CONSTRUCTION OF AN ELEMENTARY MODEL FOR THE DYNAMIC ANALYIS OF A PRESSURIZED WATER REACTOR

Chiung-Wen Tsai[1], Chunkuan Shih[1,2] and Jong-Rong Wang[3]

[1] *1Institute of Nuclear Engineering and Science, National Tsing Hua University, Taiwan*

[2]*Department of Engineering and System Science, National Tsing Hua University, Taiwan*

[3]*Institute of Nuclear Energy Research, Taiwan*

E-mail: d937121@oz.nthu.edu.tw

ICETI 2012-I1706_SCI

No. 13-CSME-46, E.I.C. Accession 3504

ABSTRACT

A lumped-parameter numerical model was constructed based on the conservation laws of mass and energy and the point neutron kinetics with 6 groups of delayed neutron to represent the dynamics of a pressurized water reactor core. On the viewpoint of control theory, the coupled phenomenon of neutron kinetics and thermal-hydraulics can be recognized as a dynamic system with feedback loops by the Doppler effect and the coolant temperature. Scilab was implemented to construct the equivalent transfer functions and associated feedback loops of a PWR core. The dynamic responses were performed by the perturbations of positive reactivity insertion, coolant flow rate, and coolant inlet temperature. This elementary PWR core model has been qualitatively assessed against independent numerical data to ensure the models and codes have a certain level of confidence or validation.

Keywords: PWR core model; Scilab; dynamic analysis.

CONSTRUCTION D'UN MODÈLE ÉLÉMENTAIRE D'UN RÉACTEUR À EAU SOUS PRESSION POUR UNE ANALYSE DYNAMIQUE

RÉSUMÉ

On a construit un modèle numérique de type lumped parameter en se basant sur les lois de la masse et de l'énergie, la cinétique ponctuelle du neutron et la thermodynamique pour représenter la dynamique du cœur d'un réacteur à eau sous pression. Du point de vue de la commande théorique, le phénomène de la cinétique du neutron et de la thermodynamique peut être reconnu comme un système avec boucles de rétroaction par l'effet Doppler et la température de l'agent de refroidissement. *Scilab* a été implanté pour la construction des fonctions de transferts équivalents et des boucles de rétroaction associées du cœur d'un réacteur à eau sous pression. La réponse dynamique se produit par les perturbations de l'insertion de la réactivité positive, l'indice d'écoulement du liquide réfrigérant, la température d'entrée de l'agent de refroidissement. La qualité de ce modèle du cœur d'un réacteur élémentaire à eau sous pression a été comparée à des données numériques indépendantes pour assurer que les modèles et les codes ont un certain niveau de certitude ou de validation.

Mots-clés : réacteur à eau sous pression ; Scilab ; analyse dynamique.

NOMENCLATURE

A	total heat transfer area (m^2)
C	delay neutron precursor or specific heat capacity (J/kg)
G	Transfer function
h	convection heat transfer coefficient (W/(m^2K))
k	constant
m	mass
n	neutron flux
P	thermal power
T	temperature
Greek symbols	
β	delay neutron yield fraction
δ s	mall perturbation term
Λ	neutron generation time (s)
λ	decay constant (s^{-1})
ρ	reactivity
Subscripts	
c	coolant
cav	averaged property of coolant
f	fuel
i	group number
in	inlet
th	quanty generated by fission reaction
out	outlet

1. INTRODUCTION

Typically, the system dynamics of a reactor is simulated by the complex codes consisting of the one-dimensional or three-dimensional neutronics [1], the correlations of two-phase follow [2], critical flow and thermal properties, and the built in control system blocks. Those codes such as RELAP, RETRAN, and TRACE, are applied for safety analyses, plant design, and predictions of postulated tests [3–5]. Although those advanced codes are programmed to calculate the systematic parameters during significant transients or severe accidents, the computing time is proportional to the nodolization of reactor models and corresponding transients. Instead, for those simulations associated with reactor control during normal operation and abnormal transients induced by perturbations, simplified models may be adopted for the investigation of dynamic responses and control system design to reduce the computing cost. One of the approaches derives the equivalent transfer functions of a reactor system, investigating the reactor kinetics and associated control systems. This approach has been implemented since 1950. A brief review of the history of transfer function on nuclear engineering is referred to the textbook by Harrer [6]. So far, this approach is widely utilized in nuclear applications such as the determination of neutron generation time [7] and boiling water reactor (BWR) stability analysis [8].

This study applied the above approach and Scilab to construct an elementary pressurized water reactor (PWR) core model which simulates the dynamic responses induced by the perturbations of reactivity insertion, coolant flow rate, and coolant inlet temperature. Scilab [9] is a Matlab-like free software, developed by the French National Institute for Research in Computer Science and Control (INRIA) in 2003. Scilab provides a platform with numerous embedded tool boxes for scientific computations [10] to represent transfer functions and simulate frequency-domain as well as time-domain analyses. This PWR core model was

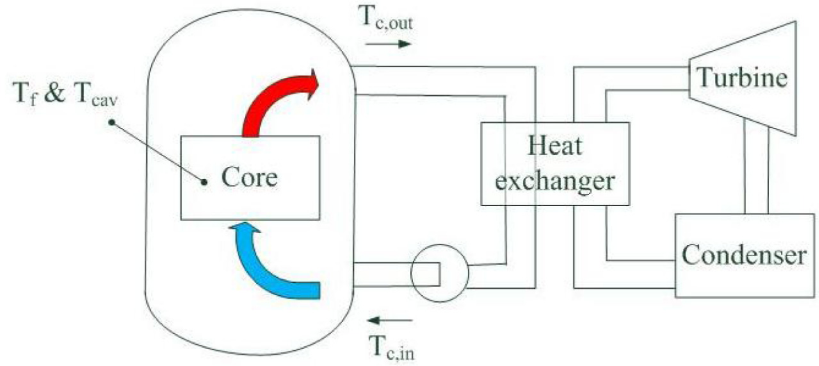


Fig. 1. An elementary block diagram of a PWR.

assessed against independent data generated by a reference PWR model to ensure that it has a certain level of confidence.

2. ASSUMPTIONS AND MODELING

An elementary block diagram of a PWR is shown in Fig. 1. The present model simulates the reactor core, and the thermal-hydraulic conditions of inlet and outlet are taken as boundary conditions. This simplified mathematical model lumps the reactor as a single point, and incorporates point kinetics and energy conservation equations to represent the neutron dynamics, energy transfer between fuel and coolant, and reactivity feedback loop. The method to derive the reactor transfer functions from time derivatives is based on the small perturbation theorem and Laplace transform, obtaining a linear approximation for these departures form a steady operation state.

2.1. Neutron Kinetics

Inserting perturbation terms of neutron flux and delayed neutron precursors into point kinetic equation, we obtain

$$\frac{d(n_0+\delta n)}{dt} = \frac{(\rho_0+\delta\rho-\beta)}{\Lambda}(n_0+\delta n)+\sum_i \lambda_i(C_{i,0}+\delta C_i), \tag{1}$$

$$\frac{d(C_{i,0}+\delta C_i)}{dt} = \frac{\beta_i}{\Lambda}(n_0+\delta n)-\lambda_i(C_{i,0}+\delta C_i) \tag{2}$$

where n is the neutron flux, ρ is reactivity, C_iu is the delayed neutron precursor of group i, Λ is the neutron regeneration time, β_i is the delayed neutron yield fraction of group i, β is the total delayed neutron yield fraction, and λ_i is the decay constant of group i.

Taking Laplace transform of Eqs. (1)–(2), and further substitution yields the transfer function,

$$\frac{\delta\tilde{n}}{n_0} = \left[\Lambda s+\sum_i\left(\frac{\beta_i s}{s+\lambda_i}\right)\right]^{-1}\delta\tilde{\rho} = G_n\delta\tilde{\rho}. \tag{3}$$

2.2. Reactor Core

It is supposed that the fuel coolant temperatures in core are represented by average temperatures, and the thermal-dynamic properties such as specific heat capacity and density are constant. The heat transfer between fuel and coolant and the energy balance of coolant inventory are represented by Eqs. (4)–(5), respectively. The coolant temperature in core is assumed to be the average value of inlet temperature and outlet temperature, as in Eq. (6).

$$m_f C_f \frac{d(T_f + \delta T_f)}{dt} = P_{\text{th},0} + \delta P_{\text{th}} - hA(T_{f,0} + \delta T_f - T_{\text{cav}}), \tag{4}$$

$$\begin{aligned} m_c C_{\text{cav}} \frac{d(T_{\text{cav},0} + \delta T_{\text{cav}})}{dt} &= hA(T_{f,0} + \delta T_f - T_{\text{cav},0} - \delta T_{\text{cav}}) \\ &+ (\dot{w}_0 + \delta \dot{w}) C_{\text{cav}} (T_{c,\text{in},0} + \delta T_{c,\text{in}} - T_{c,\text{out},0} - \delta T_{c,\text{out}}), \end{aligned} \tag{5}$$

$$T_{\text{cav}} = \frac{1}{2}(T_{c,\text{in}} + T_{c,\text{out}}). \tag{6}$$

where m_f and m_c are the total fuel mass and total coolant mass, respectively, C_f and C_{cav} are the specific heat capacities of fuel and coolant, respectively, T_f and T_{cav} are the average temperatures of fuel and coolant, respectively, P_{th} is the average fission power, h is the convection heat transfer coefficient, A is the total heat transfer area, $T_{c,in}$ and $T_{c,out}$ are the inlet and outlet temperatures, respectively.

The transfer functions of fuel and coolant Eqs. (7)–(8) are generated by Laplace transform of Eqs. (4)–(5),

$$\delta \tilde{T}_f = \frac{1}{(m_f C_f s + hA)} \delta \tilde{P}_{\text{th}} = G_{\delta P \text{th}} \delta \tilde{P}_{\text{th}} \tag{7}$$

$$\begin{aligned} \delta \tilde{T}_{\text{cav}} &= \frac{hA}{J_1} \delta \tilde{T}_f + \frac{2C_{\text{cav}} \Delta T_0}{J_1} \delta \dot{w} + \frac{2C_{\text{cav}} \dot{w}_0}{J_1} \delta \tilde{T}_{c,\text{in}} \\ &= G_{\delta T f} \delta \tilde{T}_f + G_{\delta w} \delta \dot{w} + G_{\delta T c,\text{in}} \delta \tilde{T}_{c,\text{in}}, \end{aligned} \tag{8}$$

where $J_1 = m_c C_{\text{cav}} s + hA + 2C_{\text{cav}} \dot{w}_0$, $\Delta T_0 = T_{c,\text{in},0} - T_{\text{cav},0}$.

The fission power, P_{th}, is assumed to be proportional to neutron flux, n, as

$$(P_{\text{th},0} + \delta P_{\text{th}}) = k(n_0 + \delta n), \tag{9}$$

where k is a proportional constant.

The value of k is calculated by fission power and neutron flux in steady state, $P_{\text{th},0}$ and n_0. Therefore, the perturbation term of fission power is represented by

$$\delta P_{\text{th}} = k \delta n = \left(\frac{P_{\text{th},0}}{n_0}\right)\left(\frac{\delta \tilde{n}}{n_0}\right) = P_{\text{th},0} G_n \delta \tilde{\rho}. \tag{10}$$

2.3. Control Blocks

Combining Eqs. (3), (7)–(8), (10), the closed loop transfer function with three external perturbation sources representing the coupling of neutronics and thermal-hydraulics of reactivity feedback is shown in Fig. 2. Those transfer functions were programmed by Scilab for perturbation analyses.

3. PERTURBATION ANALYSIS AND ASSESSMENT

3.1. Perturbation Analysis

The technical parameters of a reference PWR applied for perturbation analysis are listed in Table 1 [11]. The delayed neutron data listed in Table 2 for thermal fission in U-235 are referred to [12]. The dynamic responses are performed by the perturbation sources including reactivity, coolant inlet flow, and coolant inlet temperature.

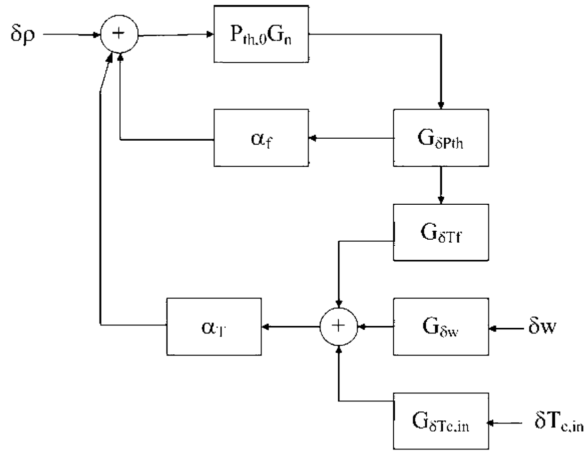


Fig. 2. The closed loop transfer function of reactor core.

Table 1. Technical parameters of a reference PWR.

Parameters	Values	Parameters	Values
Rated thermal power	1933 [MWt]	Primary circuit volume	239 [m^3]
Primary coolant flow rate	9190 [kg/s]	Heat transfer surface	4170 [m^2]
Coolant inlet temperature	279.5 [°C]	Fuel mass	66.9e3 [kg]
Coolant outlet temperature	315.6 [°C]	Fuel Specific heat	247 [J/kg°C]
Average coolant temperature	297.55 [°C]	Average fuel temperature	652 [°C]
Coolant Specific heat	5680 [J/(kg*^{0}k)]	Coolant density	718.4 [kg/m^3]

Table 2. Delayed neutron data.

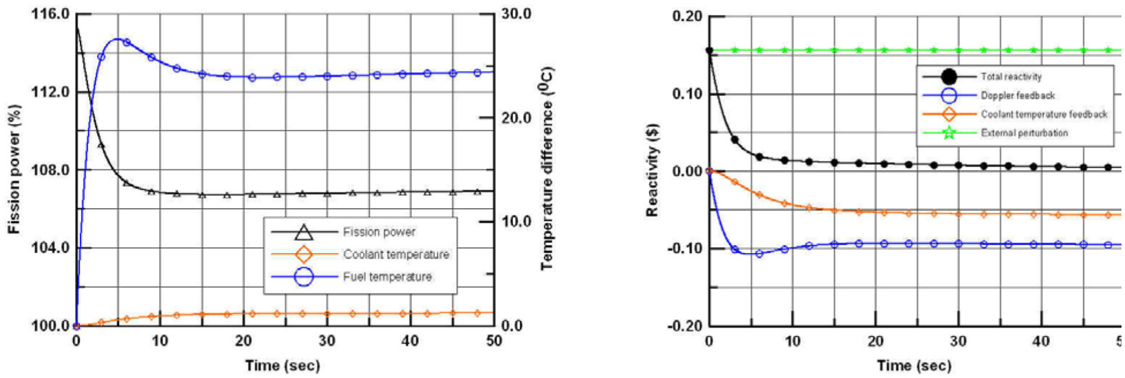
Group	Decay constant	Yield	Yield fraction
1	0.0124 [s^{-1}]	0.00052	0.000215
2	0.0305 [s^{-1}]	0.00346	0.001424
3	0.111 [s^{-1}]	0.00310	0.001274
4	0.301 [s^{-1}]	0.00624	0.002568
5	1.14 [s^{-1}]	0.00182	0.000748
6	3.01 [s^{-1}]	0.00066	0.000273

3.1.1. Step Increase of Reactivity

This transient is initiated by a step increase of reactivity of 0.001 which causes the rapid power increase (Fig. 3a). The fuel and coolant temperatures increase with a time delay due to thermal inertia (Fig. 3b). Consequently, as shown in Fig. 3b, the negative reactivity feedback by Doppler effect (fuel temperature) and coolant temperature mitigate the total reactivity as well as fission power.

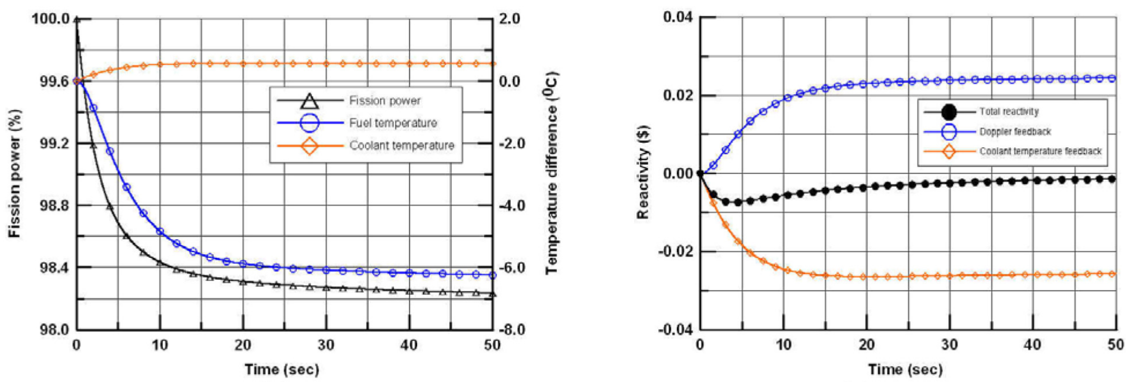
3.1.2. Step Decrease of Inlet Flow Rate

The step decrease of inlet flow rate of 10% leads to the negative reactivity feedback as a result of the increase of coolant temperature. Therefore, the neutron flux is decreased due to the negative reactivity feedback (Fig. 4a). Figure 4a also represents the decrease of fuel temperature because of the reduction of neutron flux (fission power). Consequently, the decrease of fuel temperature causes a positive reactivity feedback, as



(a) (b)

Fig. 3. (a) The fission power and temperature differences; (b) the reactivity values.



(a) (b)

Fig. 4. (a) The fission power and temperature differences; (b) the reactivity values.

shown in Fig. 4b. Thus, the negative reactivity caused by the perturbation of inlet flow rate is compensated by the Doppler effect.

3.1.3. Step Decrease of Coolant Inlet Temperature

Compared with the results by the step decrease of inlet flow rate, those by the step decrease of inlet coolant temperature lead to the increase of neutron flux (Fig. 5a) as a result of positive reactivity caused by the reduction of coolant temperature (Fig. 5a). However, the increase of fuel temperature caused by fission power leads to the negative reactivity by Doppler effect (Fig. 5b). Therefore, as shown in Fig. 5b, the total reactivity is the competition of positivity reactivity by coolant temperature and negative reactivity by Doppler effect.

3.2. Assessment

Assessment of numerical models and codes against independent data sources ensures the models and codes have a certain level of confidence or validation. Because of the lack of experimental data, one of the approaches is to compare the numerical models and codes with other similar numerical simulations qualitatively. Therefore, the numerical results of a PWR model [13] which simulates the transient induced by the perturbation of positivity reactivity is employed as the reference data to assess our model. However, the simulated reactors and corresponding design values and thermal-hydraulic conditions are different. The as-

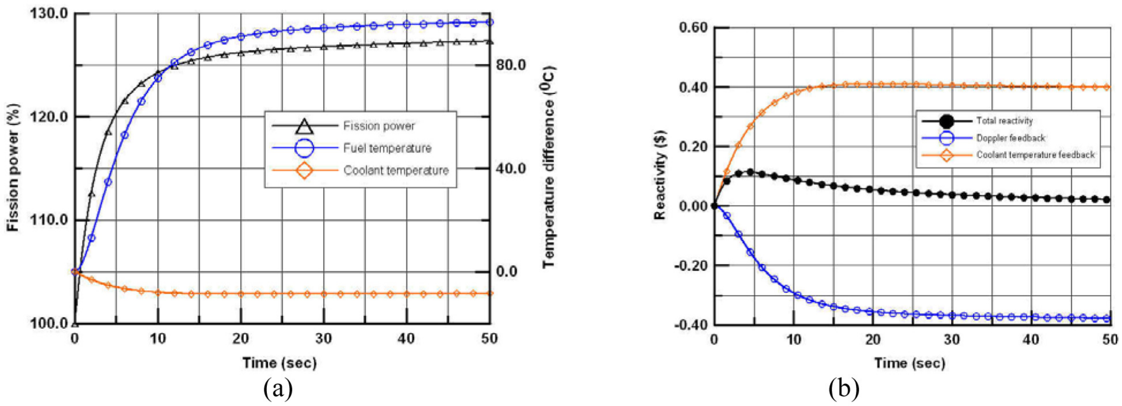


Fig. 5. (a) The fission power and temperature differences; (b) the reactivity values.

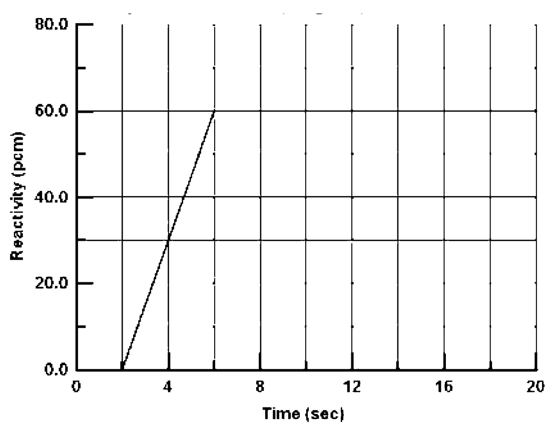


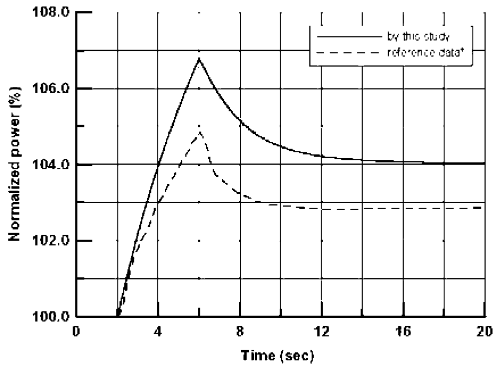
Fig. 6. Positive reactivity insertion.

sessment focuses on the comparison of the trends of normalized power and fuel temperature increase caused by the same positive reactivity insertion (Fig. 6).

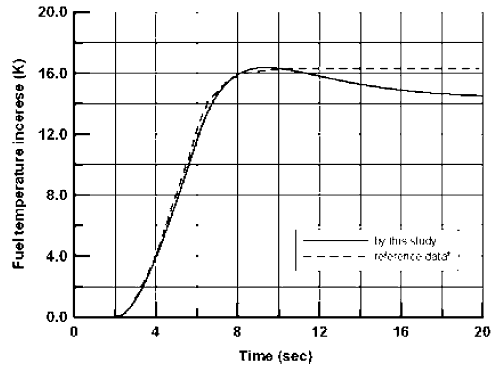
The comparisons of normalized power and fuel temperature increases are shown in Fig. 7. The maximum difference of normalized power is around 2% (Fig. 7a). The relatively large normalized power by this study may be caused by the less temperature and Doppler reactivity feedbacks which depress the effect of positivity reactivity insertion. The fuel temperature increases (Fig. 7b) before 8 s are almost the same, but a deviation of around 2 K is observed at 20 s. That is because the fuel temperature by this study has not reach another steady state before the end of calculation. In other words, the behavior of fuel temperature is dominated by thermal inertia, and the corresponding parameters include the coolant inventory, conductivity and specific heat of fuel. Therefore, the different material properties and thermal conditions cause the slight deviations in fuel temperature increase. The assessment results show that the PWR core model constructed by Scilab in this study may represent the fundamental dynamics of a PWR core properly in trend during the transients induced by small perturbation of reactivity.

4. CONCLUSIONS

This study implements Scilab to construct a numerical model which represents the equivalent transfer function of a PWR core. Compared with complex codes of safety analysis of nuclear reactors, the numerical



(a)



(b)

Fig. 7. Comparison results: (a) normalized power; (b) fuel temperature increase.

model in this study is designed to solve the core dynamics during abnormal transients induced by small perturbation and save computing time without sacrificing the precision of numerical results.

The demonstrations are performed by the perturbation sources of reactivity insertion, coolant flow rate, and coolant inlet temperature to represent the dynamic responses of a reference PWR core. Besides, this elementary PWR core model has been qualitatively assessed against independent numerical data to ensure the models and codes have a certain level of confidence or validation. The maximum differences of normalized power and fuel temperature are around 2% and 2 K, respectively. It supposes that the different material properties and thermal conditions cause the slight deviations in normalized thermal power and fuel temperature increase.The assessment results indicate that the PWR core model is capable to represent the fundamental dynamics of a PWR core well in trend during the transients induced by small perturbation of reactivity.

REFERENCES

1. Downar, T. et al., *PARCS, U.S. NRC Neutronics Simulator User Manual*, v3.0, NSNRC, 2010.
2. Tong, L.S. and Tang, Y.S., *Boiling Heat Transfer and Two-Phase Flow*, 2nd ed., Taylor & Francis, 1997.
3. Salah, A.B. and D'Auria, F., "Analysis of the peach bottom flow stability test number 3 using coupled RELAP/PARCS code", *Annals of Nuclear Energy*, Vol. 33, pp. 646–652, 2006.
4. Tsai, C.W., Shih, C., Wang, J.R. and Cheng, S.C., "The design assessment of limiting transient event by RETRAN-02 and VIPRE-01 for Lungmen ABWR", *Nuclear Engineering and Design*, Vol. 241, pp. 942–949, 2011.
5. Freixa, J. and Manera, A., "Verification of a TRACE EPRTM model on the basis of a scaling calculation of an SBLOCA ROSA test", *Nuclear Engineering and Design*, Vol. 241, pp. 888–896, 2011.
6. Harrer, J.M., *Nuclear Reactor Control Engineering*, pp. 275–276, D. Van Mostrand Company, 1963.
7. Hainoum, A. and Khamis, I., "Determination of neutron generation time in miniature neutron source reactor by measurement of neutronics transfer function", *Nuclear Engineering and Design*, Vol. 195, pp. 299–305, 2000.
8. Morishima, N., Tashiro, S. and Takeuchi, Y., "BWR core-wide stability analysis with respect to dominant feedback mechanisms and characteristic poles", *Annals of Nuclear Energy*, Vol. 33, pp. 43, 2006.
9. Information on http://www.scilab.org
10. Campbell, S.L., Chancelier, J.P. and Nikoukhah, R., *Modeling and Simulation in Scilab/Scicos*, Springer, 2000.
11. International Atomic Energy Agency, *Pressurized water reactor simulator*, IAEA, 2003.
12. Lamarsh, J.R. and Baratta, A.J., *Introduction to Nuclear Engineering*, 3rd ed., Prentice Hall, 2001.
13. Marseguerra, M., Zio, E. and Canetta R., "Using genetic algorithms for calibrating simplified models of nuclear reactor dynamics", *Annals of Nuclear Energy*, Vol. 31, pp. 1219–1250, 2004.