APPLICATION OF THE TAGUCHI QUALITY METHOD TO DESIGN NAVIGATION CONSTANTS THAT ARE ROBUST TO TARGET MANEUVERS

Chien-Chun Kung[1], Feng-Lung Chiang[2], Ciann-Dong Yang[2]

[1] *Department of Mechatronic, Energy and Aerospace Engineering, National Defense University, ROC.*

[2] *Department of Aeronautics and astronautics, National Cheng Kung University, ROC.*

E-mail: [1]cckung@ndu.edu.tw; [2]flyjason12001@yahoo.com.tw

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ABSTRACT

The object of this paper is to present a quality control approach to design the navigation constants for missile guidance law that are robust for target's maneuverability and missile's performance. Target's maneuvers and missile's time constants are considered to be noise factors and Taguchi Quality Method (TQM) is used to conduct matrix experiments and determine the robust navigation constants. From the simulation results for three kinds of guidance laws, we find that robust navigation constants have a greater ability to enhance the interception of targets that are performing uncertain maneuvers with less interception time and smaller miss distances in the hitting phase.

Keywords: taguchi quality method; matrix experiments; robust navigation constant; uncertain maneuver.

APPLICATION DE LA MÉTHODE TAGUCHI POUR LA CONCEPTION DE CONSTANTES DE NAVIGATION ROBUSTES POUR DES MANŒUVRES DE POINTAGE

RÉSUMÉ

L'objectif de cet article est de présenter une approche de contrôle de la qualité pour la conception de constantes de navigation robustes, selon la loi d'orientation de missile, pour la maniabilité de la cible et la performance du missile. La maniabilité de la cible et les constantes de temps du missile sont considérées comme des facteurs de bruit, la méthode Taguchi de contrôle de la qualité (TQM) est utilisée pour faire des expériences sur la matrice, et déterminer des constantes de navigation robustes. Selon les résultats des simulations pour trois types de lois d'orientation, nous remarquons que les constantes de navigation robustes ont une plus grande capacité de renforcer l'interception de cibles en train de faire des manœuvres incertaines, avec moins de temps d'interception et moins de tirs qui ratent leur cible dans la phase de frappe.

Mots-clés : méthode Taguchi de contrôle de la qualité; expériences sur la matrice; constante de navigation robuste; manœuvre incertaine

NOMENCLATURE

a_p , a_{pc}	missile pitch-axis acceleration and its command signal.
a_y , a_{yc}	missile yaw-axis acceleration and its command signal.
a_{pT} , a_{yT}	target pitch-axis and yaw-axis acceleration, respectively.
D_m , D_T	missile and target drags, respectively.
g	gravity.
K_p , K_y	missile pitch-axis and yaw-axis effective navigation constants, respectively.
k_1 , k_2	missile drag coefficients.
m_m	missile mass.
S_T	step size of Runge–Kutta method.
T	tracking time.
T_m	missile thrust.
V_m	missile velocity.
β_1 , β_2	constants of Jink and Split-S maneuvers, respectively.
γ_m , Ψ_m	missile flight path angle and azimuth angle, respectively.
γ_T , Ψ_T	target flight path angle and azimuth angle, respectively.
λ	missile flight pre-angle, that is, the angle between the missile velocity vector and line-of-Sight.
τ	missile time constant.
θ	line-of-sight azimuth angle.
ϕ	line-of-sight altitude angle.

1. INTRODUCTION

In realistic air combat, missiles exist to destroy enemy aircraft, and then the aircraft employs various maneuvers to avoid being hit itself. This is a pursuit-evasion game between human intelligence and missile guidance. Therefore, how to design an excellent robust missile guidance law is a popular subject of discussion. Many studies have been published in [1–3], however, most of them have attempted to simplify fighter maneuvers to get analytical solutions for the missile guidance law. For this reason, there are still no practical assessments for verifying the robustness of a given guidance law to a target's maneuvers. On the other hand, navigation constants affect the probability of a kill and the performance of a guidance law. Previous studies [4–7] had directly set the values but did not contain a clear description of the reasons for them. Non-optimum navigation constants might reduce the performance of an interceptor missile. For the above reasons, this paper proposes a new viewpoint in that a target's maneuvers, such as Jink, Split-S and long acceleration, in addition to performance, are considered to be noise factors, and an experimental solution for navigation constant selection by exploiting a robust design method used in quality control.

The Taguchi Quality Method (TQM) developed by Dr. Taguchi is a technique by which to optimize the process of quality design and perform a matrix experiment to design the robust quality into the product before it is built [8–10]. TQM has been used widely to search robust parameters in literature. Studies [11,12] proposed the applications of a genetic-Taguchi algorithm to find the best combination of parameters for different flight-control problems in F-16 fighters and helicopters Chen et al., [13] applied TQM to design and implement a robust, fuzzy controlled photovoltaic (PV) power inverter to achieve a fast transient response, small steady-state errors and system robustness. An application of TQM to optimize the design parameters of a light-emitting diode (LED) backlight unit for wide color gamut liquid crystal displays (LCDs) was studied by Lin et al., [14]. The Taguchi design is an efficient and useful approach for optimization of sandwich immunoassay microarrays for detecting breast cancer biomarkers [15]. The above mentioned papers have proved that TQM is widely used for optimization of products and complex production processes in many different industries [16].

In recent years, the study of the robust guidance law has attracted considerable attention to improve missile performance and to overcome environmental disturbances. In [17–19], robust guidance laws based on H_∞ control were considered. The most difficult and challenging task involved in applying the nonlinear H_∞ control theory was the solution of the associated Hamilton-Jacobi partial differential inequality. Many nonlinear control schemes have been proposed for a robust guidance law, such as the sliding-mode control [20–22] and fuzzy logic control [23,24]. Although the sliding mode control can meet robust stability, it cannot guarantee the satisfaction of pre-specified bounds on the system performance under unknown disturbances. Fuzzy logic control (FLC) is a good alternative to overcome the difficulties in the requirement of exact mathematical models for plants with unexpected complex dynamics and external disturbances. Therefore, the use of FLC for robust guidance is usually motivated by the need to deal with nonlinear flight control and nonlinear aerodynamic problems.

Gurfil [25] proposed a robust guidance law that applied interval transfer functions to model the structured uncertainties in missile dynamics and defined a navigation constant to design a transfer function. This method showed that the navigation constant is one of the solutions that satisfy the boundary conditions. However, it does not possess an optimal mode. Performance of a missile that is based on varied guidance laws is discussed in [26,27]. However, how to determine a robust navigation constant of a guidance law by use of an effective and systematic approach to improve missile performance has been somewhat neglected in the study area. In this paper, a Taguchi Quality Method is presented for some missile guidance laws to obtain the navigation constants that are robust for a target's maneuvers. The object of a robust design is to improve the quality of a guidance law by minimizing the effect of the target evasion tactic variation. Four main processes are considered in this paper for robust parameter design. They are: (1) a new viewpoint of noise factors; target's maneuvers, such as Jink, Split-S and long acceleration are considered to be noise factors; (2) matrix experiments; a matrix experiment consists of a set of experiments wherein the combination of parameters set are designed to meet the orthogonal property for each parameter, and orthogonal arrays are used to determine the least number of experiments; (3) signal-to-noise ratio (S/N); is a calculated value used for studying the combination effects and quality of the design parameters; and (4) effect analysis of each parameter; after completing a matrix experiment, the data from all experiments are analyzed to determine the effects and the robust settings of the parameters. On the other hand, to test the effects of the robust navigation constants against a target's varying maneuvers, three kinds of guidance laws are applied. These are proportional-pursuit coupled guidance (PPCG), proportional navigation guidance (PNG) and pursuit guidance (PG). PPCG is formed by collecting proportional navigation guidance in the Z-R plane and pursuit guidance in the X-Y plane. The performance and robustness of these guidance laws in various interception situations with robust navigation constants will be considered in detail.

2. MISSILE DYNAMICS

Consider now the free-flight dynamic model of a missile. Assume for simplicity that the missile can be modeled as point mass. Here it is assumed that thrust is constant during engine burn and that the drag coefficient, air density, and missile mass are also constant. With reference to Figure 1, from the missile's balanced forces, we may write the following approximate expression for the missile motion [28]:

$$\dot{V}_m = \frac{1}{m_m}(T_m - D_m) - g\sin\gamma_m \tag{1}$$

$$\dot{g}_m = \frac{(a_p - g\cos g_m)}{V_m} \quad (2)$$

$$\dot{Y}_m = \frac{a_y}{V_m \cos g_m} \quad (3)$$

$$\dot{x}_m = V_m \cos g_m \cos Y_m \quad (4)$$

$$\dot{y}_m = V_m \cos g_m \sin Y_m \quad (5)$$

$$\dot{z}_m = V_m \sin g_m \quad (6)$$

The acceleration commands due to time delay are given by:

$$\dot{a}_p = \frac{(a_{pc} - a_p)}{t} \quad (7)$$

$$\dot{a}_y = \frac{(a_{yc} - a_y)}{t} \quad (8)$$

The missile drag D_m is given by the expression:

$$D_m = k_1 v_m^2 + k_2 \frac{a_p^2 + a_y^2}{v_m^2} \quad (9)$$

The above equations can be used to provide velocity (V), flight path angle (γ) and azimuth angle (Ψ) information to the tracking system location of the missile, Furthermore, we can obtain the relative range between the missile and the target by transforming the 3-D Cartesian coordinates into spherical coordinates. The missile movement is controlled by two actuators (see Eq.(7) and Eq.(8)), which receive acceleration commands a_{pc} and a_{yc} from the guidance system and cause control surfaces to move so as to attain these commanded accelerations. The guidance design of acceleration commands will be discussed in the next section.

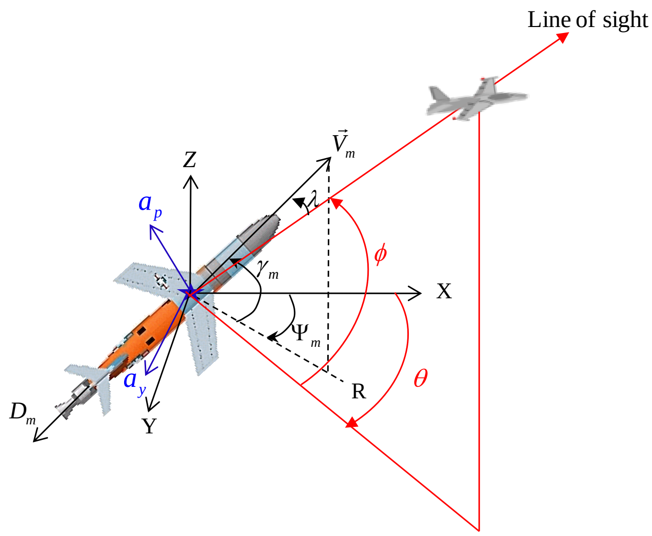

Fig. 1. Three-dimensional geometry for missile and target

3. THREE MISSILE GUIDANCE LAWS WITH MISSILE DYNAMICS

3.1 Three-dimensional Proportional Navigation Guidance (PNG) with Missile Dynamics

The three-dimensional proportional navigation guidance law can be constructed by two two-dimensional guidance laws that are employed in the horizontal (X-Y) plane and vertical (Z-R) plane, in which the O-R axis is the projection of the missile velocity vector in the horizontal plane. Referring to Figure 1, assume the variables q and ϕ are independent. The proportional navigation guidance law states that the rate of change of a missile's flight path angle is directly proportional to the rate of change of the line-of-sight (LOS) angle from the missile to the target [29]. Compared with the three-dimensional missile-body coordinate frame and the earth-centered inertial reference frame in Figure 1, two guidance equations can be written as follows:

$$\dot{\gamma}_m = K_p \dot{\phi} \tag{10}$$

$$\dot{\Psi}_m = K_y \dot{\theta} \tag{11}$$

When the pitch-axis acceleration enables γ_m to track ϕ , Eq. (10) can combine with Eq. (2) as follows:

$$\dot{\gamma}_m = \frac{\left(a_p - g\cos\gamma_m\right)}{V_m} = K_p\dot{\phi} \tag{12}$$

Hence, the pitch-axis acceleration command a_p with missile dynamics for PNG can be obtained by:

$$a_{p\_PNG} = k_p \dot{\phi} \cdot V_m + g \cdot \cos\gamma_m \tag{13}$$

Similarly, taking Eq. (11) and combining it with Eq. (3) the yaw-axis analytic solution can be derived as Eq. (14), which enables Ψ_m to track q :

$$a_{y\_PNG} = K_y \dot{q} V_m \cos\gamma_m \tag{14}$$

3.2 Three-dimensional Pursuit Guidance (PG) with Missile Dynamics

The three-dimensional pursuit guidance can be constructed by two two-dimensional ones that are employed in the X-Y and the Z-R planes. Similar to the previous subsection, we assume the variables q and ϕ are independent. In Figure 1, the interception rule of pursuit guidance is that the tail-chase intercept occurs at $\lambda = 0$ [29]. It can be considered that the missile flight path angle γ_m and azimuth angle Ψ_m has to track angle ϕ and angle q exactly. The problem can be solved by feedback from the linearization control method [27,30]. Should we wish the azimuth angle Ψ_m to track angle q , the equation of the control system is as follows:

$$\dot{\Psi}_m + K_y \Psi_m = K_y \theta \tag{15}$$

Assuming the yaw-axis acceleration is as in Eq. (16) and take it into Eq. (3), we can then get the resulting Eq. (15). Therefore, the acceleration command of PG along the yaw-axis in Eq. (15) can enable the flight path angle Ψ_m to track angle q :

$$a_{y\_PG} = K_y V_m \cos\gamma_m (\theta - \Psi_m) \tag{16}$$

Following the same procedure, the acceleration command of PG along the pitch-axis with missile dynamics can be obtained as in Eq. (17):

$$a_{p\_PG} = K_p V_m (\phi - \gamma_m) + g \cos \gamma_m \tag{17}$$

3.3 Three-dimensional Proportional-pursuit Coupled Guidance (PPCG) with Missile Dynamics

It is well known that PNG is effective in intercepting a maneuvering aircraft, but it often fails to achieve superior robustness. On the other hand, PG is the first navigation law and is easy to implement, however it is considered to be a bit impractical as it is used for interception with the requirement of ending an attack in a tail chase. In other words, the interceptor with pursuit guidance needs high acceleration. In this paper, we try to combine PNG and PG to create a new guidance, PPCG, with a balanced performance, including robustness, tracking time and acceleration. PPCG joins PNG along the pitch-axis and PG along the yaw-axis. Therefore, the pitch and yaw acceleration commands of PPCG are expressed as Eq. (18) and Eq. (19) respectively:

$$a_{p\_PPCG} = K_p \dot{\phi} \cdot V_m + g \cdot \cos \gamma_m \tag{18}$$

$$a_{y\_PPCG} = K_y V_m \cos \gamma_m (\theta - \Psi_m) \tag{19}$$

4. THE PROCEDURES TO DESIGN ROBUST NAVIGATION CONSTANTS USING TQM

The purpose of this study is to seek versatile navigation constants which are capable of dealing with highly maneuverable aircraft. Therefore, this paper proposes a new viewpoint that an experimental solution for navigation constant selection is by exploiting a robust design method used in quality control. In this paper, we consider the Taguchi Quality Method (TQM) to determine the robust navigation constants that cause a serious threat to pilots.

The central idea of TQM is to estimate the optimum setting of the control factors such that the loss in quality is minimized. The tools used in robust design include the signal-to-noise (S/N) ratio to measure quality and matrix experiments using orthogonal arrays to study the effects of minimizing performance variations, while simultaneously improving the quality of a system with many design parameters. Signal-to-noise ratio (S/N) is one of the popular approaches and has a 'larger-the-better' characteristic in that a larger S/N ratio means better robustness.

The underlying concept of TQM using orthogonal arrays is imbedded in the following construction process of qualified navigation constants. We refer readers to [16] for further details about the general theory of TQM. The strategy to find robust navigation constants consists of the following steps:

(1) Design the noise factors to consider the maneuverability of a target and the performance of a missile

In TQM, characteristic parameters include noise factors and control factors. A noise factor is a factor that cannot be controlled and makes for a worse result, due to obvious practical reasons. In air combat, a missile cannot predict what maneuvers will be applied by a pilot to dodge an attack. A pilot's opinion regarding tactics used cannot be controlled. This paper proposes a new viewpoint that a pilot's intelligence is considered to be a noise factor and maneuverability can be regarded as the intelligence of a pilot. For the above reason, this study adopts three kinds of maneuvers, Jink, Split-S and long acceleration, which are of common usage in real air combat, to be noise factors to test the robustness of guidance laws for an intercept.

Another noise factor is the time constant (τ) of a control surface actuator which can be regarded as a missile's performance. Time constant (τ) has a critical effect on missile performance, and its meaning is actuator dynamics. If the guidance laws have a smaller time constant, a missile displays better sensitivity. We therefore referred to previous studies to decide the missile's time constant. Time constant (0.3s) was adopted by Fumiaki [5]. Blakelock and Palumboet et al. [31,32] to describe bank to turn missile (BTT) and skid to turn missile (STT) in which the time constants are 0.3s and 0.4s, respectively. Yang [33] used time constant 0.1s~0.5s for simulation. In summary, most of the missile time constants are set between 0.1s and 0.5s, therefore, in this paper, we choose 0.2 to be the median. The levels of time constant thus are decided as $\tau = 0.1, 0.2, 0.3$.

This paper proposes three kinds of maneuvers as noise factors, and gives three different levels of time constant for each maneuver. This represents nine types of noise cases that will be used to search for the most robust navigation constants. From the above viewpoint, target's intelligence and missile's performance were considered to determine the robust navigation constants.

(2) Identify levels of control factors

A control factor is a factor that will be selected to meet our requirement from a certain level. In this present study, navigation constants (K_p, K_y) in Eqs. (13), (14), (16) and (17) are the control factors. Levels of control factors may be determined in accordance with some general laws and experiences, such as equal increments between levels or equal sizes of levels. It is a rule of thumb that at the start of the experiment we select 5 levels and take those levels sufficiently far apart so that a wide region can be covered by them in order to fully reflect the nonlinear relations between parameters and quality characteristics. Therefore, this paper gives 5 levels to navigation constants (K_p, K_y), which are often between 1 and 10, and are shown in table 1.

Control factors will be assigned to an experiment and more than two levels will be required. Once significant control factors are identified, multiple levels can be used for estimating nonlinear responses [8]. Curvature effects are the effects of multiple levels for nonlinearity that can inhibit the occurrence of a nonlinear response. In other words, curvature effects are based on multiple levels. If we took only two levels, curvature effects would be missed and an accurate solution may be lost, whereas such effects can be identified by selecting five levels.

(3) Define quality index

In this paper, quality index includes missile tracking time and miss distance (M.D.), where miss distance is defined as the smallest distance between the missile and the target, so a smaller miss distance indicates a higher accuracy. In air combat, the two indexes are key points for a missile to hit a target. The scoring function for the two quality indexes is as follows:

$$J = 50\% \times f(tracking\ time) + 50\% \times g(M.D.) \tag{20}$$

where $f(tracking\ time)$ and $g(M.D.)$ are quantifiable functions. The scoring table is shown in Table 2.

Table 1. Control Factor Levels

	Level 1	Level 2	Level 3	Level 4	Level 5
Navigation constant K_p	2	4	6	8	10
Navigation constant K_y	1	3	5	7	9

Table 2. Tracking time and miss distance scoring table

Tracking time (second)	Scores ($f(tracking\ time)$)	Miss distance (meter)	Scores ($g(M.D.)$)
Less than 31.0	100	0.010~0.059	100
31.0~35.9	95	0.060~0.099	95
36.0~40.9	90	0.10~0.59	90
41.0~45.9	85	0.60~1.09	85
46.0~50.9	80	1.10~1.59	80
51.0~55.9	75	1.60~2.09	75
56.0~60.9	70	2.10~2.59	70
61.0~65.9	65	2.60~3.09	65
66.0~70.9	60	3.10~3.59	60
71.0~75.9	55	3.60~4.09	55
76.0~80.9	50	4.10~4.59	50
81.0~85.9	45	4.60~5.09	45
86.0~90.9	40	5.10~5.59	40
91.0~95.9	35	5.60~6.09	35
96.0~100.9	30	6.10~6.59	30
101.0~105.9	25	6.60~7.09	25
106.0~110.9	20	7.10~7.59	20
111.0~115.9	15	7.60~8.09	15
116.0~120.9	10	8.10~8.59	10
121.0~125.9	5	8.60~9.09	5
More than 126.0	1	More than 9.10	1

(4) Definition of S/N ratio

The S/N ratio was created and used by Dr. Taguchi to get the robust parameters which cause the least variability to noise factors and improves the quality of the system. The S/N ratio, which condenses the multiple data points within a trial, depends on the type of characteristic being evaluated, and the types include smaller is better, nominal is the best and bigger is better. Consistent with its application in this paper, the value of the S/N ratio is intended to be large, hence the value of quality loss σ^2 should be small. Therefore, the quality loss σ^2 in our present case is defined as [10]:

$$\sigma^2 = \frac{1}{n}\left(\frac{1}{J_1^2} + \frac{1}{J_2^2} + \frac{1}{J_3^2} \cdots + \frac{1}{J_n^2}\right) \tag{21}$$

where the n stands for n different conditions of noise factors, and J_i is the value of the scoring function J in Eq. (20) evaluated at the i_{th} noise factor. S/N ratio η can be obtained by Eq. (22):

$$\eta = -10 \cdot \log \sigma^2 \tag{22}$$

(5) Conduct matrix experiments

The matrix experiment shown in Table 3 exploits the standard orthogonal array L_{25} of Taguchi [34]. It consists of 25 individual experiments corresponding to the 25 rows. The entries of the second block in the table represent the level values of the parameters. Accordingly, experiment 1 is to be conducted with each parameter being at its first level. In reference to Table 3, there are three types of evasive maneuvers used by a target [4,7]: Jink, Split-S and long acceleration , and we give three different levels of time constant ($\tau = 0.1, 0.2, 0.3$) for each maneuver. The pitch and yaw acceleration commands of the three evasive maneuvers are expressed in Eqs. (23), (24) and (25), respectively. This article uses these equations to create target movements that are similar to real fighter maneuvers. Descriptions of the three maneuvers are as follows:

Jink: a jinking movement is to move swiftly or make a quick turn, rather than by moving in a straight line.

$$\begin{cases} a_{pT} = g\cos(\gamma_T) \\ a_{yT} = \beta_1 \cos(T \cdot S_T) \end{cases} \tag{23}$$

where β_1 will affect the turning angle.

Split-S: in this the defender rolls in an inverted manner and dives away vertically, pulling out in a direction opposite to that of his opponent.

$$\begin{cases} a_{pT} = \beta_2 \cos(T \cdot S_T) + \beta_2 \sin(T \cdot S_T) \\ a_{yT} = \beta_2 \cos(T \cdot S_T) + \beta_2 \sin(T \cdot S_T) \end{cases} \tag{24}$$

where β_2 will affect the turning angle of dive.

Long acceleration: target flights and accelerates to a direction.

$$\begin{cases} a_{pT} = g\cos(\gamma_T) \\ a_{yT} = 0 \end{cases} \tag{25}$$

All the 25 experiments have been conducted in an air combat simulation system which implements the point-mass motion models of the missile and the aircraft. The simulation environment has been kept the same for all of the 25 experiments, except for the numerical values of the two navigation constants. Each experiment corresponds to a specific set of navigation constants. After the completion of each experiment, two quality characteristics are recorded, namely, trace time and miss distance. Since we are interested in the variation of the quality characteristics due to a target's various maneuvers and performances, all of the 25 experiments will be repeated for different target maneuvers and performances. Referring to Table 3, for example, for experiment 1 with a Jink maneuver and the two quality characteristics evaluated from Table 2, with the air combat simulation response, are Time=70 and M. D.=90. The quality J in this case is then found from the quantitative measure defined in Eq. (20), and the result is J_1=80. For experiment 1, the qualities relating to the other noise factors are:

$$J_1 = 80,\ J_2 = 80,\ J_3 = 82.5 \cdots,\ J_9 = 75 \tag{26}$$

In the same manner, the values of J for the remaining 24 experiments can be constructed and the results are listed in Table 3. Once the values of J for all 25 experiments have been established, we can estimate the optimum level for each parameter, which results in the smallest variation of quality characteristics due to various maneuvers.

Table 3. L_{25} Orthogonal array for PPCG

No	constant		Index	Noise factors																		S/N
				Jink						Split S						Long acceleration						
	K_p	K_y		$\tau_t = 0.1$		$\tau_t = 0.2$		$\tau_t = 0.3$		$\tau_t = 0.1$		$\tau_t = 0.2$		$\tau_t = 0.3$		$\tau_t = 0.1$		$\tau_t = 0.2$		$\tau_t = 0.3$		
1	2	1	Time	70	80	70	80	75	82.5	90	85	90	90	90	80	65	77.5	70	80	70	75	38.15
			M.D.	90		90		90		80		90		70		90		90		80		
											↕											
25	10	9	Time	75	87.5	75	87.5	75	87.5	100	95	100	87.5	100	92.5	70	80	70	85	70	80	38.72
			M.D.	100		95		100		90		75		85		90		100		90		

The quality loss σ^2 in our present case is defined in Eq. (21) and nine noise levels are considered, i.e., the three different maneuvers and the three different time constants. From the definition for J we know $J_i \leq 100$ (%), and the equality occurs when both specifications are satisfied. As the value of J_i becomes smaller, the quality deviates more from the desired performance, and the magnitude of σ^2 becomes larger. Hence, we can consider σ^2 as a quantitative measure for quality loss. The S/N ratio calculated from Eq. (22) is listed in the last block of Table 3. For example, in experiment 1 the quality loss σ^2 is calculated as $\sigma^2 = 1.5312 \times 10^{-4}$ and then the S/N ratio η for experiment 1 is $10 \cdot \log_{10}(1/1.5312 \times 10^{-4}) = 38.1497$ by Eq. (22). The L_{25} orthogonal arrays associated with PPCG guidance are summarized in Table 3.

(6) Estimate the effect of each parameter

After the values of the S/N ratio for each experiment are summarized, the next step is to estimate the effect of each parameter on the quality characteristics. By taking the numerical values of η listed in Table 3, the average S/N ratio for each level of the two parameters can be obtained as in Table 4. Taking $K_p = 2$ for example, its average S/N ratio can be obtained by Eq. (27):

$$\begin{aligned} \eta_{K_p=2} &= \frac{1}{5}\left(\eta_1 + \eta_2 + \eta_3 + \eta_4 + \eta_5\right) \\ &= \frac{1}{5}(38.1497 + 38.1818 + 38.3978 + 38.3131 + 38.2909) = 38.2267 \end{aligned} \tag{27}$$

and

$$\begin{aligned} \eta_{K_y=3} &= \frac{1}{5}\left(\eta_2 + \eta_7 + \eta_{12} + \eta_{17} + \eta_{22}\right) \\ &= \frac{1}{5}(38.1818 + 38.2883 + 38.1949 + 38.5626 + 38.3749) = 38.3205 \end{aligned}$$

etc.

The entries in Table 4 represent the separate effects of each parameter and are commonly called the main effects. Since the characteristic of the S/N ratio in this paper is larger-the-better, from Table 4 we can determine the optimum level for each parameter as the level that has the highest value of η (marked with a star sign in the table). Thus, from Table 1 and Table 4, the best are $K_p = 10$ and $K_y = 9$.

5. PERFORMANCE COMPARISONS OF THREE GUIDANCE LAWS WITH ROBUST NAVIGATION CONSTANTS

Navigation constants affect the robustness and the performance of a missile. In this section, the robust navigation constants will be compared with the general navigation constants during the simulations. Three kinds of pursuit-evasion scenarios, including long acceleration, Jink and Split-S, are studied and tracking time and miss distances are considered as the performance index. For simulation, this paper set β_1=8, β_2=4 and S_T=0.1 which are mentioned in Eqs. (23), (24) and (25). In order to test whether the robust navigation constants are also effective for other guidance laws, this paper applies the same situations to PNG and PG. As mentioned in the previous section, the robust navigation constants for PPCG have been given as K_p=10 and K_y=9, whose S/N ratio is 38.72, which is evaluated in reference to the procedures indicated in the previous section. The derivation of the robust navigation constants for PG and PNG follows the same process as that used above for the PPCG. Table 5 gives the list of S/N ratios and the robust navigation constants of the three kinds of guidance laws. We make the S/N ratio comparison between the three guidance laws with their corresponding robust navigation constants and discover that PPCG has the largest S/N ratio. That is, from the viewpoint of intercepting a maneuverable target, the robustness of PPCG is better than the others.

Table 4. The S/N ratio of each parameter level

Navigation constant	Level 1	Level 2	Level 3	Level 4	Level 5
K_p	38.2667	38.3755	38.4024	38.4890	38.4926*
K_y	38.2833	38.3205	38.4688	38.4693	38.4843*

Table 5. Robust navigation constants and S/N ratios of three kinds of guidance laws

	PG	PNG	PPCG
Robust navigation constants	$K_p = 8, K_y = 5$	$K_p = 8, K_y = 9$	$K_p = 10, K_y = 9$
S/N ratio	38.45	38.67	38.72

Table 6. Navigation constant S/N ratio of PPCG

		Split-S			Jink			Long acceleration		
		PPCG	PNG	PG	PPCG	PNG	PG	PPCG	PNG	PG
K_p=10 K_y=9 (Robust navigation constants)	Tracking Time (s)	22.4	22.5	41.9	54.0	53.8	55.0	60.3	56.3	70.1
	Miss Distance (m)	0.4	0.5	0.06	0.01	0.06	0.1	0.2	0.2	0.2
K_p=2 K_y=7	Tracking Time (s)	32.1	34.6	115.7	56.5	57.0	55.0	61.0	57.5	70.3
	Miss Distance (m)	1.6	0.9	0.1	0.099	0.4	0.2	0.4	0.7	0.4
K_p=4 K_y=1	Tracking Time (s)	38.0	31.5	91.4	56.5	61.5	56.5	60.7	57.3	70.3
	Miss Distance (m)	0.5	16.5	0.6	0.7	1.1	0.4	0.8	1.0	0.6
K_p=6 K_y=3	Tracking Time (s)	34.9	23.7	41.0	54.7	56.4	55.2	60.4	56.5	70.1
	Miss Distance (m)	2.0	2.1	0.5	0.057	1.0	0.2	0.4	0.7	0.3

Table 6 collates the data of simulations of the two performance indexes - tracking time and miss distance - for PPCG, PNG and PG. Three general navigation constants (K_p, K_y) = (2,7), (4,1) and (6,3) are chosen for comparison of the robust navigation constants. In the three pursuit-evasion scenarios, the three kinds of guidance laws, which own their robust navigation constants, take the least amount of time to track. For example, in the Split-S scenario, the robust navigation constant case of PPCG has the least amount of tracking time, 22.4sec; while the general navigation constant cases are 32.1, 38, and 34.9 seconds. Split-S is a highly maneuverable maneuver for disengaging from missile tracking, so it widens the tracking time gaps between the robust navigation constant case and the general ones, and the largest gap is 15.6 seconds while the least is still 9.7 seconds. These values show that the robust navigation constant case has a higher tracking ability (less tracking time) and accuracy (less miss distance) than a general navigation constant case. It is possible to arrive at a general understanding of the result involved by the S/N ratio. The S/N ratio designed in this paper has a larger-the-better characteristic and it is obvious that a larger S/N ratio demonstrates better performance. For instance, for PPCG, the S/N ratio in a robust navigation constant case is 38.72, which is larger than the general navigation constant cases that are 38.31, 38.29 and 38.19 for (K_p, K_y) = (2,7), (4,1) and (6,3), repectively. It’s noted that the tracking time gap between the robust navigation constant case and the general navigation constant case is less when the target uses a simple maneuver such as long acceleration. From the above descriptions, we give a summary that the performance of robust navigation constant cases is better than that of general navigation constant cases. The results for PNG and PG are the same as PPCG.

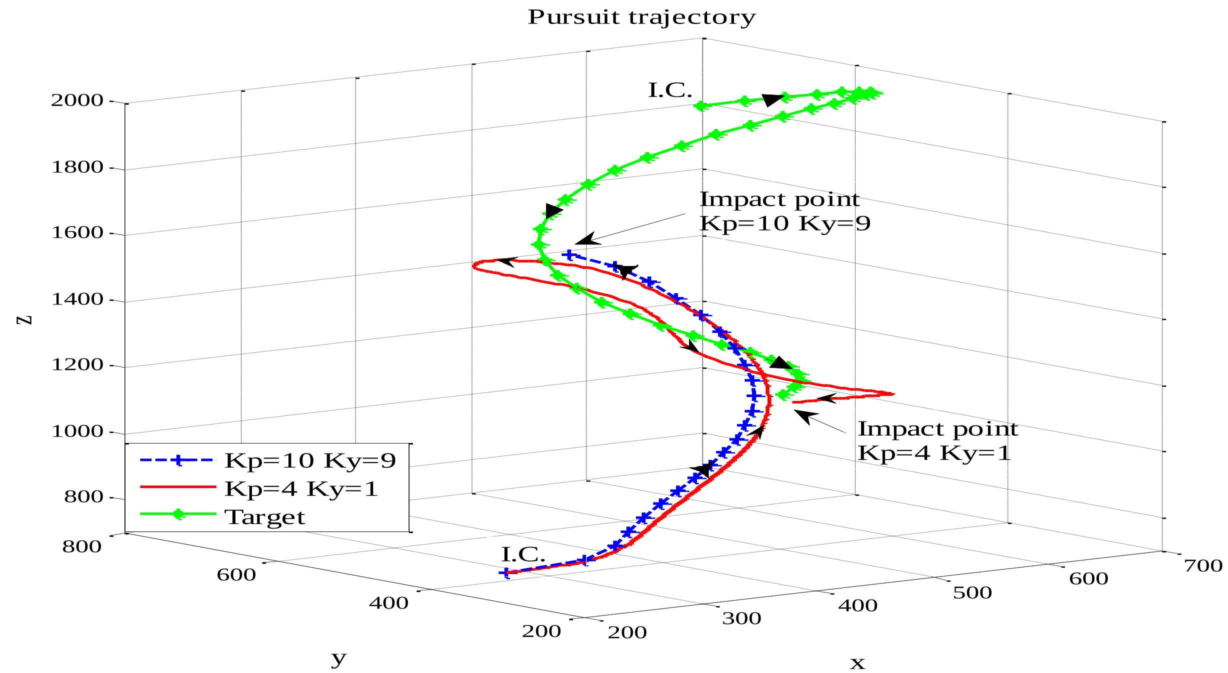

Fig. 2. The pursuit trajectories of PPCG with different navigation constants to pursue the Split-S maneuver target

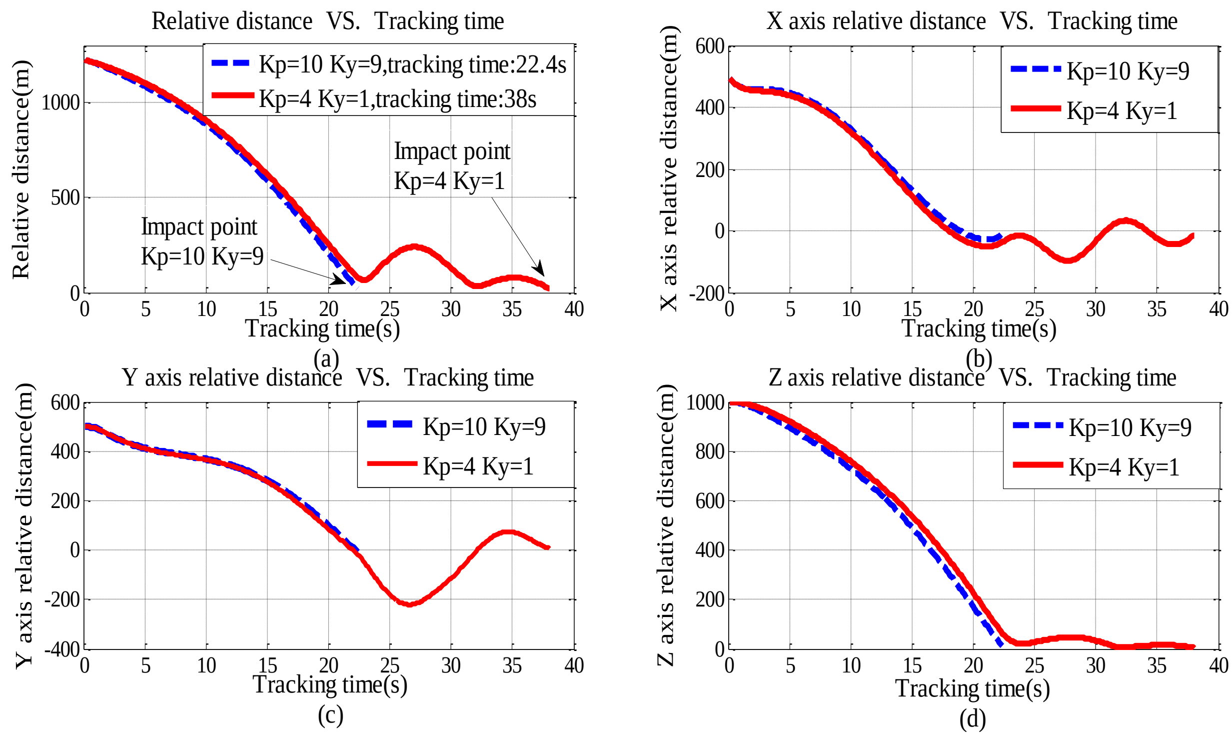

Fig. 3. The histories of PPCG tracking time for Split-S maneuver with different navigation constants

According to the pursuit trajectories in Figure 2 for a Split-S scenario, we can find that only the robust navigation constants caught the target at the first impact point and the other failed. From another point of view, a robust navigation constant case has a smaller relative distance than the general navigation constant cases with the same tracking time. The results described above are illustrated in Figure 3(a). Consequently, to pursue a highly maneuverable target, this paper demonstrates that applying TQM to obtain the robust navigation constants will increase missile tracking abilities. Figure 3(b), (c), (d) show that relative distance of Z axis changes more intense in hitting phase than that of X axis and Y axis. The phenomena give two viewpoints. One is that a target applies yaw axis movement, such as Jink maneuver, to avoid missile attack is superior to a target uses pitch axis movement, and the other one is that yaw axis movement can consume more missile's energy than pitch axis movement does. If a target just adopts maneuvers of dive or climb to avoid a missile attack, the missile has high probability to shoot down the target.

6. CONCLUSIONS

This paper proposes a new viewpoint that a target's intelligence and a missile's performance are considered to be noise factors and the selection of navigation constants that are robust to a target's maneuvers is exploited by TQM. The robust design is to improve the guidance quality to decrease the effect of the target evasion tactic variation. The proposed processes, including the design of a matrix experiment, the calculation of an S/N ratio and the effect of analysis on each parameter, are easily applied to design robust and effective navigation constants. From the simulation results of PPCG, PNG and PG with the determined robust navigation constants, the guidance laws take less tracking time to intercept the target and have a higher accuracy in the hit phase. In other words, a guidance law with robust navigation constants has better maneuverability in being robust to intercept a highly maneuverable target. Furthermore, in comparison with PNG and PG, PPCG demonstrates the balance of performance and robustness. In the future, these guidance laws and robust navigation constants will be involved in automatic pilot simulators of teams enacting air combat.

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