FUZZY SIMPLEX-TYPE SLIDING-MODE CONTROL

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ABSTRACT

In this paper, a novel fuzzy simplex sliding-mode controller is proposed for controlling a multivariable nonlinear system. The fuzzy logic control (FLC) algorithm and simplex sliding-mode control (SSMC) theory are integrated to form the fuzzy simplex sliding mode control (FSSMC) scheme which improves the system states response and reduces system states chattering phenomenon. In this paper, at first, we introduce the principle of simplex method, and then develop fuzzy controls based on the simplex method. Finally, a numerical example is proposed to illustrate the advantages of the proposed controllers, the simulation results demonstrate that the fuzzy simplex type sliding mode control scheme is a good solution to the chattering problem in the simplex sliding mode control.

Keywords: fuzzy; simplex; sliding-mode control.

LA MÉTHODE SIMPLEXE POUR LA COMMANDE EN MODE GLISSANT FLOU

RÉSUMÉ

Dans cet article, une méthode innovatrice de commande par méthode simplexe en mode glissant flou est proposée pour la commande d'un système non-linéaire multivariable. Un algorithme de commande logique floue (FLC) et la théorie de commande par méthode simplexe en mode glissant flou (SSMC) sont intégrés pour former la commande par méthode simplexe en mode glissant flou (FSSMC) pour améliorer les réponses sur l'état du système et réduire le phénomène de broutement. Premièrement, on introduit le principe de la méthode simplexe, pour ensuite développer les commandes floues basées sur la méthode simplexe, et, finalement, un exemple numérique est proposé pour illustrer les avantages des commandes proposées. Les résultats de simulation démontrent que la commande par méthode simplexe en mode glissant flou est une bonne solution au problème de broutement.

Mots-clés : flou ; simplexe ; commande en mode glissant.

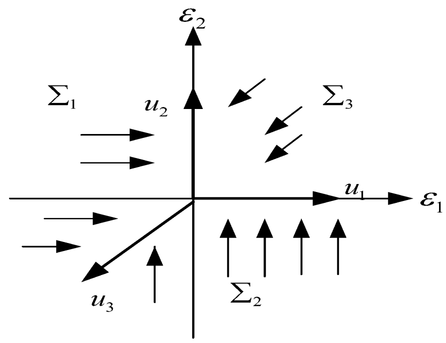


Fig. 1. The regular simplex of control vectors u_1, u_2, and u_3.

1. INTRODUCTION

The simplex sliding-mode control was first proposed by Baida and Izosimovfor [1] for the control of multi-input continuous systems. Diong [2–4] extended it to linear multivariable continuous systems. Others extended the research from multi-input linear continuous systems to multi-input nonlinear continuous systems and demonstrated the feasibility of these systems [5–8]. In this paper, we will develop the fuzzy simplex type sliding mode control (FSSMC) in a nonlinear multi-input system. Basically, the simplex SMC control design procedure is similar to that of the conventional sliding-mode control (SMC). It includes the sliding and reaching conditions. In the first step, we determine the sliding surface such that the closed-loop systems are stable in the sliding mode. Next, a set of simple hitting control vectors is chosen so that the states will move toward the sliding surface as soon as possible. The choice of control vectors of simplex-type SMC is easier than conventional SMC, suppose in an m dimensional vector space, the control vectors must satisfy the simplex definition, and the simplex dependent set contains only $m+1$ vectors. A basic feature of simplex SMC for $m=2$ is shown in Fig. 1, where σ is the sliding function, u is the designed control vector, and Σ is the corresponding region. The simplex method has a disadvantage of the constant control vectors such that the chattering phenomenon occurs when switching control take place in simplex-type SMC scheme. In order to ensure that assumption conditions are satisfied, we must select a sufficiently large magnitude control vectors. However, increasing the magnitude of the control vector aggravates the chattering problem. Hence, we integrate fuzzy algorithm into the simplex method that form a fuzzy simplex sliding mode control scheme, the proposed irregular simplex concept is transferred to a fuzzy control framework. In the design phase, a regular simplex of control vectors is modified by adding a control change quantity du obtained by fuzzy controller which yields a new irregular simplex control vector u', which is then applied to the controlled systems to steer the sliding function σ toward the origin, in the meanwhile, the chattering phenomenon can be improved.

The remainder of this paper is organized as follows: the Simplex-Type Sliding Mode Control is brief described in Section 2. In Section 3, the fuzzy Simplex Sliding Mode Control is addressed. Section 4 presents an example to demonstrate the validity of fuzz simplex sliding mode control. Finally, we conclude with Section 5.

2. SIMPLEX-TYPE SLIDING MODE CONTROL

Consider a class of nonlinear affine systems:

$$\dot{x} = f(x) + g(x)u \tag{1}$$

where $u \in R^m$ is the control input, $x \in R^n$ is system states, and $m \leq n$. Choose a manifold described by

$$\sigma(x) = Sx = 0 \tag{2}$$

so that if the system trajectories lie on the manifold, the behavior of the system satisfies a pre- specified control objective, where $\sigma \in R^m$; $s \in R^{m \times n}$, the sliding manifold is defined by the mapping σ: $R^n \rightarrow R^m$.

2.1. The Sliding Mode Dynamics Design

Linearizing with respect to the equilibrium points of the system (1), a linear multi-input system (3) is obtained as follows:

$$\dot{x} = Ax + Bu \tag{3}$$

where $x \in R^n$ and $u \in R^m$ represent the state and control vectors of the system, respectively. In this paper, the design approaches of [3, 4] are used as the method of determining the sliding-mode dynamics. Therefore, the sliding mode dynamic (3) can now be represented by M,

$$\dot{x}_r = (A_r - B_r M)x_r \tag{4}$$

where $A_r = HAH^*$, $B_r = HAB$. Choice H satisfies $HB = 0$. $M \in R^{m \times (n-m)}$ is selected such that the matrix $A_r - B_r M$ is Hurwitz.

2.2. Simplex Control Strategy

Before the introduction of the simplex sliding mode control, the definition of simplex concept is first presented in Definition 1.

Definition 1. The set $U = \{u_1, u_2, \ldots, u_{m+1}\}$, where $u_i \in R^m$ are nonzero and distinct vectors, is said to form a simplex in R^m if any m of the vectors is linearly independent and there exist $m+1$ real positive constants c_1, c_2, $\ldots$, and c_{m+1} such that $\sum_{i=1}^{m+1} c_i u_i = 0$ and $\sum_{i=1}^{m+1} c_i = 1$.

This definition means that a simplex is a set of $m+1$ affine independent vectors in R^m such that 0^m is in the interior of the convex hull of those vectors. In order to provide a control strategy accomplishing the objective of steering $x(t)$ to zero in the multi-input case, a coordinate transformation from σ to ε is performed [1, 8], namely,

$$\varepsilon = (sg)^{-1}\sigma \tag{5}$$

The system's equation which describes the variable ε will be

$$\dot{\varepsilon} = (sg)^{-1}\dot{\sigma} = (sg)^{-1}sf + u \tag{6}$$

The ε space is divided into $m+1$ regions, which are described by

$$\Sigma_i = \left\{ \varepsilon : \varepsilon = \sum_{j \neq i, j=1}^{m+1} \lambda_j u_j, \lambda_j > 0 \right\}, \quad i = 1, \ldots, m+1 \tag{7}$$

Thus, each Σ is an infinite cone, with vertex at 0^m, situated on the side of R^m opposite to u_i.

Definition 2. If the set $U = \{u_1, u_2, \ldots, u_{m+1}\}$ forms a simplex set in ε space, then the control strategy for simplex method is defined as

$$u = f(\varepsilon) = \begin{cases} u_j, & \varepsilon \in \Sigma_j \\ 0, & \varepsilon \in u_i, s \neq i \end{cases} \tag{8}$$

The significance of (8) is that when ε is in Σ, then the control u is selected as u_j; $u = 0$ when $\varepsilon \in u_i$, $s \neq i$, i.e. ε is exactly at the u_i, the system state ε will slide back or forth the region of $\varepsilon \in u_i$.

The basic configuration of Eqs. (5) and (8) for two-input case is depicted in Fig. 1, where ε is decomposed into three regions determined by the selection of the simplex set U. We refer to the figure as a regular simplex control.

2.3. Irregular Simplex Sliding Mode Control

For any partition of the ε space into $m+1$ regions by using a regular simplex, the control vector u is decided as follows:

$$u = f(\varepsilon) = \begin{cases} u'_j = u_j - \eta\varepsilon, & \varepsilon \in \Sigma_j, \eta > 0, \\ 0, & \varepsilon \in u_i, s \neq i \end{cases} \tag{9}$$

where $u = u'_j + du = u_j - \eta\varepsilon$.

The significance of (9) is that when ε is in Σ_j, then the control u is selected as u'_j; $u = 0$ when $\varepsilon \in u$, $s \neq i$, i.e. ε is exactly at the u_i, the system state ε will slide back or forth the region of $\varepsilon \in u_i$.

The control u'_j causes the origin of the ε space to be asymptotically stable in finite time, where u'_j is the new irregular simplex control vector, du is the change in quantity in control.

Assumption 1 [1]. There exist real numbers α_i and ξ such that the evolution of x determined by (1) and (9) satisfy

$$(Sg)^{-1}Sf = -\sum_{i=1}^{m+1} \alpha_i u_i, \quad \alpha_i \geq 0 \tag{10}$$

$$\sum_{i=1}^{m+1} \alpha_i = \xi, \quad 0 \leq \xi < 1 \tag{11}$$

Theorem 1. The ε space is divided into $m+1$ regions by the regular simplex set $U = \{u_1, u_2, \ldots, u_{m+1}\}$. When $\varepsilon \in \Sigma_j$, $j = 1, \ldots, m+1$, the irregular control vector can be chosen as $u'_j = u_j + du = u_j - \eta\varepsilon$; then the origin is asymptotically stable, and the closed-loop system described by (1) and (9) will achieve the sliding mode on the hypersurface $\varepsilon = 0$ in finite time, which means $\varepsilon \to 0$, from (5) $\sigma = (Sg)\varepsilon$ yields $\sigma \to 0$.

Proof. Suppose the control $u \in R^m$ and the states of the plant locate at Σ_j, i.e. ε in the Σ_j region, $j \in [1, m+1]$. Following Definition 1, the control vector u_j can be represented by

$$u_j = -\sum_{i=1, i\neq j}^{m+1} \frac{c_i}{c_j} u_i = -\sum_{i=1, i\neq j}^{m+1} \gamma_i u_i, \quad \gamma_i = \frac{c_i}{c_j} > 0 \tag{12}$$

By (7) the ε in Σ_j is represented as $\varepsilon = \sum_{i=1, i\neq j}^{m+1} \lambda_i u_i$, where $\sum_{i=1, i\neq j}^{m+1} \lambda_i = \beta$ and $\lambda_i > 0$. In fact, for any $\varepsilon \neq 0$, $\beta(\varepsilon) = \sum_{i=1}^{m+1} \lambda_i > 0$ and $\beta(\varepsilon) = 0$ as $\varepsilon = 0$. Thus, $\beta(\varepsilon)$ can be treated as a Lyapunov function. According to (9), the control law is expressed as $u'_j = u_j - \eta\varepsilon$ for $\varepsilon \in \Sigma_j$. We define

$$U_j = [u_1 \ldots u_{j-1} \quad u_{j+1} \ldots u_{m+1}] \tag{13}$$

which is nonsingular and define the following m-dimensional vectors:

$$\begin{aligned} \Lambda_j(k) &= [\lambda_1^T \ldots \lambda_{j-1}^T \quad \lambda_{j+1}^T \ldots \lambda_{m+1}^T]^T \\ G_j &= [\gamma_1^T \ldots \gamma_{j-1}^T \quad \gamma_{j+1}^T \ldots \gamma_{m+1}^T]^T \\ K_j &= [\alpha_1^T \ldots \alpha_{j-1}^T \quad \alpha_{j+1}^T \ldots \alpha_{m+1}^T]^T \\ b &= [1 \ldots 1 \quad 1 \ldots 1]^T \end{aligned} \tag{14}$$

Then we can obtain

$$\varepsilon = \sum_{i=1,i\neq j}^{m+1} \lambda_i u_i = U_j \Lambda_i$$
$$\beta = \sum_{i=1,i\neq j}^{m+1} \lambda_i = b^T \Lambda_j \quad (15)$$

Because ε is in the Σ_j region, from Definition 1, the simplex u_j can be expressed by

$$u = u_j = -U_j G_j \quad (16)$$

the irregular simplex control u'_j can be expressed by

$$u'_j = u_j - \eta\varepsilon = -U_j G_j - \eta U_j \Lambda_j \quad (17)$$

Thus, Eq. (11) can be represented as

$$(Sg)^{-1} Sf = -\sum_{i=1}^{m+1} \alpha_i u_i = -U_j K_j + \alpha_j U_j G_j \quad (18)$$

From (15) the derivative of ε is $\dot{\varepsilon} = U_j \dot{\Lambda}_j$, and then we can obtain

$$\dot{\Lambda}_j = U_j^{-1} \dot{\varepsilon} = U_j^{-1}((Sg)^{-1} Sf + u'_j) = U_j^{-1}(-U_j K_j + \alpha_j U_j G_j - U_j G_j - \eta U_j \Lambda_j) \quad (19)$$

which can be expanded as

$$\begin{bmatrix} \dot{\lambda}_1 \\ \vdots \\ \dot{\lambda}_{j-1} \\ \dot{\lambda}_{j+1} \\ \vdots \\ \dot{\lambda}_{m+1} \end{bmatrix} = -\begin{bmatrix} \dot{\alpha}_1 \\ \vdots \\ \dot{\alpha}_{j-1} \\ \dot{\alpha}_{j+1} \\ \vdots \\ \dot{\alpha}_{m+1} \end{bmatrix} - \alpha_j \begin{bmatrix} \dot{\gamma}_1 \\ \vdots \\ \dot{\gamma}_{j-1} \\ \dot{\gamma}_{j+1} \\ \vdots \\ \dot{\gamma}_{m+1} \end{bmatrix} - \begin{bmatrix} \dot{\gamma}_1 \\ \vdots \\ \dot{\gamma}_{j-1} \\ \dot{\gamma}_{j+1} \\ \vdots \\ \dot{\gamma}_{m+1} \end{bmatrix} - \eta \begin{bmatrix} \dot{\lambda}_1 \\ \vdots \\ \dot{\lambda}_{j-1} \\ \dot{\lambda}_{j+1} \\ \vdots \\ \dot{\lambda}_{m+1} \end{bmatrix} \quad (20)$$

Now, we want to prove the rate of change of the function $\beta(\varepsilon)$ should be negative. Differentiation of β yields

$$\dot{\beta} = b^T \dot{\Lambda}_j = \dot{\lambda}_1 + \cdots + \dot{\lambda}_{j-1} + \cdots + \dot{\lambda}_{m+1} \quad (21)$$

The above equation is equal to the row-by-row summation of (20), that is,

$$\dot{\beta} = -\sum_{i=1,i\neq j}^{m+1} \alpha_i + \alpha_j \sum_{i=1,i\neq j}^{m+1} \gamma_i - \sum_{i=1,i\neq j}^{m+1} -\eta \sum_{i=1,i\neq j}^{m+1} \lambda_i \quad (22)$$

because $\alpha_i \geq 0$ and $\gamma_i > 0$, thus $\sum_{i=1,i\neq j}^{m+1} \alpha_i \geq 0$ and $\sum_{i=1,i\neq j}^{m+1} \gamma_i > 0$ yield

$$\dot{\beta} = -\sum_{i=1,i\neq j}^{m+1} \alpha_i - (1-\alpha_j) \sum_{i=1,i\neq j}^{m+1} \gamma_i - \eta \sum_{i=1,i\neq j}^{m+1} \lambda_i \quad (23)$$

According to Assumption 1, $\sum_{i=1}^{m+1} \alpha_i = \xi$, $0 \leq \xi < 1$ implying $(1-\alpha_j) > 0$, and $\lambda_i > 0$, $\eta > 0$, then one can obtain

$$\dot{\beta} = -\sum_{i=1,i\neq j}^{m+1} \alpha_i - (1-\alpha_j) \sum_{i=1,i\neq j}^{m+1} \gamma_i - \eta \sum_{i=1,i\neq j}^{m+1} \lambda_i < 0 \quad (24)$$

This completes the proof.

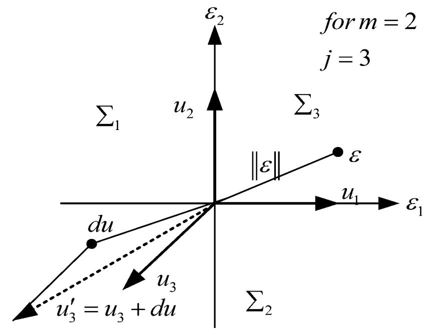


Fig. 2. The irregular simplex control $u_3' = u_3 + du$.

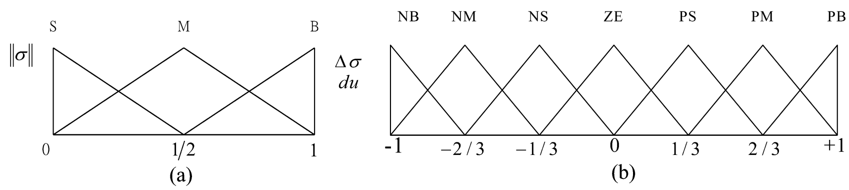


Fig. 3. Membership function (a) $\|\sigma\|$; (b) $\Delta\sigma$ and du.

3. FUZZY SIMPLE-TYPE SLIDING-MODE CONTROL

The irregular simplex controller $u_j' = u_j + \eta\varepsilon$ is addressed by Theorem 1 which inspires the fuzzy irregular simplex sliding mode control scheme; the irregular simplex concepts will be exploited to develop the fuzzy irregular simplex-type sliding-mode control. The step is to infer the control du which is viewed as $\eta\varepsilon$ by fuzzy algorithm, and then the irregular simplex control $u_j' = u_j + du$ is applied to a closed loop system. Stabilization of fuzzy simplex-type sliding-mode control scheme based on Theorem 1 is guaranteed. The fuzzy control rules modify the regular simplex control vectors such that the system states are steered to the origin. The interpretation for $u \in R^2$ and $j = 3$ (i.e. $m = 2$, $j = 3$) is shown in Fig. 2. For convenience, the following explanation assumes $m = 2$, $j = 3$.

The input variables are $\|\sigma\|$ and $\Delta\sigma$, the output variable is du. Let $\{S, M, B\}$ be the term sets of $\|\sigma\|$, where *S*, *M* and *B* are labels of fuzzy sets, which represent small, medium, big, respectively. Let us take the term sets of input variable $\Delta\sigma$ and control output du as $\{NB, NM, NS, ZE, PS, PM, PB\}$, which represent negative big, negative medium, negative small, zero, positive small, positive medium, positive big, respectively. The above fuzzy sets are triangular. Their membership functions are depicted in Fig. 3.

According to the magnitude of $\|\sigma\|$ and the change of $\Delta\sigma$, the compensative quantity of du is decided in order to achieve a expectative closed-loop behavior. The stability is guaranteed because the fuzzy rules satisfy Theorem 1. According to the above explanation, we can build the fuzzy control rules which are tabulated in Table 1.

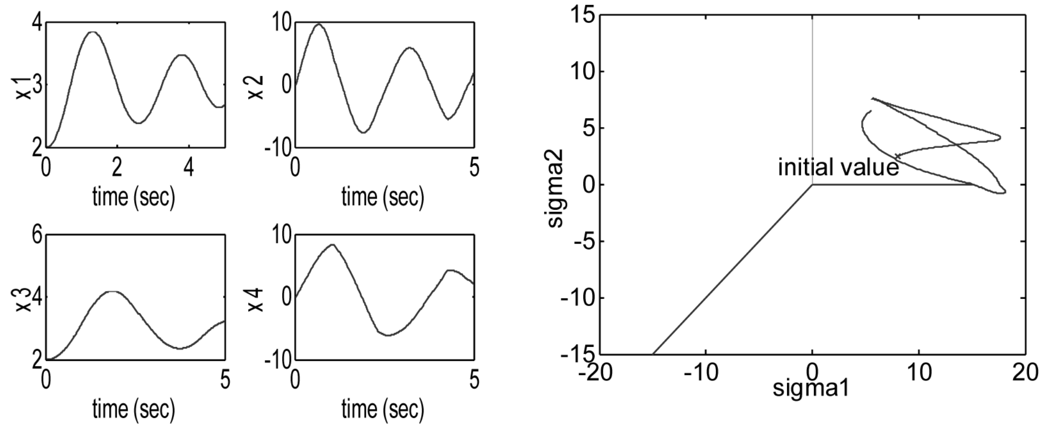
The crisp control command du is calculated by center-average defuzzification method, the output du is computed as

$$du = \frac{\sum_{i=1}^{N} \mu_i c_i}{\sum_i^N \mu_i} \tag{25}$$

where μ_i is the degree of the antecedent of the ith fuzzy rule, and c_i is the center of the membership of du in the ith fuzzy rule.

Table 1. Rule table for fuzzy-type simplex-type sliding mode control.

$\|\sigma\|$	$\Delta\sigma$						
	NB	NM	NS	ZE	PS	PM	PB
S	NS	NM	NS	ZE	PS	PS	PM
M	NS	ZE	ZE	PM	PS	PS	PB
B	PB	PB	PB	PB	PB	PB	PB



(a) (b)

Fig. 4. System response plots. (a) The time response of Case 1. (b) The phase response of Case 1..

4. COMPUTER SIMULATION AND RESULTS

Consider a nonlinear system described as follows:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = 91.375\sin(x_1) + 5u_i + 1.25\sin(x_3)$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_2 = 39.24\sin(x_3) + 2u_2 + 0.125\sin(x_2)$$

The first step is the selection of H, which must satisfy $HB = 0$, thus after some computation one can get

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}, \quad M = \begin{bmatrix} 0 & 1.6 \\ 2.5 & -4.0 \end{bmatrix}, \quad S = \begin{bmatrix} 1.6 & 0.2 & 0 & 0 \\ -1.5 & 0 & 2.5 & 0.5 \end{bmatrix}$$

The eigenvalues of $A_r - B_r M$ are designed as $\{-5 \quad -8\}$. There are three case simulations in this nonlinear system example. Case 1 is regular simplex control case for small control vectors chosen as $u_1 = [5 \quad 5]^T$, $u_2 = [-5 \;\; 5]^T$, $u_3 = [0 \quad -10]^T$. Case 2 is the same as Case 1 but has much bigger control vectors chosen as $u_1 = [50 \;\; 50]^T$, $u_2 = [-50 \;\; 50]^T$, $u_3 = [0 \quad -100]^T$ and Case 3 is a fuzzy simplex control case that chooses the same control vector as Case 1. All time and phase trajectories of the same initial value $x_0 = [2 \;\; 0 \;\; 2 \;\; 0]^T$ are depicted in Figs. 4–6, respectively. From the figures of time response, one can see that the proposed fuzzy simplex control law can achieve a smoother response. The choice of the simplex control vector not only meets Theorem 1 but is also big enough to fulfill Assumption 1. According to Figs. 4a and 4b, it is shown that the small magnitudes of the simplex control vectors, $u_1 = [5 \;\; 5]^T$, $u_2 = [-5 \;\; 5]^T$, $u_3 = [0 \quad -10]^T$,

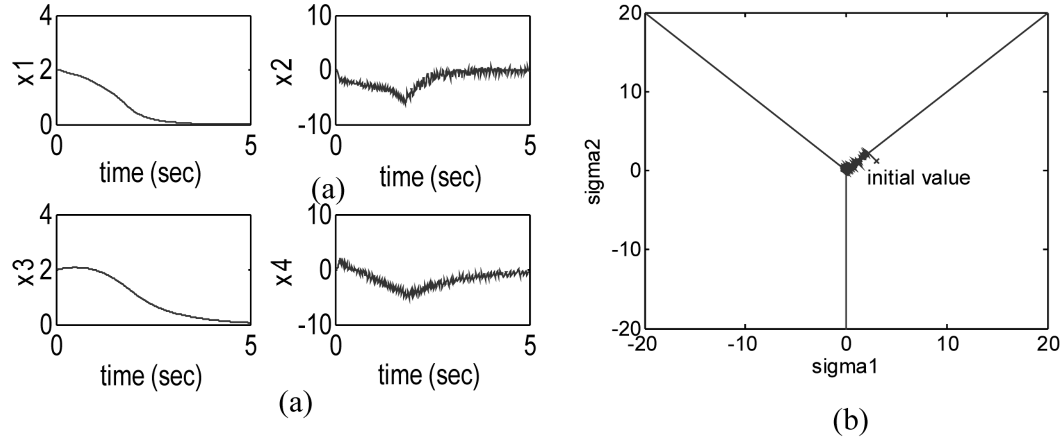


Fig. 5. System response plots. (a) The time response of Case 2. (b) The phase response of Case 2.

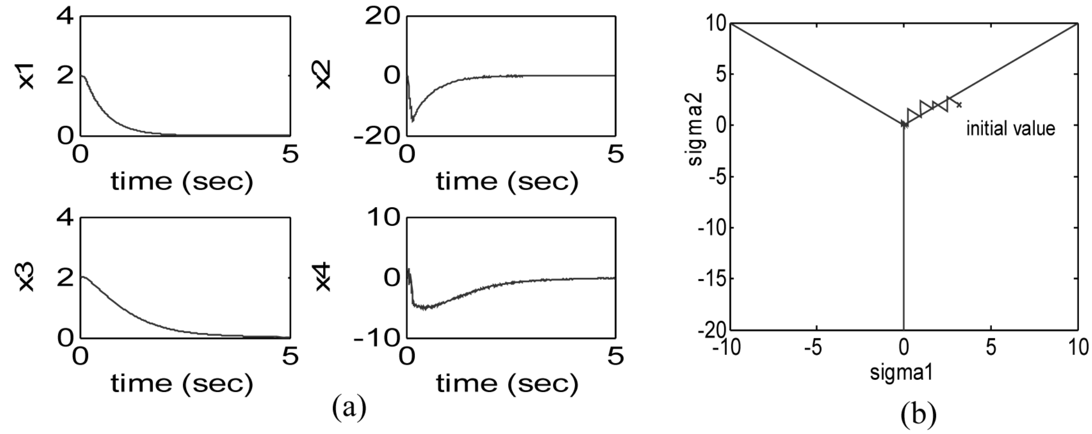


Fig. 6. System response plots. (a) The time response of Case 3. (b) The phase response of Case 3.

cannot meet Assumption 1 which results in an unstable response. Therefore, if we do not compensate u_j, the system is always unstable. The simulation results of Case 2 for time and phase trajectories are depicted in Figs. 5a and 5b, respectively. The control vectors, $u_1 = [50 \;\; 50]^T$, $u_2 = [-50 \;\; 50]^T$ and $u_3 = [0 \;\; -100]^T$, are appropriately large; hence it is big enough to satisfy the simplex definition and fulfill Assumption 1 such that it can stabilize the system but has much chattering. The time and phase trajectories of Case 3 are depicted in Figs. 6a and 6b, respectively. From these figures of time response, one can see that the proposed fuzzy simplex control law can achieve a smoother response than Case 2. In Case 2, the states of the system will repeatedly cross the switching surfaces and slide to the boundary of origin. In this situation since the control vectors are constant and have no strategy to reduce the control quantity, the chattering phenomenon occurs in the boundary of origin. However, the proposed fuzzy simplex control law can achieve a smooth response, and drive the states towards the origin with lower chattering, because the control change quantity, du can compensate by fuzzy rules, it can gradually decrease the magnitude of the simplex control vector u_j during ε's approach to the origin, which results in chattering elimination.

5. CONCLUSION

This paper considered the design of adaptive control based on simplex-type sliding-mode control philosophy for a nonlinear affair multivariable system. These procedures were illustrated on a nonlinear multi-input system; the results showed fuzzy simplex-type sliding-mode control's much improved transient responses compared with the transient responses obtained with only simplex control law. The example also indicated that a fairly large simplex control effort was required to obtain better performance in compliance with the conditions of simplex Definition 1 and Assumption 1, however, the transient responses accompanying the chattering, fuzzy simplex-type sliding-mode method can reduce the control effort required and attenuate chattering circumstances; Hence, one can conclude that the FSSMC theoretical study and the simulation results prove that the fuzzy simplex type sliding mode control proposed in this paper is a stable control scheme and is a good solution to the chattering problem in the simplex sliding mode control.

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