

A PRACTICAL TRAJECTORY TRACKING APPROACH FOR AUTONOMOUS MOBILE ROBOTS: NONLINEAR ADAPTIVE H_2 DESIGN

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ABSTRACT

A nonlinear adaptive trajectory tracking design for autonomous mobile robot and its practical implementation are presented in this paper. This approach can be applied to generate trajectory tracking control commands for autonomous mobile robot tracking predefined trajectories. The design objective is to specify one nonlinear controller with a parameter adaptive law that satisfies the adaptive H_2 optimal performance. In general, it is hard to obtain the closed-form solution from this nonlinear trajectory tracking problem. Fortunately, based on the property of the trajectory tracking error dynamic system of the autonomous mobile robot, one closed-form solution to this problem can be obtained with a very simple form for the preceding control design.

Keywords: autonomous mobile robot; nonlinear adaptive control law; H_2 performance index; closed-form solution.

UNE APPROCHE PRATIQUE DE SUIVI DE TRAJECTOIRE POUR ROBOTS MOBILES AUTONOMES : CONCEPTION DE COMMANDE NONLINAIRE ADAPTATIVE H_2

RÉSUMÉ

Une conception de suivi de trajectoire à commande nonlinéaire adaptative pour un robot mobile autonome et son application pratique, est présentée dans cet article. Cette approche peut-être appliquée pour générer des commandes de suivi de trajectoires pour un robot mobile autonome à trajectoires prédéterminées. L'objectif du concept est de déterminer pour un contrôleur nonlinéaire spécifique une loi de paramètre adaptatif qui satisfait la performance optimale adaptative H_2 . En général, il est difficile d'obtenir une solution en forme close pour ce problème de suivi de trajectoire nonlinéaire. Heureusement, en se basant sur les propriétés du système dynamique d'erreurs de suivi de trajectoire d'un robot mobile autonome, une solution en forme close peut être obtenue, par une forme très simple.

Mots-clés : Robot mobile autonome ; loi de commande nonlinéaire adaptative ; index de performance H_2 ; solution de forme close.

1. INTRODUCTION

In the past decades, autonomous designs of wheel mobile robots attract a lot of attentions and the successful cases were widely applied in various industrial and service fields which included transportation, inspection, security, etc. Hence, it becomes more and more important to precisely manipulate wheeled mobile robots, especially in the trajectory tracking subjects. Many existing studies about the trajectory tracking problems for autonomous mobile robots are discussed in this paper. Studies in [1–3] proposed an easily implemented and optimal control law for the path following designs of autonomous mobile robots without considerations of parameters uncertainties. Others combined neural network, fuzzy concept, and sliding mode control [4–6], but they are not easily realized because of the complicated control structures.

Based on these reasons, in this proposed study, authors propose an easily implemented nonlinear adaptive control law for the trajectory following design of mobile robots. The mathematical model and design objective of autonomous mobile robot will be briefly introduced in Section 2. In Section 3, problem formulation and controller law for trajectory tracking design will be described. Practical trajectory tracking verifications of the autonomous mobile robot by the proposed design are demonstrated in Section 4. Conclusions are summarized in Section 5.

2. MATHEMATICAL MODEL AND DESIGN OBJECTIVE

2.1. Model and Dynamics of an Autonomous Mobile Robot

In general, the structure of wheel mobile robot consists of two driving wheels which locate at the same axis and a passive self-adjusted supporting wheel which leads the mechanical system. Both driving wheels which are for the motion and orientation purposes are driven by two actuators (e.g., DC motors) independently.

As shown in Fig. 1, the two driving wheels with the same radius are denoted by r and separated by $2R$. The location of the vehicle in the global coordinate frame $\{O, X, Y\}$ is represented by the vector $P = [x_c \ y_c \ \theta]^T$, where x_c and y_c are the coordinates of the point C in the global coordinate frame, and θ is the orientation of the local frame $\{C, x_c, y_c\}$. The generalized coordinate of the vehicle is described as

$$q = [x_c \ y_c \ \theta]^T \quad (1)$$

For ordinary mobile robot system, the robot can just move along the direction of the axis of the driving wheels with pure rolling and nonslipping, nonholonomic condition status. Consequently, the velocity of contact points with the ground and orthogonal to the plane of the wheel is zero and can be expressed as follows [7]:

$$\dot{y}_c \cos \theta - \dot{x}_c \sin \theta - d\dot{\theta} = 0 \quad (2)$$

Based on Eq. (2), the kinematic equation can then be described as

$$\dot{q} = \begin{bmatrix} \dot{x}_c \\ \dot{y}_c \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos \theta & -d \sin \theta \\ \sin \theta & d \cos \theta \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_l \\ w \end{bmatrix} \quad (3)$$

where v_l and w are the linear and angular velocity along the robot axis.

For the purpose of finding an easily implemented control law, the mobile robot dynamic equation (3) is furthermore reformulated as

$$H(q)\ddot{q} + C(q, \dot{q})\dot{q} + F_g(q) = B(q)\tau \quad (4)$$

where $H(q) \in \mathfrak{R}^{3 \times 3}$ is a symmetric positive definite inertia matrix, $C(q, \dot{q}) \in \mathfrak{R}^{3 \times 3}$ is the centripetal and coriolis matrix, $F_g(q) \in \mathfrak{R}^{3 \times 1}$ is the gravitational vector, $B(q) \in \mathfrak{R}^{3 \times 2}$ is the input transformation matrix, $\tau \in \mathfrak{R}^{2 \times 1}$ is

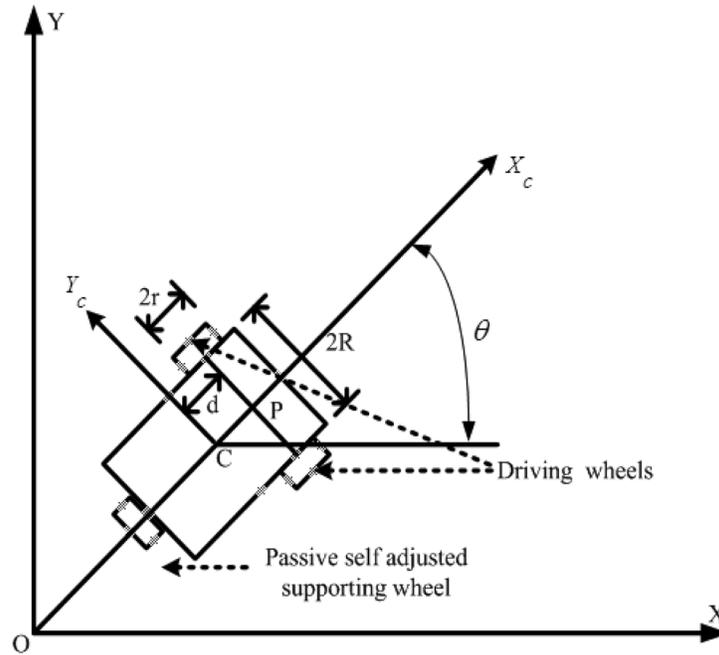


Fig. 1. Motion illustration of an autonomous mobile robot.

the input vector.

$$\begin{aligned}
 H(q) &= \begin{bmatrix} m & 0 & md \sin \theta \\ 0 & m & -md \cos \theta \\ md \sin \theta & -md \cos \theta & I \end{bmatrix} \\
 C(q, \dot{q}) &= \begin{bmatrix} 0 & 0 & md \dot{\theta} \cos \theta \\ 0 & 0 & md \dot{\theta} \sin \theta \\ 0 & 0 & 0 \end{bmatrix} \\
 B(q) &= \frac{1}{r} \begin{bmatrix} \cos \theta & \sin \theta \\ \sin \theta & \sin \theta \\ R & -R \end{bmatrix}, \quad \tau = \begin{bmatrix} \tau_r \\ \tau_l \end{bmatrix} \quad (5)
 \end{aligned}$$

where τ_r and τ_l represents right and left wheel torques, respectively. Since the autonomous mobile robot moves in the horizontal plane, the gravitational vector will be zero and can be removed for control purpose.

2.2. Problem Formulation

For treating the trajectory tracking problem of autonomous mobile robots with system parameter variations, a nonlinear adaptive H_2 trajectory tracking control design will be developed next in this paper. First, the desired tracking reference trajectory q_r is assumed as bounded time function $q_r \in C^2$ which is a twice continuously differentiable function. The velocity vector and acceleration vector of q_r can be denoted as \dot{q}_r and \ddot{q}_r respectively. Based on these arrangements, the tracking errors for the autonomous mobile robot are defined as

$$e = \begin{bmatrix} \dot{\tilde{q}} \\ \tilde{q} \end{bmatrix} = \begin{bmatrix} \dot{q} - \dot{q}_r \\ q - q_r \end{bmatrix} \quad (6)$$

and the tracking error dynamic equation is given as

$$\dot{e} = \begin{bmatrix} -H^{-1}(q)C(q, \dot{q}) & 0_{2 \times 2} \\ I_{2 \times 2} & 0_{2 \times 2} \end{bmatrix} e + \begin{bmatrix} -\ddot{q}_r - H^{-1}(q)C(q, \dot{q})\dot{q}_r \\ 0_{2 \times 2} \end{bmatrix} + \begin{bmatrix} H^{-1}(q)B(q)\tau \\ 0_{2 \times 2} \end{bmatrix} \quad (7)$$

Generally speaking, the error dynamic in Eq. (7) is difficult to directly apply for the trajectory tracking design due to its complicated system structure. For simplifying the design difficulties, the following filtered link of tracking error $l(t)$ is given to transfer tracking error dynamic equation in (7) into a more analytic formulation:

$$l(t) = \eta \dot{\tilde{q}} + \Phi \tilde{q} \quad (8)$$

where $\eta = [\eta_1 \ \eta_2]$ are some positive constants which will be adequately determined later. From Eq. (8), we have

$$\dot{l}(t) = -H^{-1}(q)C(q, \dot{q})l + \eta_2 H^{-1}(q)[- \Phi(e, t) + f] \quad (9)$$

where

$$\Phi(e, t)\zeta = \Phi \left(q, \dot{q}, \dot{q}_r - \frac{\eta_1}{\eta_2} \tilde{q}, \ddot{q}_r - \frac{\eta_1}{\eta_2} \dot{\tilde{q}} \right) \zeta = M(q) \left(\ddot{q}_r - \frac{\eta_1}{\eta_2} \dot{\tilde{q}} \right) + C(q, \dot{q}) \left(\dot{q}_r - \frac{\eta_1}{\eta_2} \tilde{q} \right)$$

From Eq. (9), the error dynamic Eq. (7) can then be rewritten as a compact form:

$$\dot{e} = T^{-1} \begin{bmatrix} \dot{l}(t) \\ \dot{\tilde{q}}(t) \end{bmatrix} = M_T(e, t)e + \eta_2 N_T(e, t)[- \Phi(e, t)\zeta + f] \quad (10)$$

where

$$M_T(e, t) = T^{-1} \begin{bmatrix} -H^{-1}(q)C(q, \dot{q}) & 0_{2 \times 2} \\ \frac{1}{\eta_2} I_{2 \times 2} & -\frac{\eta_1}{\eta_2} I_{2 \times 2} \end{bmatrix}$$

$$T, N_T(e, t) = T^{-1} V H^{-1}(q) \text{ with } V = \begin{bmatrix} I_{2 \times 2} \\ 0_{2 \times 2} \end{bmatrix}$$

and T is the state-space transformation matrix by

$$T = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ 0_{2 \times 2} & I_{2 \times 2} \end{bmatrix} = \begin{bmatrix} \eta_1 I_{2 \times 2} & \eta_2 I_{2 \times 2} \\ I_{2 \times 2} & 0_{2 \times 2} \end{bmatrix} \quad (11)$$

If the following applied torque input is chosen:

$$f = \Phi(e, t)\hat{\zeta} + \frac{1}{\eta_2} u_2 \quad (12)$$

then the tracking error dynamic equation driven by the control input u_2 will become

$$\dot{e} = M_T(e, t)e + N_T(e, t) \left[-\eta_2 \Phi(e, t)\tilde{\zeta} + u_2 \right] \quad (13)$$

where $\tilde{\zeta} = \hat{\zeta} - \zeta$ denotes the parameter estimation error.

3. DESIGN OBJECTIVE AND CONTROLLER DESIGN

3.1. The Nonlinear Adaptive H_2 Trajectory Tracking Control Problem

To consider the nonlinear autonomous mobile robot system of the form in Eq. (13), and given weighting matrices G_2 and U_2 , then the nonlinear adaptive trajectory tracking control problem with H_2 performance property is said to be solved if there exist a closed-form solution u_2^* and an adaptive law for $\hat{\zeta}$ such that the following performance index [8]:

$$\begin{aligned} J(u_2^*) &= \min_{u_2} J(u_2,) \\ &= \min_{u_2} \left[e^T(t_f)G_{2f}e(t_f) + \tilde{\zeta}^T(t_f)S_2\tilde{\zeta}(t_f) + \int_0^{t_f} [e^T(t)G_2e(t) + u_2^T(t)U_2u_2(t)] dt \right] \end{aligned} \quad (14)$$

can be achieved for all $t_f \in [0, \infty)$ and for some positive definite matrices $G_{2f} = G_{2f}^T$ and $S_2 = S_2^T > 0$. Totally speaking, the design objective is to derive a closed-form solution for control law and an adaptive law for system parameters that satisfy with performance index in Eq. (14) for all $t_f \in [0, \infty)$.

3.2. The Closed-Form Solution for Nonlinear Adaptive H_2 Control Design of Autonomous Mobile Robots

As mentioned above, a closed-form solution for control law and an adaptive law for system parameters should be derived analytically. For this purpose, a novel nonlinear adaptive control law with trajectory tracking capability for autonomous mobile robot will be described by nonlinear adaptive H_2 control theorem. The nonlinear H_2 control law and adaptive law are selected as

$$u_2^*(e, t) = -U_2^{-1}N_T^T(e, t)\Gamma_2(e, t)e(t) \quad (15)$$

$$\dot{\hat{\zeta}}(e, t) = -\eta_2 S_2^{-1}\Phi(e, t)N_T^T(e, t)\Gamma_2(e, t)e(t) \quad (16)$$

where $\Gamma_2(e, t) \geq 0$ can be obtained by solving the following nonlinear time-varying Riccati-like equation:

$$\dot{\Gamma}_2(e, t) + \Gamma_2(e, t)M_T^T(e, t) + M_T^T(e, t)\Gamma_2(e, t) + G_2 - \Gamma_2(e, t)N_T(e, t)U_2^{-1}N_T^T(e, t)\Gamma_2(e, t) = 0 \quad (17)$$

with $G_{2f} = J_2(e(t_f), t_f)$.

The solution of Eq. (17) is also the solution for the nonlinear adaptive H_2 trajectory tracking problem in Eq. (14).

In general, it is not easy to find out an analytical solution from Eq. (17). After some mathematical manipulations, the following nonlinear closed-form solution and adaptive law for Eqs. (15), (16) and (17) are derived analytically:

$$u_2^*(e, t) = -\frac{r}{a\sqrt{r^2 - a^2}}(g_{11}\tilde{q} + g_{22}\dot{\tilde{q}}) \quad (18)$$

$$\dot{\hat{\zeta}}(e, t) = -\frac{a^2 r^2 g_{22}}{b(r^2 - a^2)}\Phi(e, t)(g_{11}\tilde{q} + g_{22}\dot{\tilde{q}}) \quad (19)$$

where $0 < a < r/\sqrt{2}$ and $b > 0$. This means that the H_2 performance index in (14) can be achieved with Eqs. (14)–(15), for all $t_f \in [0, \infty)$.

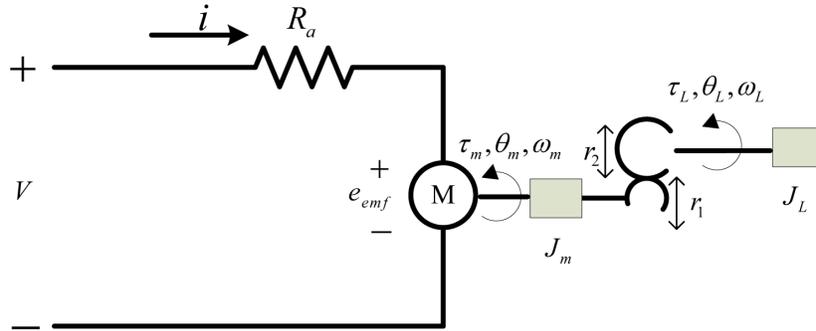


Fig. 2. The mathematical model for DC motors.

Table 1. The parameters of the autonomous mobile robot.

Parameters	Value	Units	Description
N	3.71		Gear ratio
K_a	3.49	$[\Omega]$	Terminal resistance
K_b	3.49	$[\text{mV}/\text{rpm}]$	Back-EMF constant
K_T	33.3	$[\text{mNm}/\text{A}]$	Torque constant

3.3. Actuator Dynamics

For the practical design purpose, dynamics of actuators, DC motors are briefly introduced as below. In this investigation, the wheels of autonomous mobile robot are driven by DC motors, and DC motor's circuit model is indicated in Fig. 2. By applying Kirchoff voltage law to the circuit, we have $V = iR_a + e_{emf}$ and

$$\tau_L = \frac{NK_T}{R_a}V - \frac{N^2K_T}{R_a}K_bw_L$$

where V is the input voltage, i is the motor current, R_a is the motor resistance, e_{emf} is the back emf (electromotive force), $N = r_2/r_1$ the gear ratio, τ_m and τ_L are the torque triggered by motor and on load, w_m (before gears) and w_L (after gears) are the angular velocities of the motor, θ_m (before gears) and θ_L (after gears) are the angles of the motor, and K_T and K_b are torque and back emf constant, respectively, hence the relationship between the optimal control toques and inputs of actuators can be described as

$$V = \frac{K_a}{NK_T} \left\{ \Phi(e, t) \hat{\zeta} + \frac{1}{\eta_2} u_2^*(e, t) + \frac{N^2K_T}{K_a} K_b w_L \right\} \quad (20)$$

4. PRACTICAL IMPLEMENTATION AND PERFORMANCE VERIFICATION

4.1. Overall Architecture of an Autonomous Mobile Robot

The proposed control method is realized on a wheel mobile robot with two driving wheels and one passive self-adjusted supporting system as shown in Fig. 3. The small passive self-adjusted support wheel is attached to the back of the vehicle to carry the framework of the mobile robot system. The driving wheels are driven by two individual DC motors, and the parameters of DC motor of autonomous mobile robot are shown in Table 1. In this research, the autonomous mobile robot system is practically implemented, and the overall control signals are divided into two parts: the personal computer side and the mobile robot side. The computer side consists of a personal computer and a bluetooth USB dongle. It provides an operation interface to issue preset up trajectories. The autonomous mobile robot side consists of a bluetooth



Fig. 3. Mobile robot system.

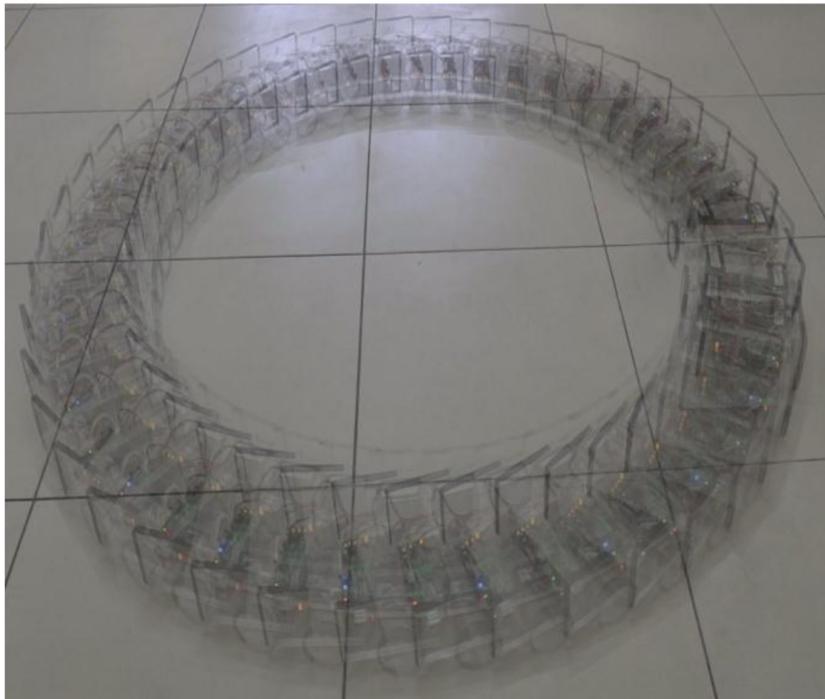


Fig. 4. Real-time tracking performance verification by tracking a predefined circular-type trajectory with radius of 60 cm.

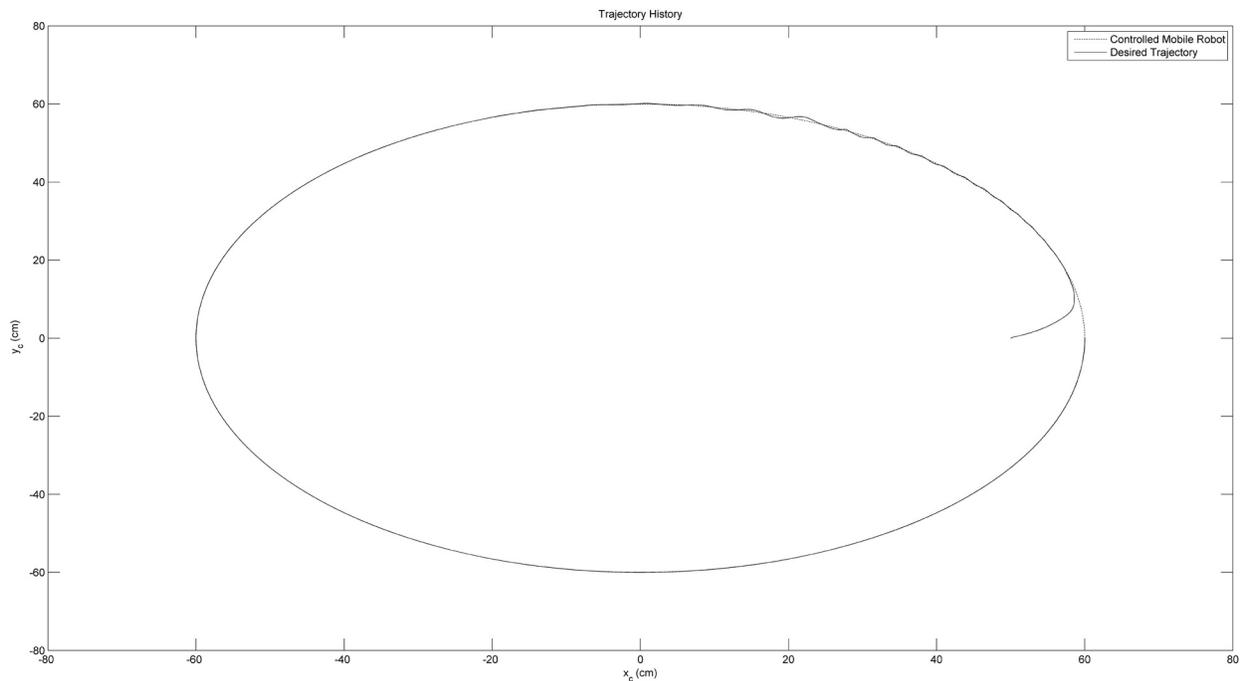


Fig. 5. Trajectories of the controlled mobile robot and the desired circular-type pattern.

RS232 adapter, a controller main board, a DC motor micro controller, two DC motors, a vehicle case and an MEMS-based inertial navigation system (INS) [9] that is utilized to fully measure the acceleration and angle rate, and then to generate velocity, position and the heading angle messages for the navigation of the autonomous mobile robot. In this study, the desk top personal computer interacts with mobile robot through a bluetooth USB dongle and bluetooth adapter. The controller main board consists of dsPIC 30F4011 microchip and performs proposed algorithm by MPLAB IDE and C30 C complier and WinPIC tool. The DC motor controller receives the control command to trigger the DC motor and then rotates the wheels of vehicle to move as the designate trajectory.

4.2. Performance Verification

In this section, performance of the proposed nonlinear adaptive H_2 control law will be verified with practical testing. In the experiment, in a scenario one circular trajectory is given to be the tracking pattern, and a radius of the above pattern is preset as 60 cm in a desktop personal computer. The proposed method is programmed in the main board, and an inertial navigation system (INS) [9] is used to measure/feedback the movement messages (position, velocity, and heading angle) of the autonomous mobile robot to the controller. Initially, the preset testing trajectory pattern is sent with the bluetooth module from the desktop personal computer, and when the controller in the main board receives this message, the proposed nonlinear adaptive H_2 controller based on this reference trajectory to control the DC motor directs the autonomous mobile robot to track the trajectory pattern until it achieves the tracking mission.

Figure 4 is the practical performance verification for the proposed method controlling the autonomous mobile robot in tracking a circular trajectory pattern, and Fig. 5 shows the recorded histories of the desired circular trajectory and controlled mobile robot trajectory. Obviously, the tracking performance in Fig. 5 matches the results of tracking errors are given in Fig. 6. For checking the proposed control performance, the same control process is repeated 10 times to the controlled mobile robot with respect to the above pattern, and a tracking error bound $|e| \leq 5$ cm always holds during the tracking period. From the testing results

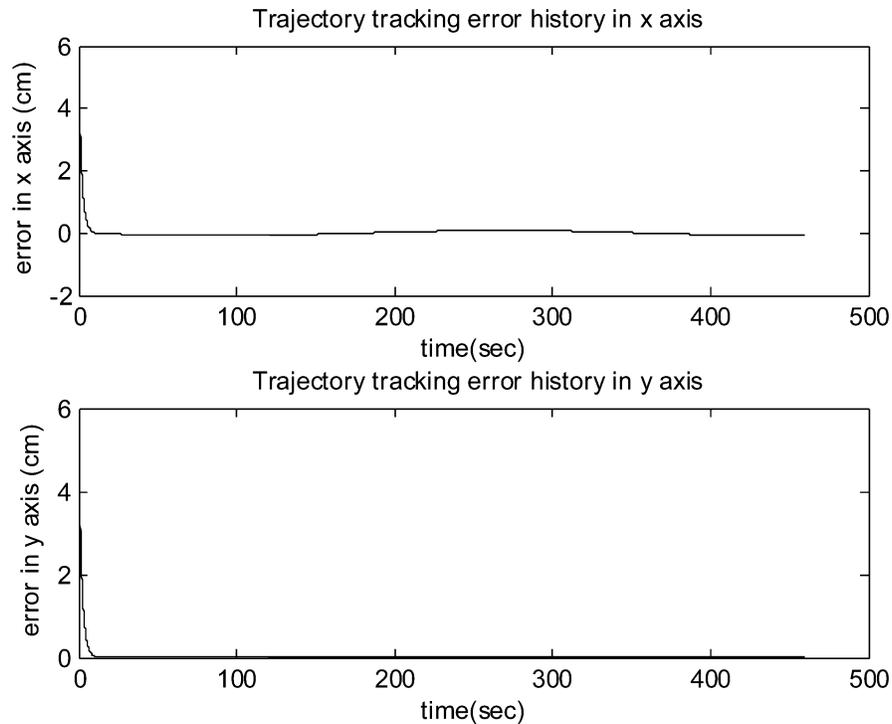


Fig. 6. Tracking error histories of the controlled mobile robot in x - y axis with respect to circular pattern.

shown in Figs. 4–6, it is obvious that the proposed adaptive nonlinear H_2 control law yields a satisfactory performance for precisely tracking any desired predefined trajectories.

5. CONCLUSIONS

A nonlinear adaptive H_2 control law is successfully developed for improving the trajectory tracking ability of autonomous mobile robots in this paper. The main contribution of this study is that an easily implemented closed-form solution is analytically derived for the nonlinear adaptive H_2 control problem of autonomous mobile robot. From practical verification it is obvious that this proposed method achieves satisfactory experimental results for tracking the desired trajectory with circular-type and possesses an excellent tracking capability for precisely directing the autonomous mobile robots to track predefined trajectories.

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