in the control input is overcome by considering a boundary layer around the sliding surface. The stability of the method is also proved using the Lyapunov theory.

This paper is organized as follows: problem statement is in Section 2; preliminary background on AFSM method is shortly reviewed in Section 3; the main idea and algorithm for the proposed observer is explained in section 4; the stability analysis is considered is Section 5; the chattering elimination method is explained in Section 6; numerical examples on applications of the proposed control method to a chaotic system is presented in Section 7; and finally the concluding remarks are given in Section 8.

2. PROBLEM STATEMENT

In this section, the general structure of the considered class of the nonlinear systems is introduced. It is important to note that this structure will be used for the purpose of stability analysis only, and not for the purpose of controller design. Consider a class of the SISO n^{th} order nonlinear system described by:

$$\begin{aligned} x^{(\mathrm{n})} &= f(X) + gu\,, \\ y &= CX\,, \end{aligned} \tag{1}$$

where $X = \left[x, \dot{x}, \ldots, x^{n-1}\right]^T = [x_1, x_2, \ldots, x_n]^T \in R^n$ is state vector, $u \in R$ is system input, $f(.)$ are unknown smooth functions of X, g is an unknown constant, with known sign, $C \in R^n$ is a row vector whose entries are Hurwitz coefficients, $y \in R$ is the output of the system and n is the system order.

The nonlinear system (Eq. 1) can be equivalently presented in the form:

$$\begin{aligned} \dot{X} &= \mathrm{A}\mathrm{X} + B\left(f(X) + gu\right)\,, \\ y &= CX\,, \end{aligned} \tag{2}$$

where $A = \begin{pmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ldots & \vdots \\ 0 & 0 & 0 & \ldots & 1 \\ 0 & 0 & 0 & \ldots & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}.$

Furthermore, it is assumed that the pair (A, C) is observable. The control objective is guarantee the closed-loop asymptotic stability and reference input tracking.

3. DIRECT AFSM CONTROLLER

The design procedure for a direct AFSM controller is reviewed in the sequel. Suppose that the governing equations of the nonlinear system (Eq. 1) are specified, then, the ideal control signal, u^*, can be specifies based on the sliding mode control theory. The sliding surface is defined as:

$$s = (D + \lambda)^{n-1}\tilde{x}\,, \tag{3}$$

where λ is a user defined positive constant, D is time derivative operator,

$$\tilde{x}(t) = x_d(t) - x(t) \tag{4}$$

and $x_d(t)$ is desired state.

The ideal control signal, u^*, is derived such that $\dot{s} = 0$, i.e.,

$$u^* = g^{-1}\left[-f(X) + x_d^{(n)} + \sum_{i=1}^{n-1} C_i^{n-1} D^{n-i}\lambda^i\tilde{x}\right]\,, \tag{5}$$

where $C_r^n = (n!/(r!\,(n-r)!))$.

The time differentiation of s is:

$$\dot{s} = D\left(D+\lambda\right)^{n-1}\tilde{x} = \left(D^n + \sum_{i=1}^{n-1} C_i^{n-1} D^{n-i}\lambda^i\right)\tilde{x} = x_d^{(n)} - x^{(n)} + \sum_{i=1}^{n-1} C_i^{n-1} D^{n-i}\lambda^i \tilde{x}. \quad (6)$$

Since the nonlinear system (Eq. 1) is assumed unknown, a fuzzy algorithm is used to approximate the ideal control signal. Consider an single-input, single-output fuzzy system with n_r fuzzy IF-THEN rules; for example:

Rule r: IF $\hat{s}$ is A^r THEN $u_{fuz}^r = \theta^r$, $r = 1,\ldots,n_r$,

where A^r is a fuzzy set defined through some Gaussian membership functions, and θ^r is a fuzzy singleton output [12]. Also $\hat{s}$ is obtained as:

$$\hat{s} = \left(D+\lambda\right)^{n-1}\hat{\tilde{x}}, \quad (7)$$

where $\hat{\tilde{x}}$ is the estimated value of $\tilde{x}$.

By singleton fuzzification, product inference scheme and center average deffuzification method, the output of the fuzzy algorithm will be:

$$u_{fuz} = \Theta^T W, \quad (8)$$

where $\Theta = [\theta^1,\ldots,\theta^{n_r}]^T$ is the vector of output fuzzy singletons and $W = [w^1,\ldots,w^{n_r}]^T$ is the vector of firing strengths of rules.

It is well-known that a fuzzy system of the general form (Eq. 8) is a universal approximator [32], i.e., it can uniformly approximate any real continuous function in a compact domain to any degree of accuracy. The estimation error of the fuzzy system is denoted by ψ as:

$$u^* = u_{fuz} + \psi, \quad (9)$$

which is assumed to be bounded, i.e.:

$$|\psi| < \Psi, \quad (10)$$

where Ψ is estimation error bound. An adaptive algorithm is used for tuning the fuzzy singleton outputs as:

$$\dot{\hat{\Theta}} = -\dot{\tilde{\Theta}} = \alpha_1 \hat{s}(t)\, W sgn(g), \quad (11)$$

where α_1 is adaptation rate and $\hat{\Theta}$ is the estimated value of Θ, and:

$$\tilde{\Theta} = \Theta - \hat{\Theta}. \quad (12)$$

Therefore, the fuzzy algorithm output can be rewritten as:

$$\hat{u}_{fuz}\left(\hat{s},\hat{\Theta}\right) = \hat{\Theta}^T W. \quad (13)$$

To compensate the fuzzy estimation error, u_{rb} is designed as:

$$u_{rb} = \hat{\Psi}\mathrm{sgn}\left[\hat{s}(t)\right]\mathrm{sgn}(g), \quad (14)$$

where $\hat{\Psi}$ is error bound estimation which is adjusted adaptively by:

$$\dot{\hat{\Psi}} = -\dot{\tilde{\Psi}} = \alpha_2 \left|\hat{s}(t)\right|, \quad (15)$$

where α_2 is user defined positive constant and:

$$\tilde{\Psi}(t) = \Psi - \hat{\Psi}(t). \tag{16}$$

Therefore, the total control signal is formulated in the form of:

$$u = \hat{u}_{fuz}\left(\hat{s}, \hat{\Theta}\right) + u_{rb}(\hat{s})\,. \tag{17}$$

Substitution of Eq. (17) into Eq. (1) yields:

$$x^{(n)} = f(X) + g\left[\hat{u}_{fuz}\left(\hat{s}, \hat{\Theta}\right) + u_{rb}(\hat{s})\right]\,. \tag{18}$$

Multiplying Eq. (5) with g, added to Eq. (18) and using Eqs. (6) and (7), the error dynamic is obtained as:

$$\left(D^n + \sum_{i=1}^{n-1} C_i^{n-1} D^{n-i} \lambda^i\right) \hat{\tilde{x}} = g\left[u^* - \hat{u}_{fuz} - u_{rb}\right] = \hat{\dot{s}}\,. \tag{19}$$

4. PROPOSED AFSM OBSERVER

An AFSM observer is proposed to estimate the state tracking error vector. The dynamic of the state tracking error can be expressed by using Eqs. (1), (5) and (14), as:

$$\dot{\tilde{X}} = A\tilde{X} + Bh\left(X, \tilde{X}, u\right)\,, \tag{20}$$

where

$$h\left(X, \tilde{X}, u\right) = -gu_{rb} - \sum_{i=1}^{n-1} C_i^{n-1} D^{n-i} \lambda^i \tilde{x} \tag{21}$$

and

$$\tilde{X} = X_d - X\,, \tag{22}$$

where X_d is the desired state vector.

The state tracking error observer is designed in the form of:

$$\begin{gathered} \dot{\hat{\tilde{X}}} = A\hat{\tilde{X}} + Bh(X, \hat{\tilde{X}}, u) + Q(e - \hat{e})\,, \\ e = y_d - y = C\tilde{X}\,, \\ \hat{e} = C\hat{\tilde{X}}\,, \end{gathered} \tag{23}$$

where Q is the observer gain and $\hat{\tilde{X}}$ is the estimated value of the state tracking error vector.

The observation error dynamic is obtained by subtracting Eq. (23) from Eq. (20), i.e.:

$$\dot{\tilde{\tilde{X}}} = (A - QC)\tilde{\tilde{X}} + B\tilde{h}\left(X, \tilde{X}, \hat{\tilde{X}}, u\right)\,, \tag{24}$$

where $\tilde{\tilde{X}} = \tilde{X} - \hat{\tilde{X}}$ and $\tilde{h}(X, \tilde{X}, \hat{\tilde{X}}, u) = h(X, \tilde{X}, u) - h(X, \hat{\tilde{X}}, u)$. The gain Q is determined such that the characteristic polynomial of $A - QC$ be strictly Hurwitz. Figure 1 shows the observer block diagram.

Once again, the AFSM algorithm is used to approximate $h(X, \hat{\tilde{X}}, u)$ which is the unknown term of the observer. Consider the sliding surface, p, such that:

$$p = \gamma\tilde{e}\,, \tag{25}$$

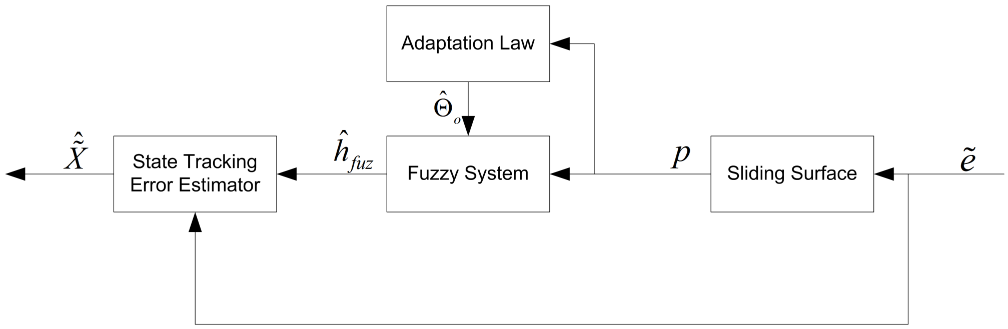


Fig. 1. Observer block diagram.

where

$$\tilde{e} = e - \hat{e} = C\tilde{\tilde{X}} \tag{26}$$

and γ is user defined positive constant. Time differentiation of p yields:

$$\dot{p} = \gamma\dot{\tilde{e}} = \gamma \mathrm{C}\dot{\tilde{\tilde{X}}}\,. \tag{27}$$

Considering the assumption that the entries of C are Hurwitz coefficients, the property $\dot{p} = 0$ guarantees that $\tilde{\tilde{X}}$ goes to zero and h is equal to its ideal value, h^*, which can be derived as:

$$Bh^* = \dot{\tilde{X}} - (A - QC)\hat{\tilde{X}} - QC\tilde{X}\,. \tag{28}$$

Now, the fuzzy system is used to estimate h^*. The fuzzy rules are in the form of:

Rule r : IF p is A_o^r THEN $h_{fuz}^r = \theta_o^r$,

where A_o^r is the fuzzy set and θ_o^r is the fuzzy singleton output. The membership functions and the inference system are the same as those used in the controller fuzzy system.

The output of fuzzy observer is obtained as:

$$h_{fuz} = \Theta_o^T W_o\,, \tag{29}$$

where $\Theta_o = [\theta_o^1, \ldots, \theta_o^r]^T$ is the vector of output fuzzy singletons and $W_o = [w_o^1, \ldots, w_o^r]^T$ is the vector of firing strengths. The vector of fuzzy output singletons is tuned by an adaptive law as:

$$\dot{\hat{\Theta}}_o = -\dot{\tilde{\Theta}}_o = \alpha_3 p(t) W_o sgn(CB)\,, \tag{30}$$

where α_3 is adaptation rate and $\hat{\Theta}_o$ is the estimated value of Θ_o and:

$$\tilde{\Theta}_o = \Theta_o - \hat{\Theta}_o\,. \tag{31}$$

Therefore, the observer fuzzy system output can be rewritten as:

$$\hat{h}_{fuz}\left(p, \hat{\Theta}_o\right) = \hat{\Theta}_o^T W_o\,. \tag{32}$$

Substitution of $\hat{h}$ into Eq. (23) yields:

$$\dot{\hat{\tilde{X}}} = A\hat{\tilde{X}} + B\hat{h}(X, \hat{\tilde{X}}, u) + Q(e - \hat{e})\,. \tag{33}$$

By adding Eq. (28) to Eq. (33), the observer error dynamic is resulted as:

$$\gamma CB\left(h^* - \hat{h}\right) = \gamma C\dot{\tilde{\tilde{X}}} = \dot{p}\,. \tag{34}$$

The block diagram of the proposed algorithm is depicted in Fig. (2).

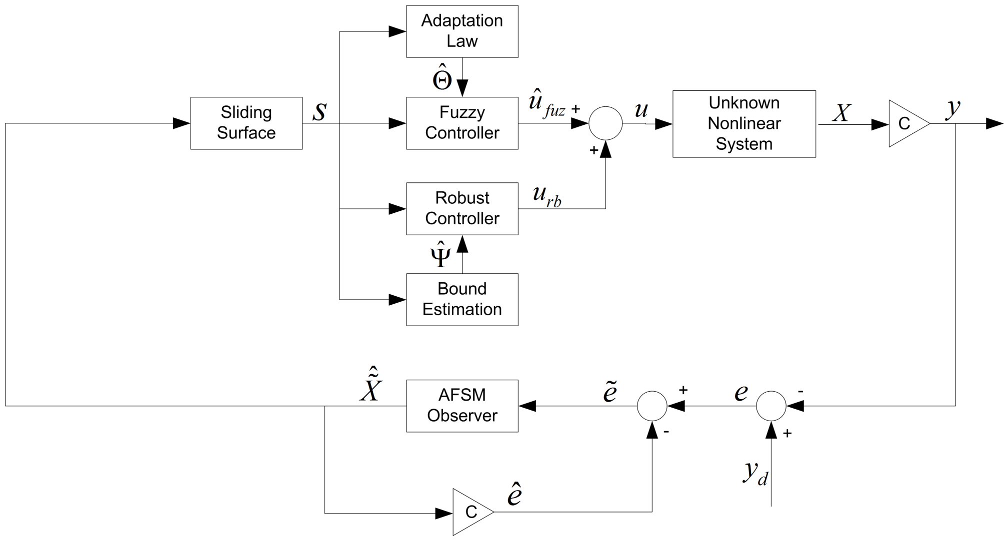


Fig. 2. Block diagram of the proposed algorithm.

5. STABILITY ANALYSIS

Theorem 1: Consider the unknown nonlinear system described by Eq. (1). If the input signal is applied as Eq. (17), where the fuzzy system (Eq. 8) is used to estimate u^* and the vector of fuzzy rule outputs is adaptively tuned according to Eq. (11) and the robust controller is designed as Eq. (14) with bound estimation algorithm given by Eq. (15), and the observer is designed according to Eq. (23), in which fuzzy algorithm given by Eq. (29) is used for estimation of h^* and the parameter vector of the fuzzy system output is adjusted by Eq. (30), then the stability of algorithm is guaranteed.

Proof:

The positive definite candidate Lyapunov function is defined as:

$$V(\hat{s},\tilde{\Theta},\tilde{\Psi},p,\tilde{\Theta}_o) = \frac{1}{2}\hat{s}^2 + \frac{|g|}{2\alpha_1}\tilde{\Theta}^T\tilde{\Theta} + \frac{|g|}{2\alpha_2}\tilde{\Psi}^2 + \frac{1}{2}p^2 + \frac{\gamma|CB|}{2\alpha_3}\tilde{\Theta}_o^T\tilde{\Theta}_o\,. \tag{35}$$

By differentiating Eq. (35) and by using Eqs. (11–16), (19), (30), (31) and (34), it is obtained that:

$$\begin{aligned}
\dot{V} &= \hat{s}\dot{\hat{s}} + \tfrac{|g|}{\alpha_1}\tilde{\Theta}^T\dot{\tilde{\Theta}} + \tfrac{|g|}{\alpha_2}\tilde{\Psi}\dot{\tilde{\Psi}} + p\dot{p} + \tfrac{\gamma|CB|}{\alpha_3}\tilde{\Theta}_o^T\dot{\tilde{\Theta}}_o \\
&= \hat{s}g(\tilde{\Theta}^T W + \psi - u_{rb}) + \tfrac{|g|}{\alpha_1}\tilde{\Theta}^T\dot{\tilde{\Theta}} + \tfrac{|g|}{\alpha_2}\tilde{\Psi}\dot{\tilde{\Psi}} + \gamma CBp(\tilde{\Theta}_o^T W_o) + \tfrac{\gamma|CB|}{\alpha_3}\tilde{\Theta}_o^T\dot{\tilde{\Theta}}_o \\
&= |g|\tilde{\Theta}^T(\hat{s}W sgn(g) + \tfrac{\dot{\tilde{\Theta}}}{\alpha_1}) + \hat{s}g(\psi - u_{rb}) + \tfrac{|g|}{\alpha_2}\tilde{\Psi}\dot{\tilde{\Psi}} + \gamma|CB|\tilde{\Theta}_o^T(pW_o sgn(CB) - \tfrac{\dot{\tilde{\Theta}}_o}{\alpha_3}) \\
&= \hat{s}g\psi - |\hat{s}||g|\Psi \le |\hat{s}||g||\psi| - |\hat{s}||g|\Psi = -|\hat{s}||g|(\Psi - \psi) \le 0\,.
\end{aligned} \tag{36}$$

Since $\dot{V}(\hat{s},\tilde{\Theta},\tilde{\Psi},p,\tilde{\Theta}_o)$ is negative semi-definite, the stability is proved in the Lyapunov sense. Now, define:

$$\Gamma(t) = |\hat{s}|\,|g|\,(|\psi| - \Psi) \le -\dot{V} \tag{37}$$

and integrate $\Gamma(t)$ with respect to time, then it can be shown that:

$$\int_0^t \Gamma(\tau)d\tau \leq V\left(\hat{s}(0),\tilde{\Theta},\tilde{\Psi},p(0),\tilde{\Theta}_o\right) - V\left(\hat{s}(t),\tilde{\Theta},\tilde{\Psi},p(t),\tilde{\Theta}_o\right) . \tag{38}$$

Since $V\left(\hat{s}(0),\tilde{\Theta},\tilde{\Psi},p(0),\tilde{\Theta}_o\right)$ is bounded and $V\left(\hat{s}(t),\tilde{\Theta},\tilde{\Psi},p(t),\tilde{\Theta}_o\right)$ is nonincreasing and bounded, it implies that [12]:

$$\lim_{x\to\infty}\int_0^t \Gamma(\tau)d\tau \leq \infty . \tag{39}$$

Furthermore, $\dot{\Gamma}$ is bounded, so by Barbalat's Lemma [33], it can be shown that $\lim_{x\to\infty}\Gamma(t) = 0$, i.e., $\lim_{x\to\infty} s(t) \to 0$. Therefore, the proposed control system is asymptotically stable.

6. CHATTER ELIMINATION

The presence of a discontinuous term in the control law leads to the well known chattering phenomenon. In order to overcome the undesirable chattering effects, a thin boundary layer, ϕ, in the neighborhood of the switching surface can be adopted [34] as:

$$S_\phi = \left\{ \hat{\hat{X}} \in R^n \middle| \left| \hat{s}(\hat{\hat{X}}) \leq \phi \right| \right\} , \tag{40}$$

where ϕ is a strictly positive constant that represents the boundary thickness.

The boundary layer is achieved by replacing the sign function by a continuous interpolation inside S_ϕ. For this purpose the saturation function is used as [35]:

$$sat\left(\frac{\hat{s}}{\phi}\right) = \begin{cases} sgn(\hat{s}), & \text{if } |\hat{s}/\phi| \geq 1 \\ \frac{\hat{s}}{\phi}, & \text{if } |\hat{s}/\phi| < 1 \end{cases} . \tag{41}$$

A measure of distance of the current state to the boundary layer, $\hat{s}_\phi$, can be obtained as:

$$\hat{s}_\phi = \hat{s} - \phi\, sat(\hat{s}/\phi) . \tag{42}$$

In order to establish the attractiveness and invariant properties of the defined boundary layer, $\hat{s}$ is replaced by $\hat{s}_\phi$ in the candidate Lyapunov function (Eq. 35) and adaptation laws (Eqs. 11 and 15). Also $sat(\hat{s}/\phi)$ is used instead of $sgn(\hat{s})$ in the equation of robust controller (Eq. 14). By the procedure as in Eqs. (36–39) it is proved that $\hat{s}_\phi$ converges to zero asymptotically.

7. NUMERICAL EXAMPLE

The performance of the proposed output feedback direct AFSM methodology is examined by applying it to the model of an atomic force microscopy (AFM), using MATLAB/Simulink. AFM is widely used for nano-scale imaging and surface manipulation. In AFM, a micro-cantilever is brought to a distance close enough to the sample to allow for surface interactions between the tip of the micro-cantilever and the sample [34,35]. One approach to measuring of the surface forces is by monitoring the deflection of the micro-cantilever through a photodiode (contact mode). Another approach is performed by vibrating the micro-cantilever close to its resonance frequency and monitoring the changes in its effective spring contact (tapping mode) [12].

The dynamics of a micro-cantilever-tip-sample interaction in tapping mode is studied in [36]. The micro-cantilever-tip-sample interaction is modeled by the Lennard-Jones interaction potential. The equations of the system can be written in non-dimensional form as [12,37]:

$$\begin{aligned} \dot{x}_1 &= x_2\,, \\ \dot{x}_2 &= -x_1 - \delta x_2 + \beta cos(\Omega t) + f_{IL}(x_1) + u(t)\,, \\ y &= x_1 + x_2\,, \end{aligned} \tag{43}$$

where x_1 and x_2 are the non-dimensional position and velocity of the micro-cantilever tip, t is non-dimensional time, β and Ω are the amplitude and the frequency of the forcing term, δ is the damping factor, $u(t)$ is control signal, y is the system output, and:

$$f_{IL}(x_1) = \frac{\sigma^6 d}{30(\nu + x_1)^8} - \frac{d}{(\nu + x_1)^2} \tag{44}$$

is the attraction/repulsion interaction force derived from Lennard-Jones interaction potential [35]. AFM equations can be equivalently described as:

$$\begin{aligned} \dot{X} &= AX + B(-x_1 - \delta x_2 + \gamma cos(\Omega t) + f_{IL}(x_1) + u(t))\,, \\ y &= CX\,, \end{aligned} \tag{45}$$

where $X = [x_1 \;\; x_2]^T$, $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, $B = [0 \;\; 1]^T$ and $C = [1 \;\; 1]$, i.e., $y = x_1 + x_2$.

If the constants are chosen as $\delta = 0.04$, $\beta = 2$, $\sigma = 0.3$, $\nu = 0.8$, $d = 4/27$ and $\Omega = 1$ with the initial conditions $x_1(0) = -0.5$ and $x_2(0) = 0$, the AFM will have the chaotic behavior as shown in Fig. 3. The control objective is $y_d = \sin t$.

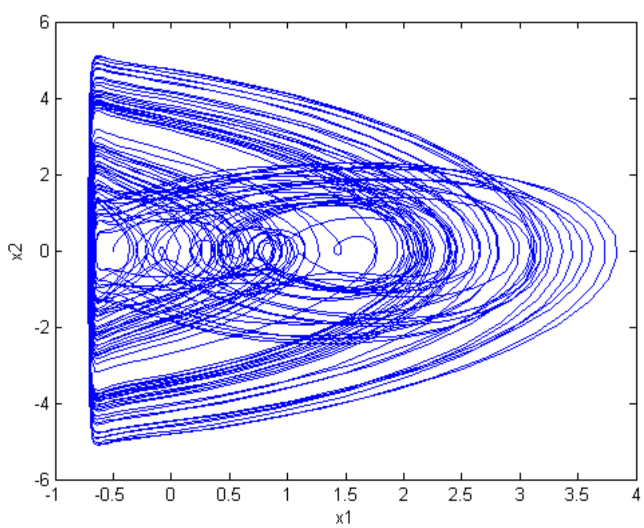


Fig. 3. Chaotic behavior of the AFM.

The AFSM controller is specified as follows: the sliding surface is defined as:

$$s = \dot{\tilde{x}} + \lambda \tilde{x}\,, \tag{46}$$

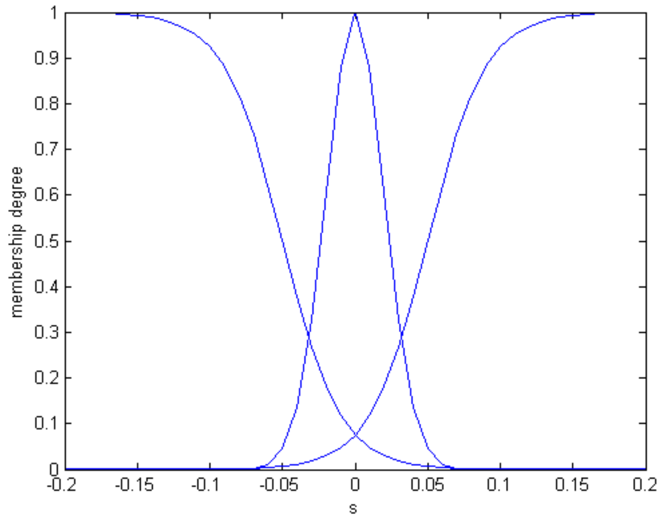


Fig. 4. Membership functions of fuzzy system.

with $\lambda = 20$. The membership functions are chosen as shown in Fig. 4. The initial vector of fuzzy singleton output and initial value of uncertainty bound are arbitrary selected as $\hat{\Theta}(0) = [-1,\ 0,\ 1^T$ and $\widehat{\Psi}(0) = 0.1$, respectively. Also, the adaptation rates are set to $\alpha_1 = 10$ and $\alpha_2 = 5$. Then, the AFSM observer sliding surface is defined as:

$$p = \gamma \tilde{e}\,, \tag{47}$$

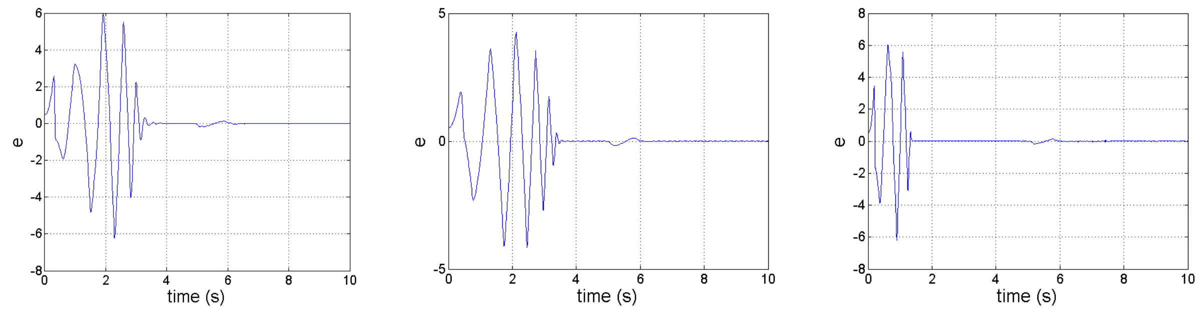
where $\gamma = 5$. The membership functions are the same as those of the controller and the initial vector of fuzzy singleton output is arbitrarily selected as $\hat{\Theta}_o(0) = [0,\ 0,0]^T$. The adaptation rate is set to $\alpha_3 = 40$ and the vector of observer gain is $Q = [-3\ ,\ 10]^T$.

Here, the simulation results are shown for three cases: (1) the non-sliding fuzzy, where, the robust switching part of the controller is omitted, (2) the non-fuzzy switching control, and (3) the complete direct AFSOC method. The comparison for the tracking error is shown in Fig. 5, and for the corresponding control inputs is Fig. 6.

It should be mentioned that the final tuned membership function singletons for the controller is $\hat{\Theta}(10) = [27.94,\ 0.40,\ -43.51]^T$ and for the observer is $\hat{\Theta}_o(10) = [79.65,\ 0.37,\ -80.52]^T$. Also the final value of uncertainty bound is obtained as $\hat{\Psi}(10) = 39.5$.

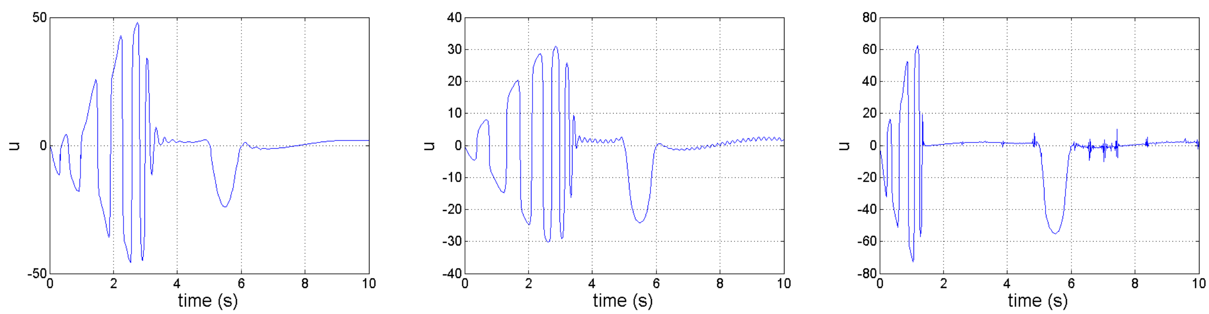
Figures 5 and 6 show that the combination of adaptive fuzzy and adaptive switching strategies yields to a more effective control action. It is interesting to note the local divergence occurring at about $t_c = 5$ sec, shown in Figs. 5 and 6. The reason refers to the chaotic behavior of the AFM system, where, for specific occurrence of values of states x_1 and x_2 at about 5, a highly chaotic behavior is appeared. This divergence is successfully controlled by the corrective action of the proposed control strategy. Also, the output response and observation error of the AFSOC are shown in Fig. 7.

As a second try to show the effectiveness of the proposed method, comparison is made with a conventional PID controller, tuned by trials and errors using the Ziegler-Nichols method. The PID controller and the proposed AFSM algorithm are applied to the AFM mathematical model. The tracking performance is compared in Fig. 8, depicting a superior performance of the AFSOC method except during the first two seconds. The reason is that the AFSM method is model-free and needs a few seconds to for control parameters adaptation, while the PID method was designed off-line, enjoying the exact mathematical model of the plant.



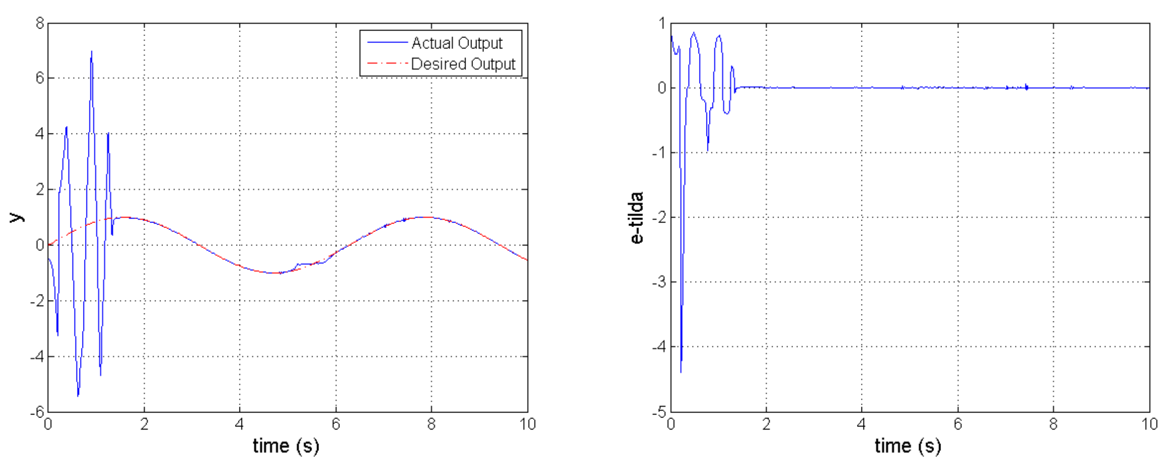
(a) (b) (c)

Fig. 5. Tracking error: (a) non-sliding fuzzy, (b) non-fuzzy switching, (c) AFSOC.



(a) (b) (c)

Fig. 6. Control input: (a) non-sliding fuzzy, (b) non-fuzzy switching, (c) AFSOC.



(a) (b)

Fig. 7. AFSOC: (a) Output response, (b) Observation error.

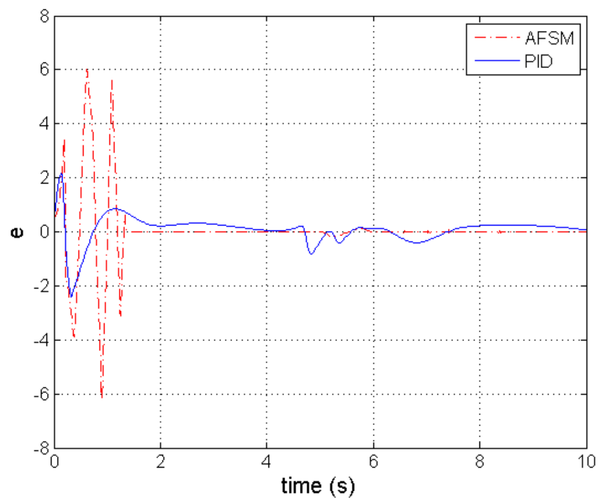


Fig. 8. Tracking error: AFSOC (model-free) vs. PID (model-based).

8. CONCLUSIONS

An AFSM observer was proposed for the direct AFSM control methodology. The proposed method is used for approximation of nonlinear part of the observer, where the output tracking error was used to define the required sliding surface, assuming that the output matrix gain has Hurwitz coefficients.

The proposed observer was used in a closed-loop control system which included an adaptively tuned fuzzy controller together with a switching sliding controller with an adaptable uncertainty bound. The application of the method to a benchmark chaotic system shows the exceptional performance of the method. Further numerical studies showed the individual contributions of the fuzzy and switching control parts of the algorithm, which was enhanced by concurrent application of those methods, as proposed by the AFSOC method.

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