ROBUST FINITE TIME CONTROLLER DESIGN FOR SECOND ORDER NONLINEAR UNDERACTUATED MECHANICAL SYSTEMS

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ABSTRACT
For a class of second order underactuated mechanical systems, a robust finite time control strategy is developed in this paper. The robust finite time controller is to drive the tracking error to be zero at the fixed final time. In order to assure system stability, we present a generalized Lyapunov stability proof for the second order underactuated mechanical system. By utilizing a Lyapunov stability theorem, we can achieve finite time tracking of desired reference signals for underactuated systems, which are subject to both external disturbances and system uncertainties. The proposed control scheme is demonstrated by actual experiments on a Furuta pendulum system.

Keywords: underactuated mechanical systems; finite time control; Furuta pendulum system.

CONCEPTION D'UN CONTRÔLEUR ROBUSTE EN TEMPS FINI POUR LES SYSTÈMES MÉCANIQUES NONLINÉAIRES

RÉSUMÉ
Pour une classe de systèmes mécaniques sous-actionnés de seconde catégorie, une stratégie de commande robuste en temps fini est développée dans cette étude. Le rôle de la commande robuste en temps fini est d'amener l'erreur de trajectoire à zéro en temps déterminé final. Afin d'assurer la stabilité du système, nous présentons une preuve de stabilité générale Lyapunov pour système mécanique sous-actionné de seconde catégorie. En utilisant le théorème de stabilité Lyapunov, nous pouvons réaliser en temps fini la trajectoire des signaux de référence désirés. Le schème de commande proposé est démontré par des expériences courantes sur un système de pendule Furuta.

Mots-clés : système mécanique sous-actionné ; commande en temps fini ; pendule de Furuta.

NOMENCLATURE

I_o	Inertia of the arm (kg·m^2)
L_o	Total length of the arm (m)
M_1	Mass of the arm (kg)
M_2	Mass of the pendulum (kg)
l_1	Distance to the center of gravity of the pendulum (m)
J_1	Inertia of the pendulum around its center of gravity (kg·m^2)

1. INTRODUCTION

Stabilization and tracking control of nonlinear uncertain underactuated systems are always challenging problems because underacturated systems have fewer independent control actuators than degrees of freedom to be controlled. The precise description is given as $\ddot{q} = f(q,\dot{q}) + b(q)u$ This system is described to be underactuated if it is satisfied by the condition $\mathrm{rank}(b) < \dim(q)$, where q is the state vector of generalized coordinates on the configuration manifold, $\dot{q}$ is the generalized velocity vector, and $b(q)$ is the input state vector.

The underactuated system generally exhibits non-minimum phase property [1]. The non-minimum phase property has long been recognized to be a major obstacle in many control problems. It is well known that unstable zeros cannot be moved with state feedback while the poles can be arbitrarily placed (if completely controllable). Much research discussing the control of underactuated systems have been published in the past years. Wang, and other researchers presented a new sliding mode controller for second order underacturated systems [2, 3]. In [4, 5] the authors introduced an IDA-PBC approach to the underactuated mechanical systems. In [6] some nonlinear control schemes, such as feedback linearization, inverse dynamics, and adaptive controller design, have been proposed for the control of underactuated systems. In order to achieve better performance, possibly unknown dynamics must be also considered in controller design. The sliding mode control was proposed to be a robust control scheme applicable to power systems, electronic motors, and robot manipulators [7]. In [8] the nonlinear control design was presented, but it requires the solution of a Hamilton–Jacobian type differential equation. It indeed renders the solutions to the system robustly asymptotically stable. In order to reach even better tracking performance, our goal in this paper is to develop a robust finite time controller is to drive the tracking error to be zero ($e(t) = 0$) at a fixed final time. In fact, finite time convergence implies non-smooth or non-Lipschitz continuous autonomous systems with non-uniqueness of solutions. The problem of finite time stabilization was raised in [9]. This characterization is used to develop a class of second order finite time systems which can be used as controller design. Some researches dealt with the finite time stability by using the Lyapunov function approach [10, 11]. Bhat and Bernstein presented a finite time stabilizing feedback controller for the translational and rotational double integrators [12, 13]. In [14] the finite time stability by using the Lyapunov function was discussed. In [15] an integral backstepping control system using adaptive recurrent neural network uncertainty observer (RNNUO) was proposed to control a PMLSM drive for the tracking of periodic reference inputs. With the proposed integral backstepping control system, the mover position of the PMLSM drive possesses the advantages of good transient control performance and robustness to uncertainties for the tracking of periodic reference trajectories. In [16] a new fuzzy-skyhook controller of an active suspension system has been presented. This theoretical investigation includes a suggestion of an active suspension system controller using fuzzy-skyhook control theory, which offers new opportunities for the improvement of vehicle ride performance. Oscillation systems have been widely used in many areas of engineering. In [17] the Max-Min Method was utilized for solving the nonlinear oscillation problems. Manipulator and robot systems possess several specific qualities in both mechanical and control senses. To this end, a computer algebra system VIBRAN was used. In [18] it is shown that such application could drastically reduce the number

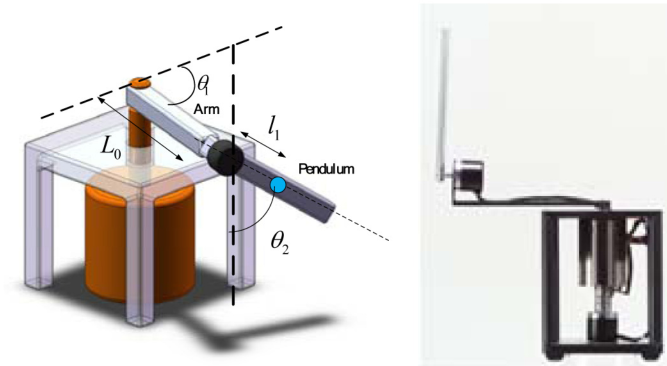

Fig. 1. The Furuta pendulum system.

of floating point product operations which are required for efficient numerical simulation of piezoelectric robots.

In this paper, a new control scheme is presented to achieve the globally finite time stability in uncertain environment for a class of underactuated mechanic systems. Its applications to the Furuta pendulum system (Fig. 1) are chosen to demonstrate the validity and the effectiveness of the proposed control approach.

2. DIFFERENTIAL INCLUSIONS AND STABILITY

The finite time convergence in controller design only exists in non-smooth or non-Lipschitz continuous autonomous systems that own uniqueness of solution. The objective of this section is to provide a finite time differential equation for finite time stability and finite stabilization, which will be applied to design of the finite time controller. The finite time stabilization is given below for non-Lipschitzian autonomous systems.

Definition [19]. Consider an autonomous system $\dot{x} = f(x)$, where $f : D \to R^n$ is non-Lipschitz continuous on an open neighborhood $D \subseteq R^n$ of the origin in R^n. The origin of $\dot{x} = f(x)$ is finite time convergent if there exists an $N \subseteq D$ of the origin and a function $T_x : N\backslash\{0\} \to (0,\infty)$ called the setting time function such that every solution trajectory $x(x_0,t)$ of $\dot{x} = f(x)$ starting from the initial point $x_0 \in [0, T_x(x_0))$, brings about $\lim\limits_{t \to T_x(x_0)} = x(x_0,t)$.

To illustrate the finite time stability for the unified sliding mode control scheme presented later, two examples with exponential stability and the finite time stability are discussed below. Consider the unique solution of an autonomous system $\dot{x} = f(x) = -x$; $f(0) = 0$. The function f appears to be locally Lipschitz continuous. By using the Lyapunov function $V = x^2$, the system is assured to be asymptotically stable but not finite time stable. The solution trajectory of the locally Lipschitz continuous system is described by $x(t) = e^{-t}$. To achieve the finite time stability, there is a philosophical issue associated with the design of the controller for non-Lipschitz systems that is uniqueness of solutions of the closed system. Consider a system expressed by $\dot{x} = -|x|^{1/n}\,\mathrm{sgn}(x)$ [20] and $x(0) = C \in R$ is a finite time differential equation and the first derivative of its right hand side is continuous everywhere except zero. The solution can be described by

$$x(t) = \left\{ \begin{array}{ll} [|C|^{\frac{n-1}{n}} - \frac{(n-1)t}{n}]^{\frac{n}{n-1}}\,\mathrm{sgn}(C) & 0 \le t \le \frac{n|C|^{\frac{n-1}{n}}}{n-1} \\ 0 & t > \frac{n|C|^{\frac{n-1}{n}}}{n-1} \end{array} \right\}. \tag{1}$$

The corresponding setting time function T_x is $n\left|C\right|^{\frac{n-1}{n}}/(n-1)$. It can be shown that a non-Lipschitz system is finite time asymptotically stable, if it holds continuous property. Therefore, in order to reach the finite time stability, the finite time controller needs to have the closed loop system either non-smooth or non-Lipschitz.

3. MODELING OF SECOND ORDER UNDERACTUATED SYSTEMS

The motivation in this paper is to continue the development of the control theory for underactuated systems. Underactuated mechanical systems have fewer independent control actuators than degrees of freedom to be controlled. For this reason, a second order dynamic model for mechanical systems with uncertain parameters has the following form:

$$\begin{aligned} \dot{x}_1 &= x_2, \quad \dot{x}_2 = (f_1(x)+\Delta f_1(x)) + (b_1(x)+\Delta b_1(x))u + d_1(t), \\ \dot{x}_3 &= x_4, \quad \dot{x}_4 = (f_2(x)+\Delta f_2(x)) + (b_2(x)+\Delta b_2(x))u + d_2(t), \end{aligned} \tag{2}$$

where $f_1(x)$, $b_1(x)$, $f_2(x)$ and $b_2(x)$ are nonlinear functions, x is defined as $x=[x_1,x_2,x_3,x_4]^T$. The function $d(t)=[d_1(t)\ \ d_2(t)]^T$ is an unknown bounded disturbance with a known magnitude bound vector ε, i.e., $\sup_{t\geq 0}\|d(t)\|\leq\varepsilon$. The uncertainties are bounded by known nonlinear functions as follows:

$$\|\Delta f_1(x)\| \leq \sum_{i=1}^{l} \alpha_{1i}^T \omega_{1i}(x), \tag{3}$$

$$\|\Delta f_2(x)\| \leq \sum_{k=1}^{p} \alpha_{3k}^T \omega_{3k}(x), \tag{4}$$

$$\|\Delta b_1(x)\| \leq \sum_{j=1}^{r} \beta_{2j}^T \omega_{2j}(x), \tag{5}$$

$$\|\Delta b_2(x)\| \leq \sum_{h=1}^{w} \beta_{4h}^T \omega_{4h}(x), \tag{6}$$

where the coefficients $\alpha_{1i,}$ $\alpha_{3k,}$ β_{2j} and β_{4h} are real uncertain parameters that $(\lambda_1,\lambda_2,\lambda_3,\lambda_4)\in R$ and $\|\alpha_{1i}\|\leq\lambda_1, \|\alpha_{3k}\|\leq\lambda_2, \left\|\beta_{2j}\right\|\leq\lambda_3, \|\beta_{4h}\|\leq\lambda_4$ hold. The bounded functions $\omega_{1i}(x)^T$, $\omega_{2j}(x)^T$, $\omega_{3k}(x)^T$ and $\omega_{4h}(x)^T$ are known positive functions of x with appropriate dimensions.

4. CONTROLLER DESIGN

For the robust finite time underactuated controller it is assumed that all the system states and the bounded functions are available. Let the error signal vector be defined as

$$\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} x_1 - x_{1d} \\ x_2 - \dot{x}_{1d} \end{bmatrix}, \quad \begin{bmatrix} e_3 \\ e_4 \end{bmatrix} = \begin{bmatrix} x_3 - x_{3d} \\ x_4 - \dot{x}_{3d} \end{bmatrix}, \tag{7}$$

where $x_{1d}, \dot{x}_{1d}, x_{3d}$ and $\dot{x}_{3d}$ are bounded reference commands. The error dynamics is given by a new manifold and let $1/\Lambda \in]0,1[$

$$S = \dot{e} + ce^{1/\Lambda} = \begin{bmatrix} \dot{e}_1 + c_1\left|e_1\right|^{1/\Lambda}\mathrm{sgn}(e_1) \\ \dot{e}_3 + c_2\left|e_3\right|^{1/\Lambda}\mathrm{sgn}(e_3) \end{bmatrix} = \begin{bmatrix} S_1 \\ S_2 \end{bmatrix}, \tag{8}$$

where c_1 and c_2 are positive constants. Using Eq. (7), (2) can be rewritten as

$$\begin{aligned} \dot{e}_1 &= e_2, \quad \dot{e}_2 = (f_1(x)+\Delta f_1(x)) + (b_1(x)+\Delta b_1(x))u + d_1(t) - \dot{x}_{1d}, \\ \dot{e}_3 &= e_4, \quad \dot{e}_4 = (f_2(x)+\Delta f_2(x)) + (b_2(x)+\Delta b_2(x))u + d_2(t) - \dot{x}_{3d}. \end{aligned} \tag{9}$$

Now, let the control law be defined as $u = u_1 + u_2 + u_{sw}$. Both u_1 and u_2 are nonlinear compensations, and u_{sw} is used to estimate the underactuation and parametric uncertainties. Finally the control laws are presented as

$$\begin{cases} u_1 = b_1(x)^{-1}(-f_1(x) - \frac{d}{dt}\Upsilon_1 - k_a S_1 - \operatorname{sgn}(S_1)\left[\sum_{i=1}^{l} \alpha_{1i}^T \omega_{1i}(x) + C_{d1}\right] + \dot{x}_{1d}) & \dot{e}_1 \neq 0, \\ u_1 = b_1(x)^{-1}(-f_1(x) - k_a S_1 - \operatorname{sgn}(S_1)\left[\sum_{i=1}^{l} \alpha_{1i}^T \omega_{1i}(x) + C_{d1}\right] + \dot{x}_{1d}) & \dot{e}_1 = 0, \end{cases} \tag{10}$$

$$\begin{cases} u_2 = b_2(x)^{-1}(-f_2(x) - \frac{d}{dt}\Upsilon_2 - k_b S_2 - \operatorname{sgn}(S_2)\left[\sum_{k=1}^{p} \alpha_{3k}^T \omega_{3k}(x) + C_{d2}\right] + \dot{x}_{3d}) & \dot{e}_3 \neq 0, \\ u_2 = b_2(x)^{-1}(-f_2(x) - k_b S_2 - \operatorname{sgn}(S_2)\left[\sum_{k=1}^{p} \alpha_{3k}^T \omega_{3k}(x) + C_{d2}\right] + \dot{x}_{3d}) & \dot{e}_3 = 0, \end{cases} \tag{11}$$

The function $[\Upsilon_1 \Upsilon_2]^T$ is continuous but non-smooth and differentiable, although the absolute value and sign operators are involved. Its generalized derivative can be expressed by

$$\frac{d}{dt}\begin{bmatrix} \Upsilon_1 \\ \Upsilon_2 \end{bmatrix} = \begin{bmatrix} (\frac{c_1}{n})\,|e_1|^{(\frac{1}{\Lambda}-1)}\,\mathrm{SGN}(e_1)\dot{e}_1 \\ (\frac{c_2}{n})\,|e_3|^{(\frac{1}{\Lambda}-1)}\,\mathrm{SGN}(e_3)\dot{e}_3 \end{bmatrix}, \quad e_1 \neq 0, e_3 \neq 0. \tag{12}$$

The SGN$(\cdot)$ function is defined as

$$\mathrm{SGN}(e) = \begin{cases} -1 & e < 0, \\ [-1, 1] & e = 0, \\ +1 & e > 0, \end{cases} \tag{13}$$

and

$$u_{sw} = k(S_1 + S_2). \tag{14}$$

The control gain k is designed to estimate the underactuation and parametric uncertainties which leads to

$$\dot{k} = \begin{cases} \Gamma k^{(1/2n+1)}\Omega & k > 0, \\ \sigma & k = 0, \end{cases} \tag{15}$$

where

$$\Omega = -u_{sw}(b_1 S_1 + b_2 S_2) - (S_1 b_1 u_2 + S_2 b_2 u_1) - |S_1| \sum_{j=1}^{r} \beta_{2j}^T \omega_{2j}(x)|u| - |S_2| \sum_{h=1}^{w} \beta_{4h}^T \omega_{4h}(x)|u|, \tag{16}$$

From Eq. (16) σ is a small positive constant and a measure zero exists for k, i.e., $\{t_i\}_{i=1}^{\infty}$ such that $k(t_i) = 0$, $i = 1, 2, 3, \ldots, \infty$.

Theorem. Consider a class of second order underactuated mechanical systems and the system parameters are not precisely known. Then, by applying the control laws given in (10)–(16), globally finite time convergence tracking results can be guaranteed in the sense that all trajectories inside the closed-loop system are bounded $S \to 0$ and $e(t, x, x_d) = 0$ in finite time.

In order to prove the theorem, we present a generalized Lyapunov stability proof for the second order underactuated mechanical system in the next section.

5. STABILITY ANALYSIS

The Lyapunov function is chosen as below, which satisfies the properties $V(0)=0$, $V(S)\geq 0$ and $V(\infty)\to\infty$

$$V(S_1,S_2)=\begin{cases} 1/2S_1^2+1/2S_2^2+(2n+1)/2n\Gamma^{-1}k^{\frac{2n}{2n+1}}, & \text{if } e_1\neq 0, e_3\neq 0,\\ 1/2(\dot{e}_1)^2+1/2S_2^2+(2n+1)/2n\Gamma^{-1}k^{\frac{2n}{2n+1}}, & \text{if } e_1=0, e_3\neq 0,\\ 1/2S_1^2+1/2(\dot{e}_3)^2+(2n+1)/2n\Gamma^{-1}k^{\frac{2n}{2n+1}}, & \text{if } e_1\neq 0, e_3=0,\\ 1/2(\dot{e}_1)^2+1/2(\dot{e}_3)^2+(2n+1)/2n\Gamma^{-1}k^{\frac{2n}{2n+1}}, & \text{if } e_1=0, e_3=0. \end{cases} \tag{17}$$

Taking the time derivative of V can lead to

$$\begin{aligned}\dot{V} \leq\ & -k_aS_1^2+S_1\sum_{i=1}^{l}\alpha_{1i}^T\omega_{1i}(x)-|S_1|\sum_{i=1}^{l}\alpha_{1i}^T\omega_{1i}(x)+S_1b_1(x)(u_2+u_{sw})\\ & +\sum_{j=1}^{r}\beta_{2j}^T\omega_{2j}(x)uS_1-|S_1|C_{d1}+d_1(t)S_1-k_bS_2^2+S_2\sum_{k=1}^{p}\alpha_{3k}^T\omega_{3k}(x)\\ & -|S_2|\sum_{k=1}^{p}\alpha_{3k}^T\omega_{3k}(x)+S_2b_2(x)(u_1+u_{sw})+\sum_{h=1}^{w}\beta_{4h}^T\omega_{4h}(x)uS_2-|S_2|C_{d2}+d_2(t)S_2\\ & +\left[-u_{sw}(b_1S_1+b_2S_2)-(S_1b_1u_2+S_2b_2u_1)-|S_1|\sum_{j=1}^{r}\beta_{2j}^T\omega_{2j}(x)\,|u|-|S_2|\sum_{h=1}^{w}\beta_{4h}^T\omega_{4h}(x)\,|u|\right]\\ \leq\ & -C_{d1}\,|S_1|-C_{d2}\,|S_2|-k_aS_1^2-k_bS_2^2. \end{aligned} \tag{18}$$

It can be clearly concluded that $\dot{V}(t)\leq 0$ whenever C_{d1} and C_{d2} are positive definite. The solution $k(t)$ of Eq. (15) is well defined and is continuous for all $t\geq 0$. Suppose that $k(t)=0$ will occur abruptly at t_i, it appears that $V(t)$ remains to be continuous at t_i. From the hypothesis, $\dot{V}(t_i^-)<0$ and $\dot{V}(t_i^+)<0$, we hence can conclude that $V(t)$ is non-increasing in t including t_i. Integrating both sides yields

$$\int_0^{t_i}\dot{V}d\tau=-\int_0^{t_i}C_{d1}|S_1|d\tau-\int_0^{t_i}C_{d2}|S_2|d\tau-\int_0^{t_i}k_aS_1^2d\tau-\int_0^{t_i}k_bS_2^2d\tau, \tag{19}$$

such that

$$V(t_i)=V(0)-\int_0^{t_i}C_{d1}|S_1|d\tau-\int_0^{t_i}C_{d2}|S_2|d\tau-\int_0^{t_i}k_aS_1^2d\tau-\int_0^{t_i}k_bS_2^2d\tau\leq V(0)<\infty. \tag{20}$$

Based on Eqs. (19) and (20), $S=0$ can be implied. Therefore, it can be concluded that $\dot{e}+c(|e|^{1/\mathrm{n}}\,\mathrm{sgn}(e))=0$ is a conventional non-Lipschitz system. This system trajectory will converge to the origin with a property of finite time stability. It is clear that there exists a C^1 generalized Lyapunov function for this system and the system has the property of a finite time convergence.

6. EXPERIMENT

The experiment is performed for demonstration of the effectiveness of the proposed controller. In this underactuated mechanical system, the actuator is a DC motor mounted on an arm (link #1) and coupled to links through the power MOSFET chopper amplifier. A 500 pulse/rev shaft encoder is used to sense the arm (link #1) position and the pendulum (link #2) position. A 12-bit A/D and D/A converter provides the required signal. The control objective here is to move the arm from the position 0° to 23° where the link #2

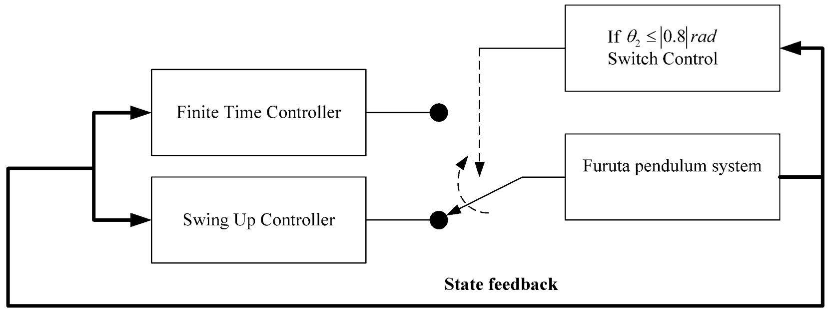

Fig. 2. Block diagram of the control system for the experiments.

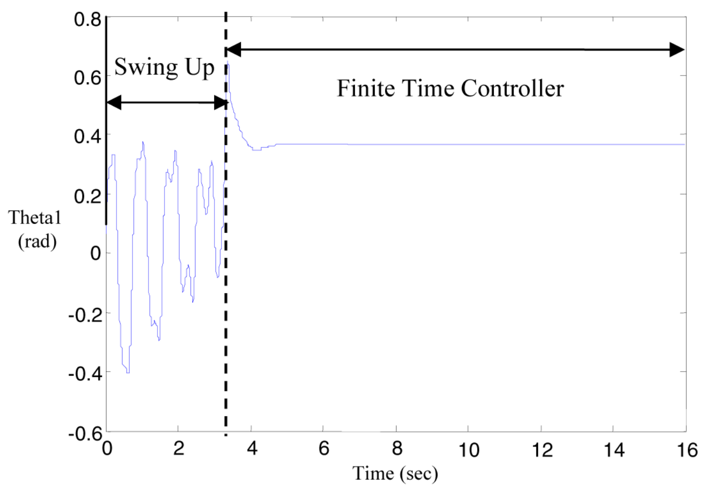

Fig. 3. Position of the link #1 (arm).

is vertically downward ± 180° to an unstable equilibrium position 0° where the link #2 is vertically upward. In order to balance the pendulum on the top unstable equilibrium point, the swing up controller must make sure the pendulum's position is close to ± 46°. When the pendulum's position is within the range of ± 46° with respect to the vertical direction, the finite time controller will be switched on so that the pendulum will be balanced at the unstable equilibrium point. At that moment, the swing up controller will be turned off. If the finite time controller is not successful, the swing up controller will be automatically switched on when the pendulum is beyond the range of ±46°.The swing up control law [1] can be written as

$$u_{sw}(t) = \begin{cases} 0 & t = 0 \\ sw_1 & t_{i+1} > t_i > 0 \\ -sw_2 & t_{i+2} > t_{i+1} \end{cases} \quad \text{where } sw_1 \text{ and } sw_2 \text{ are positive constants.}$$

As shown in Fig. 2, when the pendulum is within ±46° (±0.8 rad) with respect to the vertical position, the balancing controller is activated. Otherwise, the system is operating in the swing up control mode. Figures 3 and 4 depict the position responses of the arm and the pendulum. These curves clearly show that the arm converges to the desired position at 5 sec, and the pendulum reaches at the top unstable position at 4 sec. Finally, the response of the control input is illustrated in Fig. 5. It is seen that the finite time controller is activated when the pendulum is within ±0.8 rad apart from the vertical position. For this reason, if the swing up controller is switched to the nonlinear finite time controller, it might bring large control input to

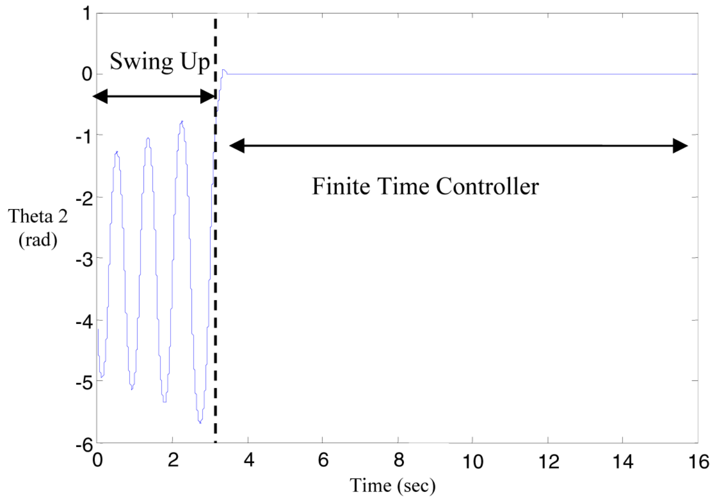

Fig. 4. Position of the link #2 (pendulum).

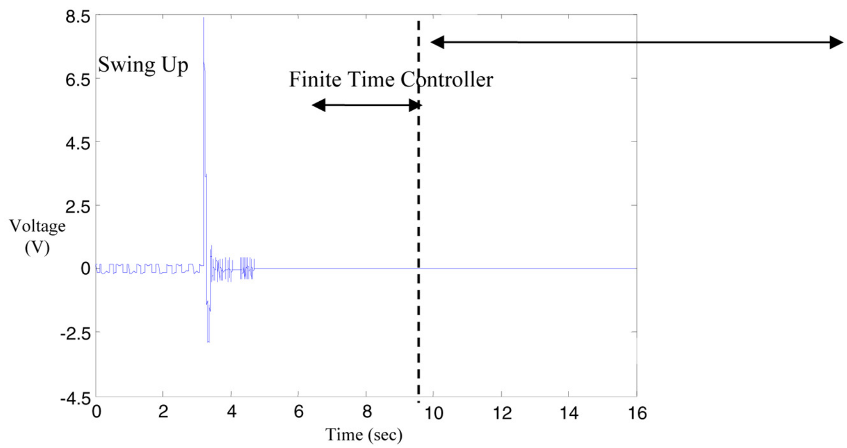

Fig. 5. Control input.

balance the pendulum as shown in Fig. 5. It appears that the proposed finite time controller and the swing up control provide stable performance over a wide range of parameter variations:

I_0: Inertia of the arm (0.001569 kg·m^2)
L_0: Total length of the arm (0.16 m)
M_1: Mass of the arm (0.056 kg)
M_2: Mass of the pendulum (0.022 kg)
l_1: Distance to the center of gravity of the pendulum (0.08 m)
J_1: Inertia of the pendulum around its center of gravity (0.0001785 kg·m^2)

7. CONCLUSIONS

For a class of second order underactuated mechanical systems, a robust finite time control framework is developed in this paper. By utilizing Lyapunov-based stability, the finite time tracking of desired reference commands can be guaranteed, which is subject to both underactuation and parametric uncertainties. The control scheme is also implemented in the Furuta pendulum system to verify the performance of the proposed robust finite time controller. Based on the experimental results, the finite time convergence of system errors can be assured.

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