PASSIVITY-BASED INTEGRAL CONTROL AND STABILITY ANALYSIS OF A CONSTRAINED SINGLE-LINK FLEXIBLE ARM

Liang Y. Liu[1] and Hsiung C. Lin[2]
[1]*Department of Automation Engineering, Chienkuo Technology University, Changhua, Taiwan*
[2]*Department of Electronic Engineering, National Chin-Yi University of Technology, Taichung, Taiwan*
E-mail: lliu@ctu.edu.tw;hclin@ncut.edu.tw

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ABSTRACT
The design of flexible manipulator is complicated due to inherently infinite dimension in nature. The sequential challenge is the problem such a non-minimum phase that is the cause of system instability. In this paper, a constrained single-link flexible arm is fully investigated using a linear distributed parameter model. In order to overcome the inherent limitations, a new input induced by the joint angular acceleration and an output generated using the contact force and root shear force are defined. A necessary and sufficient condition is thus derived so that all poles and zeros of the new transfer function lie on the imaginary axis. Also, the passive integral control is designed to accomplish the regulation of the contact force. The excellent performance of the passive integral controller is verified through numerical simulations.

Keywords: joint angular acceleration; contact force; root shear force; integral controller.

COMMANDE INTÉGRALE PASSIVE ET ANALYSE DE STABILITÉ D'UN BRAS FLEXIBLE CONTRAINT À MAILLON SIMPLE

RÉSUMÉ
La conception d'un bras manipulateur flexible se complique à cause de sa dimension infinie naturellement inhérente. Le problème est le défi séquentiel, une telle phase non-minimale est la cause de l'instabilité du système. Dans cet article, nous investiguons un bras flexible contraint à maillon simple en utilisant un modèle linéaire à paramètre distribué. Pour surmonter les limitations inhérente définies, une nouvelle donnée d'entrée induite par l'accélération due au joint angulaire, et une donnée de sortie, sont générées, utilisant la force de contact et la force de contact tangentielle. Une condition nécessaire et suffisante est dérivée à tous les pôles et zéros pour faire en sorte que la nouvelle fonction de transfert se base sur un axe imaginaire. également, la commande passive intégrale est conçue pour accomplir la régulation de la force de contact. L'excellente performance du contrôleur passif intégral est vérifiée par des simulations numériques.

Mots-clés : accélération de joint angulaire ; force de contact tangentielle ; contrôleur intégral.

NOMENCLATURE

$a_n,\ c_n,\ \beta_n(\varepsilon)$	roots of transcendental equations defined in the Appendix
EI	bending stiffness of the beam, N-m^2
$G_{\lambda\tau}$, etc.	transfer functions
I_h	mass moment of inertia of the hub, kg-m^2
k	a dimensionless parameter used to define the new output $y(t)$
k_I	controller gain constants
ℓ	length of the beam, m
N, n	positive integers
$s, \hat{s}$	Laplace transform variable
t	time variable, s
$u(t)$	a new input defined by Eqn. (9), N-m
$v(x,t)$	transverse deflection, m
$w(x,t)$	actual location of the neutral axis of the beam, m
x	coordinate axes
$y(t)$	a new output defined by Eq. (8), N
$y_d(t)$	desired output trajectory, N
ε, β	dimensionless parameters defined by Eq. (6)
$\theta(t)$	joint angle, rad
$\lambda(t)$	contact force, N
ρ	mass density of the beam, N-m^{-1}
$\tau(t)$	joint torque, N-m

1. INTRODUCTION

Force control of constrained flexible manipulators has continuously attracted much attention in both academia and industry in recent years. The issue is very important in connection with practical applications especially for light weight industrial robots in assembly, deburring and grinding tasks. The controller design for such a force control is, however, quite difficult due to the inherent limitations like nature distributed parameter of flexible arms, the noncollocation of torque actuation and contact force sensing. Based on finite-dimensional approximate models, a pioneer research in a single-link constrained flexible manipulator was carried out in [1]. The outcome showed that the link flexibility is the major source of control system instability. Later, Li [2] indicated that an inherent limitation on the achievable bandwidth occurs from the presence of infinitely non-minimum phase zeros. Some methods based on a lumped parameter model for a single-link flexible robot were thus developed [3, 4] to simplify the dynamics of a flexible arm with the tip forces. However, it was suitable only for one or two degrees of freedom flexible robots [3, 4]. Accordingly, a variety of nonlinear hybrid force-position controllers was proposed using nonlinear finite-dimensional dynamic models [5, 6] for constrained flexible robots. However, these proposed methods may not guarantee the stability of the original distributed parameter systems because of spillover problems. Matsuno and Kasai [7] then derived the distributed parameter model for a constrained one-link flexible arm with a concentrated tip mass, a finite-dimensional model for force feedback and compliance control. More recently, Bazaei and Moallem [8] used distributed parameter model for a constrained flexible beam actuated at the hub. The maximum control bandwidth was obtained by applying the output redefinition. In order to compensate the spillover instability caused by residue modes which are not included in the controller design, an optimal controller with low-pass property and a robust H_∞ controller were proposed in [7, 8]. Further, Bazaei and Moallem [9] improved force control bandwidth of the constrained one-link arm through outputs redefinition. The distributed parameter models were derived in [9], but finite-dimensional models were still used for controller designs.

As we know that the flexible arm is an inherently infinite-dimensional system, the controller design using distributed parameter model becomes more complicated. In order to avoid the spillover from finite-dimensional approximation, the distributed parameter model from [10, 11] was applied to resolve the force control problem for a constrained one-link flexible manipulator. Unfortunately, the system stability was found only in a sufficient condition [10, 11]. Similarly, the stability of the switching collision was also involved into a sufficient condition [12]. Additionally, the exact solutions for the closed-loop system can not be obtained [12]. Recently, the infinite product formulation has been utilized to design a PD controller for a linear distributed parameter model in the constrained one-link flexible arm [13]. Liu and Lin [14] further worked on the constrained one-link flexible arm with internal material damping. However, the passive property was not proved in these studies [13, 14].

In this paper we develop a linear distributed parameter model to obtain an exact solution to the above contact force regulation problem. Based on this proposed model, the globally stable infinite-dimensional closed loop system can be achieved. The paper is divided into four sections. Section 2 describes the proposed linear distributed parameter model mathematically. The non-minimum phase transfer function, condition for the nonexistence of right half-plane zeros, passivity proof, and passivity-based integral control are also included. The simulation results using a numerical example are provided and discussed in detail in Section 3. Conclusions are presented in Section 4.

2. MATHEMATICAL MODEL

Consider that the constrained one-link flexible arm is a uniform, homogeneous, Euler–Bernoulli beam [15–17] of length ℓ, mass per unit length ρ, and flexural rigidity EI. The hub is modelled by a single-mass moment of inertia I_h, where the driven torque $\tau(t)$ is applied. The contact force exerted by the smooth rigid constraint surface is $\lambda(t)$. The $\theta(t)$ designates the hub rotation angle, and $v(x,t)$ denotes the small elastic deflection of the link. The constraint equations of motion and boundary conditions are well-established [13], and introduce the new variable $w(x,t)$ as

$$w(x,t) = x\theta(t) - v(x,t) \tag{1}$$

Then, the dynamic equations of the constrained arm can be obtained as

$$\rho \ddot{w}(x,t) + EIw_{xxxx}(x,t) = 0 \tag{2}$$

$$w(0,t) = 0, \quad w(\ell,t) = 0, \quad w_x(0,t) = \theta(t), \quad w_{xx}(\ell,t) = 0 \tag{3}$$

$$EIw_{xxx}(\ell,t) = \lambda(t) \tag{4}$$

$$I_h\ddot{\theta}(t) = \tau(t) + EIw_{xx}(0,t) \tag{5}$$

2.1. Non-Minimum Phase Transfer Function

The transfer function can be derived by taking the Laplace transform of Eqs. (2)–(5) with zero initial conditions. Let s be the Laplace transform variable, and define the dimensionless parameters β, ε, and $\hat{s}$ as

$$\beta^4 = -\frac{\rho\ell^4}{EI}s^2 = -\hat{s}^2, \quad \varepsilon = \frac{I_h}{\rho\ell^3} \tag{6}$$

The solution of Eq. (2) can be written in the Laplace transform domain as

$$w(x,\hat{s}) = C_1 \cosh\frac{\beta}{\ell}x + C_2 \cos\frac{\beta}{\ell}x + C_3 \sinh\frac{\beta}{\ell}x + C_4 \sin\frac{\beta}{\ell}x \tag{7}$$

where $C_i(\beta)$, $i = 1, 2, 3, 4$ are unknown parameters. Substitution of Eq. (7) into Eqs. (3)–(5) and solving for C_2, C_3, C_4, θ, λ, τ and u yield

$$C_2 = -C_1 \tag{8}$$

$$C_3 = -\frac{\cosh\beta}{\sinh\beta}C_1, \quad C_4 = \frac{\cos\beta}{\sin\beta}C_1 \tag{9}$$

$$\theta(\hat{s}) = C_1\frac{\beta}{\ell}\cdot\frac{\cos\beta\sinh\beta - \cosh\beta\sin\beta}{\sin\beta\sinh\beta} \tag{10}$$

$$\lambda(\hat{s}) = -C_1EI\frac{\beta^3}{\ell^3}\cdot\frac{\sin\beta + \sinh\beta}{\sin\beta\sinh\beta} \tag{11}$$

$$\tau(\hat{s}) = C_1EI\frac{\beta^2}{\ell^2}\cdot\left[2 + \varepsilon\beta^3\cdot\frac{\cos\beta\sinh\beta - \cosh\beta\sin\beta}{\sin\beta\sinh\beta}\right] \tag{12}$$

$$u(\hat{s}) = -C_1EI\frac{\beta^5}{\ell^2}\varepsilon\cdot\frac{\cos\beta\sinh\beta - \cosh\beta\sin\beta}{\sin\beta\sinh\beta} \tag{13}$$

Using Eq. (7), one further obtains

$$v_{xx}(0,\hat{s}) = -w_{xx}(0,\hat{s}) = -2C_1\frac{\beta^2}{\ell^2} \tag{14}$$

$$v_{xxx}(0,\hat{s}) = -w_{xxx}(0,\hat{s}) = C_1\frac{\beta^3}{\ell^3}\frac{\cos\beta\sinh\beta + \cosh\beta\sin\beta}{\sin\beta\sinh\beta} \tag{15}$$

After algebraic manipulations, one obtains

$$\frac{\lambda(\hat{s})}{\tau(\hat{s})} = G_{\lambda\tau}(\hat{s}) = \frac{\beta}{\ell}\cdot\frac{\sinh\beta + \sin\beta}{2\sinh\beta\sin\beta - \varepsilon\beta^3(\cosh\beta\sin\beta - \sinh\beta\cos\beta)} \tag{16}$$

By applying the infinite product expansions of transcendental functions (see A1 to A4 of the Appendix), Eq. (16) can be rewritten as

$$\begin{aligned} G_{\lambda\tau}(\hat{s}) &= \frac{\lambda(\hat{s})}{\tau(\hat{s})} = \frac{\beta}{\ell}\frac{\sinh\beta + \sin\beta}{2\sinh\beta\sin\beta - \varepsilon\beta^3(\cosh\beta\sin\beta - \sinh\beta\cos\beta)} \\ &= \frac{1}{\ell}\cdot\frac{\prod_{n=1}^{\infty}\left(1 - \frac{\hat{s}^2}{\omega_{zn}^2}\right)}{\prod_{n=1}^{\infty}\left(1 + \frac{\hat{s}^2}{\omega_{\theta n}^2}\right)} \end{aligned} \tag{17}$$

In Eq. (17), it is found that $G_{\lambda\tau}(\hat{s})$ has infinitely many zeros in $\mathrm{Re}(\hat{s}) > 0$. Thus, $G_{\lambda\tau}(\hat{s})$ is a non-minimum phase. It is well established hat the existence of non-minimum phase zeros imposes fundamental limitations in the achievable performance of the closed-loop system. To alleviate the non-minimum phase problem, the real zeros of $G_{\lambda\tau}(\hat{s})$ can be replaced by the zeros on the imaginary $\hat{s}$-axis using the method of redefinition of output [18]. With the new output, the transfer function is a marginal minimum phase but not necessarily passive. Fortunately, the poles of $G_{\lambda\tau}(\hat{s})$ can be made to move along the imaginary $\hat{s}$-axis by using an appropriate feedback. Combining the feedback and the output redefinition, it is possible to find a new transfer function which satisfies the so-called the interlacing property. A transfer function with a simple pole at the origin is said to satisfy the interlacing property if all its poles and zeros lie on the imaginary $\hat{s}$-axis, are distinct and alternate each other. Such transfer functions are known as passive transfer functions.

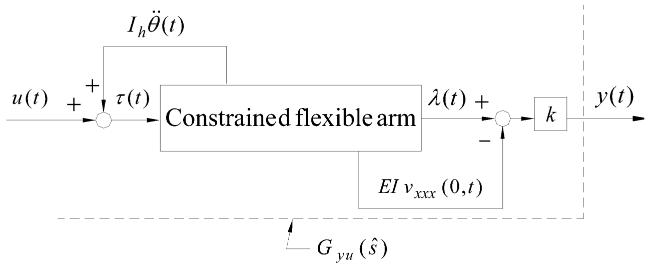

Fig. 1. The construction of a new input and a new output.

2.2. Condition for the Nonexistence of Right Half-Plane Zeros

It is well known that for nonminimum phase systems, perfect asymptotic tracking of output trajectories with internal stability cannot be achieved. To alleviate the non-minimum phase problem, the right half-plane zeros can be replaced by the left half-plane zeros by the method of redefinition of output. Let the new output be chosen as

$$y(t) = (k-1)[EIv_{xxx}(0,t) + \lambda(t)] + \lambda(t) \tag{18}$$

where the contact force $\lambda(t)$ can be measured by a force sensor [8, 9] located at the tip of the arm. Also, the shear force signal $v_{xxx}(0,t)$ can be detected by a similar full-bridge strain gauge [8, 9] cemented at the pinned end of the arm, where k is a real constant whose permissible values will be determined to satisfy the minimum phase condition. Let the joint torque $\tau(t)$ be given by

$$\tau(t) = I_h\ddot{\theta}(t) + u(t) \tag{19}$$

where the joint angular acceleration $\ddot{\theta}(t)$ can be measured [19]. The construction of a new input and a new output is shown schematically in Fig. 1. One obtains

$$G_{yu}(\hat{s},k) := \frac{y(\hat{s})}{u(\hat{s})} = \frac{\beta}{2\ell} \times \frac{(1-k)(\cosh\beta\sin\beta + \sinh\beta\cos\beta) + k(\sin\beta + \sinh\beta)}{\sin\beta\sinh\beta} \tag{20}$$

Now, using Eq. (6) for the zeros of $G_{yu}(\hat{s},k)$ are given by the roots of

$$\frac{k}{1-k} \cdot \frac{\prod_{n=1}^{\infty}\left[1 + \frac{4\hat{s}^2}{\omega_{zn}^2}\right]}{\prod_{m=1}^{\infty}\left[1 - \frac{\hat{s}^2}{\omega_{zm}^2}\right]} = -1 \tag{21}$$

where A1 and A5 from the appendix have been used in deriving Eq. (21). The root locus of Eq. (21) for $-\infty < k < 1$ is shown in Fig. 2 based on a 6 pole-zero pair approximation. The approximate breakaway points on the imaginary $\hat{s}$-axis are $\pm 15.449j$, $\pm 105.826j$, $\pm 285.327j$ corresponding to $k = 0.592$, 0.631, 0861, respectively. Note that the breakaway points on the imaginary $\hat{s}$-axis actually corresponds to the real positive double roots of $N(\beta,k) = 0$, or equivalently,

$$\cosh\beta\sin\beta + \sinh\beta\cos\beta = -\frac{k}{1-k}(\sinh\beta + \sin\beta) \tag{22}$$

Further, the asymptotic behavior of Eq. (22) is governed by

$$\sin\beta + \cos\beta = -\frac{k}{1-k} \tag{23}$$

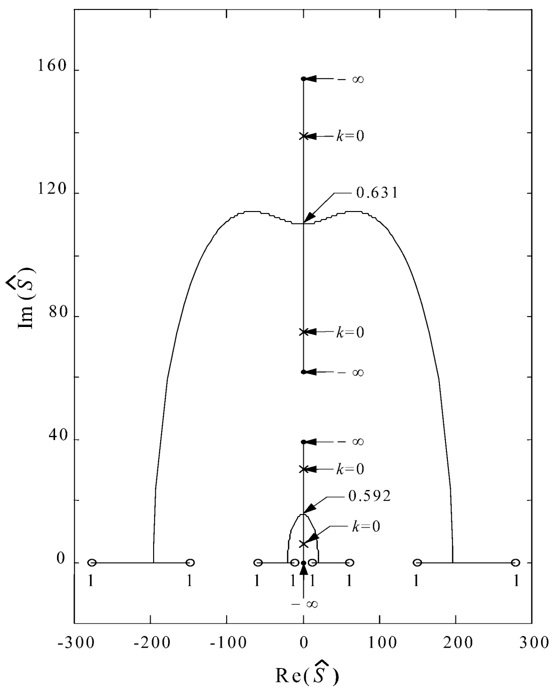

Fig. 2. Root locus of Eq. (21) for $-\infty < k < 1$ using a 6 pole-zero pair approximation.

It is easy to verify from the numerical solutions of Eq. (22) and the simple graph of Eq. (23) that (i) the smallest real positive double roots $\beta = 3.921$ ($\hat{s} = \pm 15.372j$) occur when $k = 0.592$, (ii) a larger value of real positive double roots occur at a larger value of k, and (iii) there are no real positive double roots for $k < 0.592$. Therefore it is impossible to have any breakaway point on the imaginary $\hat{s}$-axis for $k < 0.592$. We conclude that, with the previously defined new input and output, the constrained one-link flexible manipulator is a minimum phase iff $-\infty < k \leq 0.592$. Obviously, this necessary and sufficient condition is obtained via the numerical solution of the exact transcendental equation $N(\beta, k) = 0$, not by the finite-dimensional approximation of the root locus. One can write

$$(1-k)(\cosh\beta\sin\beta + \sinh\beta\cos\beta) + k(\sin\beta + \sinh\beta) = 2\beta\prod_{n=1}^{\infty}\left(1 + \frac{\hat{s}^2}{\omega_{\alpha n}^2(k)}\right) \tag{24}$$

The numerical values that can be computed using $\omega_{\alpha n} = \beta_n^2$, where $\beta_n(k)$, $n = 1,\ 2, \cdots$ is the real positive roots of the numerator of Eq. (24). The numerical solutions of $N(\beta, k) = 0$, (i.e. $\omega_{\alpha n}(k)$) for several values of k are listed in Table 1. According to Eq. (23) and confining ourselves to $\hat{s} = \pm j\beta^2$, there must exist one and only one $\hat{s} = \pm j\omega_{\alpha n}(k)$ (i) with $-\infty < k < 0.5$ in every internal $0 < \beta < \pi$ for $n = 1$, and $(n - \frac{1}{2})\pi < \beta < n\pi$ for $n \geq 2$, (ii) with $0.5 < k \leq 0.592$ in every internal $n\pi < \beta \leq (n + \frac{1}{4})\pi$ for n=odd, and $(n - \frac{3}{4})\pi \leq \beta < (n - \frac{1}{2})\pi$ for n = even. Thus, a minimum phase stable transfer function can be deduced as

$$G_{yu}(\hat{s}, k) = \frac{1}{\ell} \cdot \frac{\prod_{n=1}^{\infty}\left(1 + \frac{\hat{s}^2}{\omega_{\alpha n}^2(k)}\right)}{\prod_{n=1}^{\infty}\left(1 + \frac{\hat{s}^2}{n^4\pi^4}\right)} \tag{25}$$

Table 1. Values of $\omega_{\alpha n}(k)$ vs. different values of k.

k	$\omega_{\alpha 1}$	$\omega_{\alpha 2}$	$\omega_{\alpha 3}$	$\omega_{\alpha 4}$	$\omega_{\alpha 5}$	$\omega_{\alpha 6}$
$-\infty$	0	6.283^2	7.853^2	12.566^2	14.137^2	18.850^2
-1000	0.394^2	6.282^2	7.854^2	12.565^2	14.138^2	18.849^2
-100	0.699^2	6.273^2	7.863^2	12.557^2	14.147^2	18.840^2
-10	1.222^2	6.196^2	7.941^2	12.479^2	14.224^2	18.762^2
-1	1.918^2	5.858^2	8.278^2	12.142^2	14.561^2	18.426^2
-0.7	2.010^2	5.792^2	8.344^2	12.076^2	14.627^2	18.360^2
-0.5	2.085^2	5.735^2	8.401^2	12.019^2	14.685^2	18.302^2
-0.3	2.176^2	5.661^2	8.475^2	11.945^2	14.759^2	18.228^2
0	2.365^2	5.498^2	8.639^2	11.781^2	14.923^2	18.064^2
0.3	2.686^2	5.193^2	8.947^2	11.473^2	15.230^2	17.756^2
0.5	3.142^2	4.730^2	9.425^2	10.996^2	15.708^2	17.279^2
0.592	3.927^2	3.927^2	10.141^2	10.305^2	15.935^2	17.148^2

2.3. Proof of Passivity

Let $\frac{G_{yu}(\hat{s})}{\hat{s}}$ be expressed as

$$\frac{G_{yu}(\hat{s})}{\hat{s}} = \frac{1}{\ell} \cdot \frac{\prod_{n=1}^{N}\left(1+\frac{\hat{s}^2}{\omega_{\alpha n}^2}\right)}{\hat{s}\prod_{n=1}^{N}\left(1+\frac{\hat{s}^2}{n^4\pi^4}\right)} = \frac{A_0}{\hat{s}} + \sum_{i=1}^{N} \frac{A_i \hat{s}}{\hat{s}^2 + i^4\pi^4} \tag{26}$$

where by assumption $N \to \infty$, $0 < \omega_{\alpha 1} < \omega_{\alpha 2} < \cdots < \omega_{\alpha N} < \cdots$. It is easy to show that $A_0 = \frac{1}{\ell}$ and

$$A_i = -\frac{A_0}{\omega_{\alpha i}^2} \prod_{n=1}^{N} \left(\frac{n^2\pi^2}{\omega_{\alpha n}}\right)^2 \frac{\prod_{n=1}^{N}\left(\omega_{\alpha n}^2 - i^4\pi^4\right)}{\prod_{\substack{n=1\\ n\neq i}}^{N}\left(n^4\pi^4 - i^4\pi^4\right)} \tag{27}$$

for $i = 1, 2, \ldots, N$. Note that if $A_0 > 0$ and $A_i > 0$ for $i = 1, 2, \ldots$ $(N \to \infty)$, then $G_{yu}(\hat{s})/\hat{s}$ is a passive transfer function since it is the sum of passive transfer functions (see Eq. (26)). We now proceed to prove the assertion by induction that if $A_i > 0$, $i = 1, 2, \ldots$ $(N \to \infty)$, then the interlacing property holds for $\frac{G_{yu}(\hat{s})}{\hat{s}}$. Using the $\hat{s} = j\omega_{\alpha n}(k)$ with $-\infty < k < 0.5$ in every internal $0 < \omega_{\alpha n}^2(k) < \pi^2$ for $n = 1$ and $(n-\frac{1}{2})^2\pi^2 < \omega_{\alpha n}^2(k) < n^2\pi^2$ for $n \geq 2$, and referring to Eq. (26), it can be easily shown that (i) for $N = 1$, $A_1 > 0$ implies $\omega_{\alpha 1} < \pi^2$, and (ii) for $N = 2$ and $\omega_{\alpha 1} < \pi^2$, $A_1 > 0$ implies $\pi^2 < \omega_{\alpha 2}$ and $A_2 > 0$ implies $\omega_{\alpha 2} < 4\pi^2$. Assume that for each positive integer N, $A_i > 0$, $i = 1, 2, \ldots, N$ imply $0 < \omega_{\alpha 1} < \pi^2 < \omega_{\alpha 2} < 4\pi^2 < \cdots < \omega_{\alpha N} < N^2\pi^2$. Then for the positive integer $N+1$, it can be shown using Eq. (27) that $A_i > 0$, $i = 1, 2, \ldots, N$ imply $N^2\pi^2 < \omega_{\alpha,N+1}$ and $A_{N+1} > 0$ implies $\omega_{\alpha,N+1} < (N+1)^2\pi^2$. This proves the above assertion for all positive integers N. One concludes that $\frac{G_{yu}(\hat{s})}{\hat{s}}$ satisfies the interlacing property. Therefore, $\frac{G_{yu}(\hat{s})}{\hat{s}}$ is a passive transfer function for $-\infty < k < 0.5$.

2.4. Passivity-Based Integral Control

Consider the control structure as shown in Fig. 3. With the integral control principle,

$$u(\hat{s}) = \frac{\frac{k_I\ell}{\hat{s}}}{1+\frac{k_I\ell}{\hat{s}}G_{yu}(\hat{s},k)} y_d(\hat{s}) \tag{28}$$

where $y_d(\hat{s})$ is the Laplace transform of the desired output trajectory, and k_{I} is a positive constant.

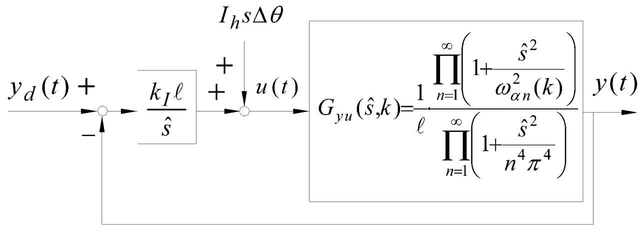

Fig. 3. Passive integral control structure.

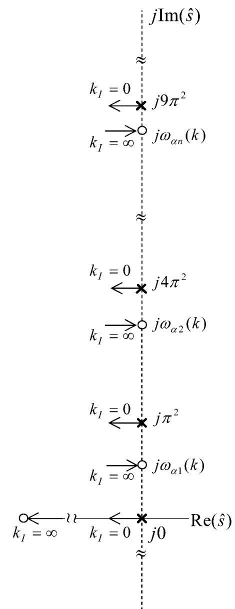

Fig. 4. Effects of passive integral control: $0 \le k_I < \infty$.

The poles of the closed-loop system are given by the roots of the characteristic equation as

$$1 + \frac{k_I \ell}{\hat{s}} G_{yu}(\hat{s}, k) = 0 \tag{29}$$

With $G_{yu}(\hat{s}, k)$ given by Eq. (25), Eq. (29) becomes

$$\frac{k_I}{\hat{s}} \cdot \frac{\prod_{n=1}^{\infty}\left(1+\frac{\hat{s}^2}{\omega_{\alpha n}^2(k)}\right)}{\prod_{n=1}^{\infty}\left(1+\frac{\hat{s}^2}{n^4\pi^4}\right)} = -1 \tag{30}$$

It can be easily verified (see Fig. 4) using a simple root locus plot that the I-control suffices to stabilize the closed-loop system for all $0 < k_I < \infty$.

3. SIMULATION

The effectiveness of the proposed control approach is evaluated using the same parameters of an experimental apparatus described in [20]. These parameters are set as: $\rho = 0.307$ kg/m, $\ell = 0.85$ m, $EI = 5.425$ N-m^2, and $I_h = 0.01$ kg-m^2. The contact force parameter and controller gain were selected as $k = 0.4$ and

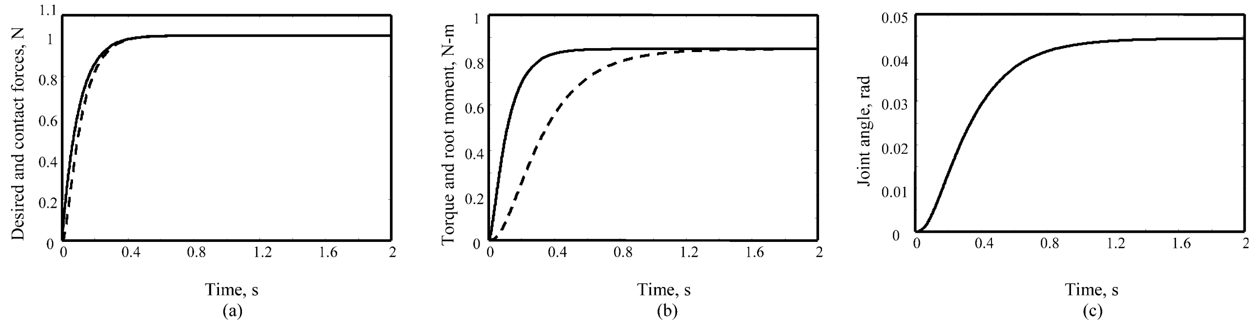

Fig. 5. Contact force regulation: (a) $\lambda(t)$ —, $y_d(t)$ - - -; (b) $\tau(t)$ —, $EIv_{xx}0,t)$ - - -; and (c) $\theta(t)$.

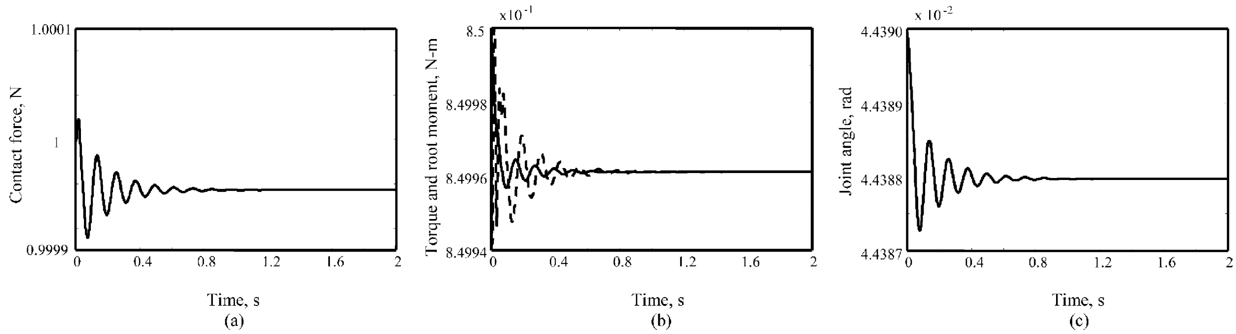

Fig. 6. Disturbance rejection: (a) $\lambda(t)$; (b) $\tau(t)$ — $EIv_{xx}(0,t)$ - - -; and (c) $\theta(t)$.

$k_I = 7.533$. The desired contact force trajectory was selected as

$$y_d(t) = 1 - e^{-10\,t} \tag{31}$$

which results in $\lim_{t\to\infty}\lambda(t) = \lim_{t\to\infty} y(t) = 1\,N$, $\lim_{t\to\infty} v_{xx}(0,t) = 0.157\,m^{-1}$, $\lim_{t\to\infty}\theta(t) = 4.439\times10^{-2}$ rad and $\lim_{t\to\infty}\tau(t) = 0.85$ N-m. The simulation results, shown in Fig. 5, is found hardly discernible since $N \geq 4$. It indicates that the tracking of a desired contact force trajectory promises both quick response and high accuracy with internal stability.

Assume that the flexible link remains in the steady state, but there is an initial joint angular displacement. Since the Laplace transform of Eqs. (5) and (19) yield $I_h\,[s^2\Delta\theta\,(s) - s\,\Delta\theta(0) - \Delta\dot{\theta}(0)] = u(s)$, the initial joint angular displacement perturbed from the steady state solution can be regarded as a disturbance entering the plant as shown in Fig. 3. The responses for the disturbance $\Delta\theta := \theta(0) - 4.439\times10^{-2} = -0.2$ rad are perturbed from the steady state solution, and the results are shown in Fig. 6. It is evident that the disturbance rejection is achieved successfully.

4. CONCLUSIONS

A constrained one-link flexible arm has been fully studied using a linear distributed parameter model. The root bending moment is considered as the input, and the contact force plus its weighted value and root shear force is chosen as the output. A necessary and sufficient condition for the new transfer function to be a minimum phase (i.e., its transfer function does not have any zeros in the open right-half plane) has been approved. This minimum phase condition is especially valuable since it is independent on the physical parameters of the flexible link. Further, the passive integral controller can stabilize the infinite-dimensional closed loop system. Unlike most traditional approaches generally suffering from control and observation spillover problems, the exact solutions have been deduced for the noncollocated infinite-dimensional force control system.

APPENDIX

The infinite product expansions for transcendental functions throughout the paper are summarized as given below [13, 18]. Also, $\hat{s}^2 = -\beta^4$ is used whenever it is required.

(A1) $\sin\beta + \sinh\beta = 2\beta \prod_{n=1}^{\infty}\left(1 - \frac{\hat{s}^2}{\omega_{zn}^2}\right)$, $\omega_{zn} = 2a_n^2$, $\tanh a_n + \tan a_n = 0$, $a_n = \left(n - \frac{1}{4}\right)\pi$ as $n \to \infty$

(A2) $\sin\beta \sinh\beta = \beta^2 \prod_{n=1}^{\infty}\left(1 + \frac{\hat{s}^2}{n^4\pi^4}\right)$

(A3) $\cosh\beta \sin\beta - \cos\beta \sinh\beta = \frac{2}{3}\beta^3 \prod_{n=1}^{\infty}\left(1 + \frac{\hat{s}^2}{\omega_{\beta n}^2}\right)$, $\omega_{\beta n} = c_n^2$, $\tanh c_n - \tan c_n = 0$, $c_n = \left(n + \frac{1}{4}\right)\pi$ as $n \to \infty$

(A4) $2\sinh\beta \sin\beta - \varepsilon\beta^3(\cosh\beta \sin\beta - \sinh\beta \cos\beta) = 2\beta^2 \prod_{n=1}^{\infty}\left(1 + \frac{\hat{s}^2}{\omega_{\theta n}^2}\right)$, $\omega_{\theta n} = \beta_n^2(\varepsilon)2\sin\beta_n - \varepsilon\beta_n^3(\sin\beta_n - \cos\beta_n) = 0$, $\beta_n(\varepsilon) = \left(n - \frac{3}{4}\right)\pi + \frac{1}{\varepsilon(n-\frac{3}{4})^3\pi^3}$ as $n \to \infty$

(A5) $\cosh\beta \sin\beta + \sinh\beta \cos\beta = 2\beta \prod_{n=1}^{\infty}\left(1 + \frac{4\hat{s}^2}{\omega_{zn}^2}\right)$, $\omega_{zn} = 2a_n^2$, $\tanh a_n + \tan a_n = 0$, $a_n = \left(n - \frac{1}{4}\right)\pi$ as $n \to \infty$

Note that the asymptotic expressions are found very accurate (to three decimal places) since $n \geq 5$.

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