

## GEAR FAULT DIAGNOSIS IN TIME DOMAINS VIA BAYESIAN NETWORKS

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### ABSTRACT

In gear or rolling bearing systems, it is difficult to extract symptoms from vibration signals where shock vibration signals are present. However, the neural network method cannot provide satisfactory diagnosis results without adequate training samples. Bayesian networks provide an effective approach for fault diagnosis in cases given uncertain and incomplete information. In this study, the statistical factors of vibration signals in the time-domain were used and the diagnosis results by using Bayesian networks were superior to other neural network methods.

**Keywords:** gear fault diagnosis; statistical, neural network; Bayesian network; time domain; frequency domain.

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### DIAGNOSTIQUE DE DÉFAILLANCE DU DÉRAILLEUR DANS LE DOMAINE TEMPOREL VIA LE RÉSEAU BAYÉSIEN

#### RÉSUMÉ

Dans un système de dérailleur ou de roulement mécanique, il est difficile d'extraire des symptômes des signaux de vibration quand ceux-ci sont présents. Cependant, la méthode de réseaux neuronaux ne peut apporter des résultats de diagnostic satisfaisants sans des échantillons d'entraînement adéquats. Les réseaux bayésiens peuvent procurer une approche efficace pour le diagnostic de défaillance dans certains cas où l'information est incertaine ou incomplète. Dans cette étude, les facteurs statistiques de signaux de vibration dans le domaine temporel ont été utilisés, et les résultats ont été supérieurs à d'autres méthodes de réseaux neuronaux.

**Mots-clés :** diagnostic de défaillance du dérailleur ; réseau neuronal ; réseau bayésien ; domaine temporel ; domaine des fréquences.

## NOMENCLATURE

$\sigma$	standard deviation
$E$	evidence
$F$	the existence of a fault
$H$	hypothesis
$M_i$	the $i$ -th order moment
$O_i$	the different gear faults
$P$	the posterior probability for $H$ proving information that evidence $E$
$S_i$	six statistical factors of vibration signals in the time-domain
$X$	the existence of fault symptoms
$x(n)$	the vibration signal in time-domain
$X_A$	allowance factor
$X_C$	crest factor
$X_I$	impulse factor
$X_K$	kurtosis factor
$X_{rms}$	root mean square value of vibration signal in time domain
$X_W$	waveform
$ \bar{X} $	mean value

## 1. INTRODUCTION

Gearboxes are widely used in rotary machinery to transmit power. By definition, gear fault diagnosis is a differentiation of faults from vibration signals based on expert knowledge or experience when the gear system breaks down. Meaningful information on the machine's state can be extracted from vibration signals via accelerometers, and artificial intelligence analysis of this information can identify and diagnose symptoms to classify faults in the gear system. For rotary machinery, the most common method of fault diagnosis is frequency spectral analysis, but this method has difficulty extracting symptoms from shock vibration signals in gear train system in the frequency domain. Dyer and Stewart [1] used the statistical analysis of vibration signals in the time-domain for fault diagnosis in rolling element bearings. Symptoms of vibration signals in the time-domain could clearly display a fault diagnosis in gear train systems. Heng and Nor [2] identified different kinds of defects in monitoring rolling bearing systems by using statistical factors in the time-domain, such as kurtosis, crest, skewness factors, etc.

It is difficult to manage a rule database for a diagnosis fault system according to traditional methods such as fuzzy inference, closeness method, etc. Traditional grading outcomes are influenced by the grader's subjectivity and experience, as are the weighting parameters. Differing rules from various experts result in inconsistent diagnosis results. Back-propagation neural networks (BPNN) are widely used to solve fault diagnosis problems. Kang et al. [3] extracted frequency symptoms from vibration signals to detect faults by using BPNN in a motor bearing system. Probabilistic neural network (PNN) is different from other supervised neural networks in that its weightings don't alternate with training samples. The output values of PNN are obtained by once-forward network computation. Lin et al. [4] employed PNN to identify faults in dissolved gas content, using the gas ratios of the oil and cellulosic decomposition to create training examples. Yang et al. [5] adopted PNN to detect faults in two analog circuit examples.

A method was developed to make effective use of vibration signals to detect fault in gear train systems by combining the statistical analysis of vibration signals in the time-domain with artificial neural networks.

Based on the Bayesian principle, probability can be estimated based on the probability of previous samples. Bayesian networks incorporate expert knowledge and historical data in the revision of prior beliefs based on new evidence. This method is well used for insufficient-data problems, and has proven useful for a variety of monitoring and predictive problems. Applications have been documented mainly in medical

treatment [6], gas turbine engines testing [7], and industrial fault diagnosis [8]. A few examples have been developed for fault location in power delivery systems. Chien et al. [9] used expert knowledge and historical data as the basis for a Bayesian network for fault diagnosis on a distribution feeder.

In conclusion, identify faults in a gear train system through frequency spectrum analysis is difficult because the components of the frequency spectrum are complex and ambiguous. Insufficient training samples result in inaccurate diagnosis results. Also, the trained samples may not include diagnostic samples. Vibration signal variations in the time-domain are distinct to differentiate faults in the gear train system. This study thus applied the statistical analysis of vibration signals in the time-domain and established Bayesian networks for fault diagnosis in gear train systems. The performances of the Bayesian networks were compared with that of BPNN and PNN.

## 2. SIGNAL PARAMETERS IN THE TIME-DOMAIN

Statistical analysis of vibration signals in the time-domain (e.g., kurtosis, crest factors, etc.) delivers good classification results for shock or pulse signals.

In this study, fault diagnosis is accomplished through statistical factors of vibration signals in the time-domain including waveform, crest, impulse, allowance, kurtosis and skewness.

- (a) The waveform factor denotes the shift in the time waveform and corresponds to the ratio between the root mean square (rms) value and the mean value of the signal:

$$\text{Waveform } (X_W) = \frac{\text{rms value}}{\text{mean value}} = \frac{X_{\text{rms}}}{|\bar{X}|} \quad (1)$$

- (b) The crest factor shows the peak height in the time waveform and corresponds to the ratio between the max value and the rms value of the signal:

$$\text{Crest } (X_C) = \frac{\text{max value}}{\text{rms value}} = \frac{\max |X|}{X_{\text{rms}}} \quad (2)$$

- (c) The impulse factor indicates the shock in the time waveform and corresponds to the ratio between the max value and the mean value of the signal:

$$\text{Impulse } (X_I) = \frac{\text{max peak}}{\text{mean value}} = \frac{\max |X|}{|\bar{X}|} \quad (3)$$

- (d) The allowance factor is the indication of plenty in the time waveform and corresponds to the ratio between the max value and  $X_r$  of the signal:

$$\text{Allowance } (X_A) = \frac{\text{max peak}}{X_r}, X_r = \left( (1/N) \sum_{n=1}^N x(n)^{1/2} \right)^2 \quad (4)$$

where  $x(n)$  is time waveform of vibration signal.

- (e) The skewness and kurtosis factors are both sensitive indicators of the signal shape, and are relative to the third and fourth moments of the signal distribution in the time-domain, respectively. The skewness factor corresponds to the moment of third order norm of the vibration signal:

$$\text{Skewness } (X_3) = \frac{M_3}{\sigma_3} \quad (5)$$

$$M_3 = (1/N) \sum_{n=1}^N (x(n) - \bar{X})^3, \quad \sigma = (1/N) \sum_{n=1}^N (x(n) - \bar{X})^2)^{1/2} \quad (6)$$

where  $M_3$  is the third order moment, and  $\sigma$  is standard deviation. The kurtosis factor corresponds to the moment of fourth order norm of the vibration signal:

$$\text{Kurtosis}(X_K) = \frac{M_4}{\sigma^4}, \quad M_4 = (1/N) \sum_{n=1}^N (x(n) - \bar{X})^4 \quad (7)$$

where  $M_4$  is the fourth order moment.

Since these statistical factors have been used as inputs to the BPNN, which used sigmoid function to activated function, the inputs and outputs of the diagnostic network can be normalized into a range from 0 to 1 and are classified into several levels, except for the skewness factor which is normalized into a range from -1 to 0. If three levels are chosen, one-third and two-thirds of total sample can be designated as the level limits of the low, medium and high levels.

### 3. BAYESIAN NETWORKS

Artificial neural networks have difficulty correctly classifying faults without enough training samples, but a well-suited Bayesian inference can overcome this problem. Given a hypothesis  $H$  and evidence  $E$ , Bayes' theorem can be expressed as

$$P(H|E) = \frac{P(E|H) \cdot P(H)}{P(E)} \quad (8)$$

where  $P(H|E)$  is the posterior probability for  $H$  providing information that evidence  $E$  is true,  $P(H)$  is the prior probability of  $H$ , and  $P(E|H)$  is the probability for evidence  $E$  giving providing information that the hypothesis  $H$  is true. The Bayesian network is a directed acyclic graph that consists of probabilistic relationships among the variables. In this study, a Bayesian network is designed to diagnose faults in a gear train system. As shown in Fig. 1a, the network combines with six nodes ( $S_1 \sim S_6$ ), which represent six statistical factors of vibration signals in the time-domain, and six nodes ( $O_1 \sim O_6$ ) which indicate the different gear faults. Although the structure of Bayesian networks is similar to that of BPNN and PNN, as shown in Fig. 1b, the algorithm of Bayesian is different from that of neural networks. In BPNN and PNN, the connections between the input and hidden layers, and the hidden and output layers are weighting coefficients. The difference between BPNN and PNN is the neuron numbers of the hidden layer, which are forty and sixty, respectively. In Bayesian networks, the connections between the inputs (statistical factors of vibration in the time-domain) and outputs (gear faults) are probability. In this study, the probability of gear fault via the Bayesian theorem is expressed as

$$P(F|X) = \frac{P(X|F) \times P(F)}{P(X|F) \times P(F) + P(X|\bar{F}) \times P(\bar{F})} \quad (9)$$

where  $F$  indicates the existence of a fault, and  $X$  indicates the existence of fault symptoms. The  $P(X|F)$  is the probability for symptom  $X$  indicating that fault  $F$  is true and  $P(X|\bar{F})$  is the probability for symptom  $X$  indicating that fault  $F$  is false. The  $P(X|F)$  and  $P(X|\bar{F})$  can be respectively expressed as

$$P(X|F) = P(X_W|F)P(X_C|F)P(X_I|F)P(X_A|F)P(X_K|F)P(X_S|F) \quad (10)$$

$$P(X|\bar{F}) = P(X_W|\bar{F})P(X_C|\bar{F})P(X_I|\bar{F})P(X_A|\bar{F})P(X_K|\bar{F})P(X_S|\bar{F}) \quad (11)$$

The maximum value obtained by Eq. (9) represents the greatest possibility of gear fault.

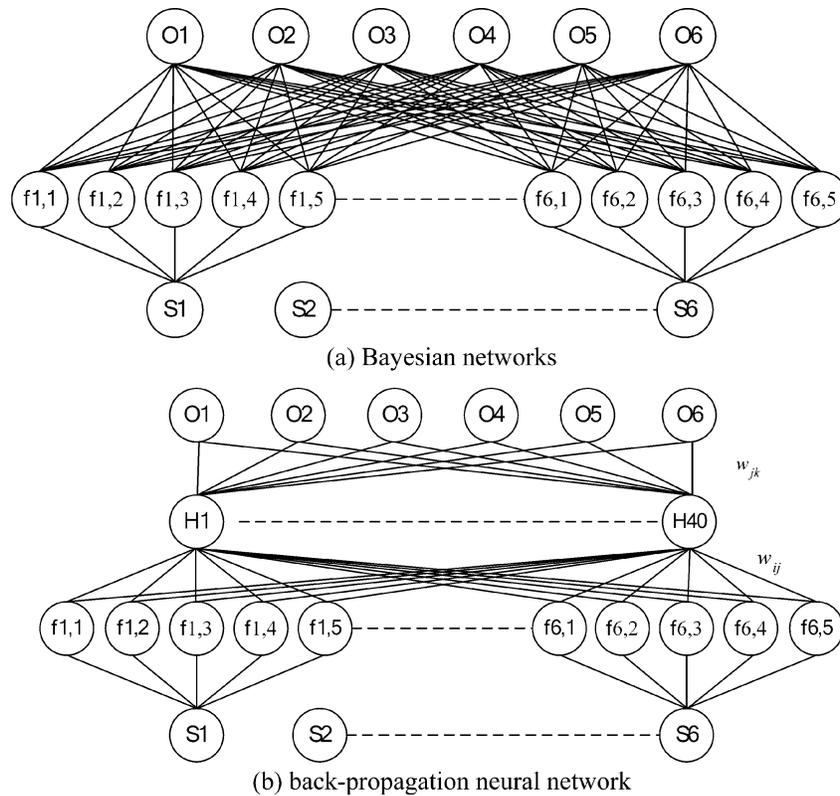


Fig. 1. The diagnostic network structure.

## 4. CASE STUDIES

### 4.1. Experiment Set-up

As shown in Fig. 2, the experimental equipment for the gear train system consists of a motor, a converter and a pair of spur gears, in which the transmitting gear has 46 teeth and the passive gear has 30 teeth. This study investigates, six kinds of health statuses in the gear train system, namely, (a) normal gear (denoted as  $O_1$ ), (b) worn gear ( $O_2$ ), (c) tooth breakage ( $O_3$ ), (d) rough surface ( $O_4$ ), (e) rough surface with tooth breakage ( $O_5$ ), and (f) rough surface with worn gear ( $O_6$ ).

### 4.2. Feature Extraction

The training samples are obtained by simulating corresponding gear faults on an experimental rotor-bearing system. The vibration signals are measured vertically by two accelerometers mounted on the bearing housing of the gear train system. These factors will be used as the model inputs after first being normalized into a range from 0 to 1 and classified into several levels. Choosing too many levels may cause redundant computation while too few may lead to ambiguous classifications. To obtain explicit classifications, in this study, the statistical factors are classified into five equal levels (i.e., one-fifth, two-fifths, etc.) which are designated as five level limits, namely, very small (VS), small (S), medium (M), large (L) and very large (VL). Taking  $X_W$  as an example, values smaller than 1.2665 are classified as VS (1 0 0 0 0), while values from 1.2665 to 1.2749, 1.2749 to 1.2942, and 1.2942 to 1.3296, are respectively designated as S (0 1 0 0 0), M (0 0 1 0 0), and L (0 0 0 1 0) respectively, and values larger than 1.3296 are designated as VL (0 0 0 0 1). Based on the membership function, for test No. 1 in diagnostic sample 1, six statistical factors 1.28990( $X_W$ ), 4.87021( $X_C$ ), 6.18745( $X_I$ ), 8.25908( $X_A$ ), 2.83640( $X_K$ ), and  $-1.37831(X_S)$  are respectively classified as levels

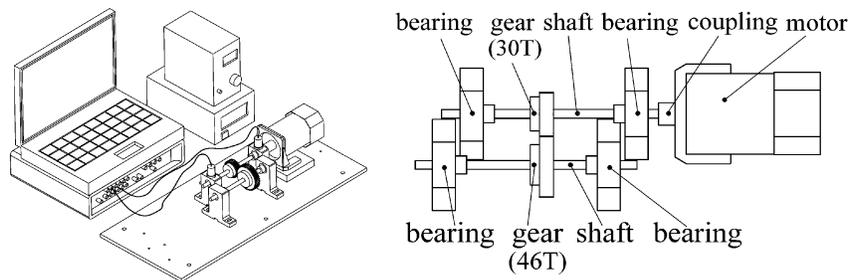


Fig. 2. Gear train system experimental setup.

Table 1. Certainty of diagnosis results due to BPNN and PNN.

Sample No.	Test No.	Faults	$\alpha_1$		$\alpha_2$		$\alpha_3$		$\alpha_4$		$\alpha_5$		$\alpha_6$	
			B <sup>▲</sup>	P*										
1	1		0.97	1	0.01	0	0.11	0	0.01	0	0.09	0	0.01	0
	2 <sup>▲*</sup>		0.13	0	0.18	1	0.29	0	0	0	0.01	0	0.02	0
	3		0.80	1	0.03	0	0.04	0	0	0	0.01	0	0.02	0
2	1		0	0	0.91	1	0.06	0	0.05	0	0.03	0	0	0
	2		0.01	0	0.76	1	0.08	0	0.01	0	0.23	0	0.01	0
	3 <sup>▲</sup>		0	0	0.23	1	0.33	0	0.12	0	0.09	0	0	0
3	1*		0	0	0.03	1	0.93	0	0.04	0	0.19	0	0	0
	2 <sup>▲*</sup>		0	0	0.03	0	0.23	0	0.64	1	0.09	0	0.01	0
	3 <sup>▲*</sup>		0.04	0	0.04	0	0.38	0	0.48	1	0.07	0	0	0
4	1*		0.04	0	0.15	1	0.25	0	0.81	0	0.04	0	0.01	0
	2 <sup>▲*</sup>		0	0	0.80	1	0.36	0	0.49	0	0.03	0	0.05	0
	3*		0.04	0	0.15	1	0.25	0	0.81	0	0.04	0	0.01	0
5	1*		0.01	0	0.13	0	0.01	0	0.04	1	0.73	0	0	0
	2		0	0	0.01	0	0	0	0.10	0	0.85	1	0	0
	3		0.07	0	0.01	0	0	0	0.06	0	0.32	1	0.03	0
6	1		0.02	0	0.04	0	0.03	0	0	0	0.04	0	0.96	1
	2 <sup>▲</sup>		0.03	0	0.32	0	0.16	0	0.05	0	0.01	0	0.23	1
	3		0	0	0.01	0	0.04	0	0	0	0.09	0	0.97	1

▲: represents wrong diagnosis result using BPNN  
 \*: represents wrong diagnosis result using PNN  
 B<sup>▲</sup>: represents BPNN diagnosis results  
 P\*: represents PNN diagnosis results

M (0 0 1 0 0), S (0 1 0 0 0), S (0 0 1 0 0), S (0 1 0 0 0), S (0 1 0 0 0), and L (0 0 0 1 0).

### 4.3. Neural Network Diagnosis

The input layer of the neural network consists of six nodes which including waveform, crest, impulse, allowance, kurtosis and skewness factors. Table 1 lists the BPNN diagnosis results (B<sup>▲</sup>) obtained with the trained weighting coefficients  $w_{ij}$  and  $w_{jk}$ . The value represents the certainty for a corresponding gear fault. Values close to 1 indicate a high possibility of fault. Based on the diagnostic samples, six diagnosis results were wrong. Similarly, the PNN diagnosis results (P\*) included eight wrong diagnosis results, as shown in Table 1. Though there are several advantages to using artificial neural networks (e.g., neural network weightings are obtained from neural computation and the diagnosis results are more objective than traditional expert experience), the resulting diagnoses are not satisfactory when the diagnostic samples are not within the range of the trained samples.

Table 2. Certainty of diagnosis result via Bayesian networks.

Sample No.	Test No.	Faults					
		$a_1$	$a_2$	$a_3$	$a_4$	$a_5$	$a_6$
1	1	0.99	0	0.19	0	0.19	0
	2	0.84	0	0	0	0	0
	3	0.72	0	0	0	0	0
2	1	0.02	0.95	0.32	0	0	0
	2	0.16	0.93	0.24	0	0	0
	3	0.02	0.87	0.30	0	0	0
3	1	0	0	0.61	0	0	0
	2	0.05	0	0.77	0	0.05	0
	3	0.21	0	0.88	0	0.01	0
4	1	0	0	0	0.95	0	0
	2	0	0	0	0.69	0	0
	3	0	0	0	0.94	0.02	0
5	1	0	0	0	0	0.88	0
	2	0	0	0	0	0.77	0
	3	0	0	0	0	0.66	0
6	1	0	0	0	0	0	1
	2	0	0	0	0	0	0.99
	3	0	0	0	0	0	1

#### 4.4. Bayesian Network Diagnosis

Based on probability calculations, according to Eq. (10), the probability  $P(X|F_1)$  of fault ( $F_1$ ) can be expressed as

$$P(X|F_1) = P(X_W|F_1)P(X_C|F_1)P(X_I|F_1)P(X_A|F_1)P(X_K|F_1)P(X_S|F_1) \quad (12)$$

where  $P(X_W|F_1)$  is the percentage of statistical factor  $X_W$  in fault ( $F_1$ ) within the training samples. Because statistical factor  $X_W$  belongs to level M for test No. 1 in diagnostic sample 1, and there are three amount level M of statistical factors  $X_W$  in training sample 1, the probability of symptom  $X_W$  in fault  $F_1$  can be expressed as  $P(X_W|F_1) = 3/10$ . The other statistical factors can be expressed as

$$P(X_C|F_1) = \frac{5}{10} \text{ (level S)}, \quad P(X_I|F_1) = \frac{5}{10} \text{ (level S)}, \quad P(X_A|F_1) = \frac{5}{10} \text{ (level S)},$$

$$P(X_K|F_1) = \frac{5}{10} \text{ (level S)}, \quad P(X_S|F_1) = \frac{3}{10} \text{ (level L)} \quad (13)$$

Thus, the probability  $P(X|F_1)$  is 0.005625 with Eq. (12). According to Eq. (11), the probability  $P(X|\bar{F}_1)$  of fault ( $F_1$ ) can be expressed as

$$P(X|\bar{F}_1) = P(X_W|\bar{F}_1)P(X_C|\bar{F}_1)P(X_I|\bar{F}_1)P(X_A|\bar{F}_1)P(X_K|\bar{F}_1)P(X_S|\bar{F}_1) \quad (14)$$

where  $P(X_W|\bar{F}_1)$  is the percentage of statistical factor  $X_W$  not in fault ( $F_1$ ) within the training samples. We have the probability

$$P(X_W|\bar{F}_1) = \frac{13 - 3}{60 - 10} = \frac{10}{50}$$

The probabilities of the other statistical factors are as follows:

$$P(X_C|\bar{F}_1) = \frac{7}{50}, \quad P(X_I|\bar{F}_1) = \frac{7}{50}, \quad P(X_A|\bar{F}_1) = \frac{7}{50},$$

$$P(X_K|\bar{F}_1) = \frac{7}{50}, \quad P(X_S|\bar{F}_1) = \frac{9}{50} \quad (15)$$

Given the probability  $P(X|\bar{F}_1)$  is 0.00001383 with Eq. (14), the probability of fault ( $F_1$ ) is  $P(F_1|X) = 0.99$  with Eq. (9). Similarly, the probability  $P(F_2|X) = 0$ ,  $P(F_3|X) = 0.19$ ,  $P(F_4|X) = 0$ ,  $P(F_5|X) = 0.19$ , and  $P(F_6|X) = 0$ . The other computation results via Bayesian networks are listed in Table 2. Using Bayesian networks, rather than BPNN or PNN, makes it easier to classify each gear fault for diagnostic samples.

## 5. CONCLUSIONS

In this study, six statistical factors of vibration signals in the time-domain were used for fault diagnosis in a gear train system. The diagnosis results were obtained via Bayesian networks and compared for six kinds of gear faults. The performance of Bayesian networks was compared against that of BPNN and PNN. Although, BPNN and PNN have strong fault detection abilities, they could not obtain superior results for diagnostic samples which were not within the range of the trained samples. Probability theory is used to calculate the certainty of diagnosis results for untrained samples with Bayesian networks. Because the connection weightings between inputs (statistical factors) and outputs (gear faults) in Bayesian networks are different from those in BPNN and PNN, the proposed method obtain diagnosis results faster than do BPNN and PNN. Additionally, based on the conclusions in Tables 1 and 2, diagnosis results via Bayesian networks are more accurate than those obtained through BPNN and PNN. For eighteen samples, the degrees of certainty were 100% (18/18), 67% (12/18) and 56% (10/18) for Bayesian networks, BPNN and PNN, respectively. Thus, fault diagnosis in gear train system via Bayesian networks not only results in higher accuracy, but also requires fewer calculations than both BPNN and PNN.

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