ON THE DESIGN OF NEW PROGRAMMABLE EXACT PATH GENERATORS

Ren-Chung Soong
Department of Mechanical & Automation Engineering, Kao Yuan University, Kaohsiung 82141, Taiwan
E-mail: t30004@cc.kyu.edu.tw

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ABSTRACT

A design method of new programmable exact path generators is proposed. The instantaneous positions of the linear input link corresponding to current angular positions of the rotational input link for each point of exact paths are also derived. By controlling the instantaneous position of linear and rotational input link, the required exact paths can be generated during a cycle. This design approach makes the proposed linkage mechanism programmable and adjustable, and increases their flexibility in practical applications.

Keywords: programmable mechanism; linkage mechanism; path generation.

LA CONCEPTION D'UN NOUVEAU GÉNÉRATEUR DE TRAJECTOIRES EXACTE PROGRAMMABLE

RÉSUMÉ

Une méthode de conception pour un nouveau générateur de trajectoire exacte est proposée dans cet article. Les positions instantanées du maillon linéaire d'entrée correspondant aux positions actuelles du maillon d'entrée rotationnel pour chaque point de la trajectoire exacte sont aussi dérivées. En contrôlant la position instantanée du maillon linéaire et rotationnel d'entrée, la trajectoire requise peut-être générée pendant un cycle. L'approche proposée rend le mécanisme de couplage programmable et ajustable, et accroît l'efficacité dans l'application pratique.

Mots-clés : mécanisme programmable ; mécanisme de couplage ; générateur de trajectoire.

1. INTRODUCTION

Path generation can be categorized into two types. One is the point-to-point path generation in which the coupler curves only specify several points on the desired path. The other is the exact path generation in which the coupler curves have to specify the entire desired path or many points on the desired path.

In industrial applications there is often a need to generate a specific exact path, even to generate several similar paths such as the concentric circles and elliptical paths with the same short axis using the same mechanism for production of similar products, for example, pipe cross section deburring machines and plate cutting machines. The coupler curves of linkage mechanisms, however, are functions of their link lengths, so if we want to generate one or more exact paths, whose number of precision points is over nine, the only way is to make at least one of its link lengths to be variable.

Some researches have been devoted to present new synthesis methods for path generation of linkage mechanisms in the past years. Angeles et al. [1] proposed an unconstrained nonlinear least-square optimal synthesis method for RRRR planar path generators. Hoeltzel and Chieng [2] proposed a pattern matching synthesis method based on the classification of coupler curves according to moment variants. Watanabe [3] presented natural equation which expresses the curvature of the path as an equation of the arc length and is independent of the location and orientation of the path. Ullah and Kota [4] presented an optimal synthesis method in which the objective function is expressed as the Fourier descriptors. Shimojima et al. [5] developed a synthesis method for straight line and L-shaped path generation by using fixed pivot positions as adjustable parameters. Unruh and Krishnaswami [6] proposed a computer-aided design technique for infinite point coupler curve synthesis of four-bar linkages. Kim and Sodhi [7] introduced a method of path generation which makes the desired path pass through five specified points exactly and other points approximately. Chuenchom and Kota [8] presented a synthesis method for programmable mechanisms using adjustable dyads. Zhou et al. [9, 10] proposed an optimal synthesis method with modified genetic optimization algorithms by adjusting position of the driven side link for continuous path generation. Russell and Sodhi [11] adjusted the fixed pivot position of RRSS-SS linkages to generate two-phase motions. Russell and Sodhi [12, 13] presented a technique for designing a slider path capable of realizing multi-phase path, function, and motion generation applications for adjustable slider-crank mechanisms, using seventh-order polynomials. Laribi et al. [14] presented a combined genetic algorithm-fuzzy logic method (GA-FL) for mechanism synthesis. Zhou and Cheung [15] proposed an optimal synthesis method that uses adjustable four-bar linkages for multi-phase motion generation by adjusting the position of a driven side-link fixed pivot. Zhou [16] proposed an optimal synthesis method for adjustable function-generation linkages, using a revolute joint to adjust the fixed pivot location of the driven link. Singh and Kohli [17] combined the complex loop closure method and envelope theory to attain exact path or motion generation for the synthesis of combined cam-linkage systems. Mundo et al. [18] presented a design method for synthesizing cam-linkage mechanisms for exact path generation. Gatti and Mundo [19] proposed an optimal synthesis method for synthesizing cam-linkage mechanisms for exact rigid-body guidance. Soong [20] suggested a design technique for adjustable mechanical forming presses with adjustable-length links. Soong and Wu [21] proposed a design method for variable coupler curve-generating mechanisms by controlling the angular displacement of the driving link and adjusting the link length of the fixed link. Soong [22] proposed a design technique for obtaining the desired characteristics of an output motion by varying the link length and speed trajectory of the driving links of four-bar mechanisms and mechanical presses. Soong and Chang [23] proposed a design method for single degree-of-freedom (DOF) linkage mechanisms with variable length input links to have exact function generations.

In this paper, we propose a new design method for programmable exact path generators to generate the required exact paths using the same mechanism. A linkage mechanism with minimal structure error corresponding to required exact closed paths and with a rotational and a linear input is introduced. The

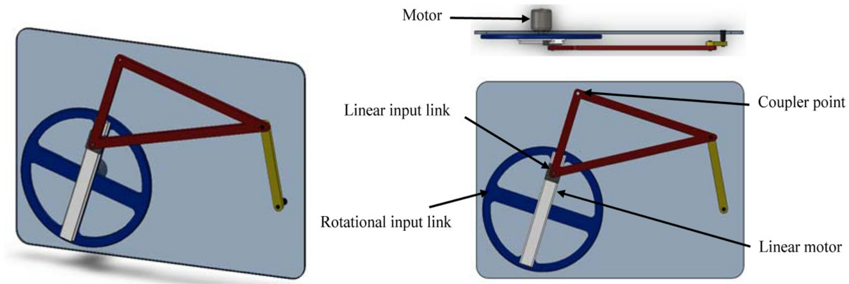

Fig. 1. The new programmable exact path generator.

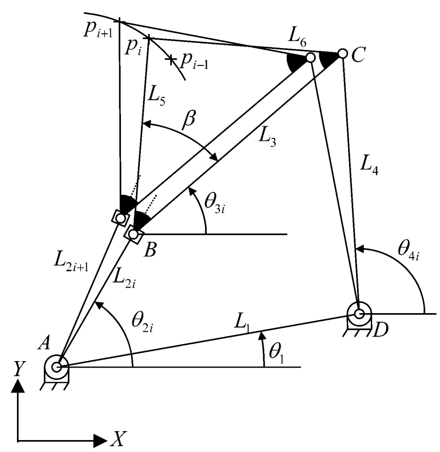

Fig. 2. The coordinate system of the new programmable exact path generator.

deviations of instantaneous displacement of linear input link corresponding to angular displacement of the rotational input link are analyzed. Examples are given to prove this design method.

2. THE NEW PROGRAMMABLE EXACT PATH GENERATOR

A new programmable exact path generator with a rotational input link and a linear input link is shown in Fig. 1. This proposed new programmable exact path generator comes from an optimal synthesized four-bar linkage with minimal structure error corresponding to required exact paths. A disk or a wheel link driven by a motor replaces the driving link of the original mechanism and serves as the rotational input link. A slider link driven by a linear motor fixed on the disk link is added to the original mechanism and serves as the linear input link. It is allowed to move along the radial direction from the center to the outer edge of the disk link. Once the replacement and addition are made, the original four-bar linkage becomes a 2 DOF five-bar linkage mechanism with a rotational input link and a linear input link. By controlling the instantaneous displacement of linear input link, the proposed linkage mechanism can generate the required exact closed paths while the rotational input link rotates with a constant speed during a cycle.

3. THE INSTANTANEOUS DISPLACEMENT OF LINEAR INPUT LINK

The coordinate system of the new programmable exact path generator is shown in Fig. 2. Since this proposed linkage mechanism comes from an optimal synthesized four-bar linkage, all link lengths are fixed except the displacement of slider B (linear input link) relative to the fixed pivot A in this linkage mechanism.

From Fig. 2 the coordinate values of the points p and D, the current position of point C can be determined as intersection between two circumferences, centered in p and D, whose radii are L_6 and L_4, respectively. The following equations can be then derived:

$$(x_C - x_D)^2 + (y_C - y_D)^2 = L_4^2 \tag{1}$$

$$(x_C - x_P)^2 + (y_C - y_P)^2 = L_6^2 \tag{2}$$

From this two couples of coordinates (x_C, y_C) are calculated as

$$x_C = Dy_C + E, \quad y_C = \frac{-H \pm \sqrt{H^2 - 4HJ}}{2G} \tag{3}$$

one of which, according to the actual mechanism configuration, represents the current position of point C, where

$$A = 2(x_D - x_P), \quad B = 2(y_P - y_D), \quad C = x_D^2 + y_D^2 + L_6^2 - x_P^2 - y_P^2 - L_4^2 \tag{4}$$

$$D = B/A, \quad E = C/A, \quad F = E - x_D, \quad G = D^2 + 1, H = 2(DF - y_D), \quad J = F^2 + y_D^2 - L_4^2 \tag{5}$$

With the same procedure, the current position of point B can be determined as intersection between two circumferences, centered in C and p, whose radii are L_3 and L_5, respectively. The following conditions also can be derived:

$$(x_B - x_P)^2 + (y_B - y_P)^2 = L_5^2 \tag{6}$$

$$(x_B - x_C)^2 + (y_B - y_C)^2 = L_3^2 \tag{7}$$

From this two couples of coordinates (x_B, y_B) are calculated as

$$x_B = D_1 y_B + E_1, \quad y_B = \frac{-H_1 \pm \sqrt{H_1^2 - 4H_1 J_1}}{2G_1} \tag{8}$$

one of which, according to the actual mechanism configuration, represents the current position of point B, where

$$A_1 = 2(x_D - x_P), \quad B_1 = 2(y_P - y_D), \quad C_1 = x_P^2 + y_P^2 + L_3^2 - x_C^2 - y_C^2 - L_5^2 \tag{9}$$

$$D_1 = B_1/A_1, \quad E_1 = C_1/A_1, \quad F_1 = E_1 - x_P, \quad G_1 = D_1^2 + 1, \quad H_1 = 2(D_1 F_1 - y_P), \quad J_1 = F_1^2 + y_P^2 - L_5^2 \tag{10}$$

Once the valves of (x_B, y_B) are determined, the instantaneous displacement of slider B relative to the fixed pivot A, L_2, corresponding to the current position of point p, can be calculated as

$$L_2 = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2} \tag{11}$$

The angular position of rotational input link θ_2, corresponding to the current position of point p, also can be calculated as

$$\theta_2 = \arctan\left(\frac{(y_B - y_A)}{(x_B - x_A)}\right) \tag{12}$$

For any required exact path, once Eqs. (11) and (12) are solved, a couple of input valves (θ_2, L_2) are determined.

Table 1. Precision points of the required path in Example 1.

Point n.	1	2	3	4	5	6	7	8	9	10	11	12	13
x-coordinate	7.03	6.95	6.77	6.40	5.91	5.43	4.93	4.67	43.8	4.04	3.76	3.76	3.76
y-coordinate	5.99	5.45	5.03	4.60	4.03	3.56	2.94	2.60	2.20	1.67	1.22	1.97	2.78
Point n.	14	15	16	17	18	19	20	21	22	23	24	25	
x-coordinate	3.76	3.76	3.76	3.76	3.80	4.07	4.53	5.07	5.45	5.89	6.41	6.92	
y-coordinate	3.56	4.34	4.91	5.47	5.98	6.40	6.75	6.85	6.84	6.83	6.80	6.58	

Table 2. Optimal original four-bar linkage according to Ref. [14].

Parameter	L_1 (cm)	L_2 (cm)	L_3 (cm)	L_4 (cm)	L_5 (cm)	(x_A, y_A) (cm)	θ_1 (deg)	β (deg)
Value	9.0	3.01	8.80	8.80	11.10	(–2.4, –4.0)	28	39

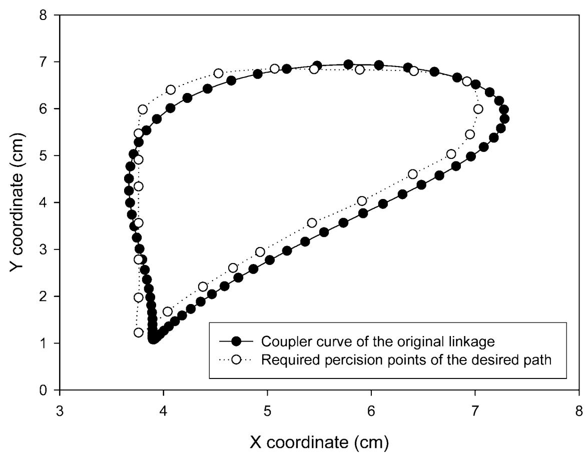

Fig. 3. Precision points of the required path and coupler curve of the original linkage.

4. ANALYSIS OF MOBILITY AND WORKING SPACE

From Figs. 1 and 2, the proposed five-bar mechanism will degenerate into a four-bar linkage when the linear input link is fixed, so the proposed mechanism can be regarded as a four-bar linkage with a variable length of input link. Therefore, in order to guarantee the rotational input link to be a crank, the first condition is that the link length relation between L_1, L_3, L_4 and L_2 has to satisfy the Grashof law, the second condition is that the shortest length link has to be L_1 or L_2 at any instantaneous moment during a cycle.

Since the proposed mechanism can be regarded as a four-bar linkage with a variable length of input link, as long as the desired exact coupler curves are in the intersection area between the two sets concentric circles, one with radii $|L_5 - L_2|$ and $|L_5 + L_2|$, the other one with radii $|L_6 - L_4|$ and $|L_6 + L_4|$, they can be generated by controlling the displacement of the linear-input link while rotational input link rotates with a constant angular speed during a cycle. Because the displacement of the linear input link, L_2, is variable, the working space of coupler point is also adjustable from $L_2 = 0$ to $L_2 = L_{2\max}$, where $L_{2\max}$ is the possible longest length of L_2 which must satisfy the Grashof law at any instantaneous moment during a cycle.

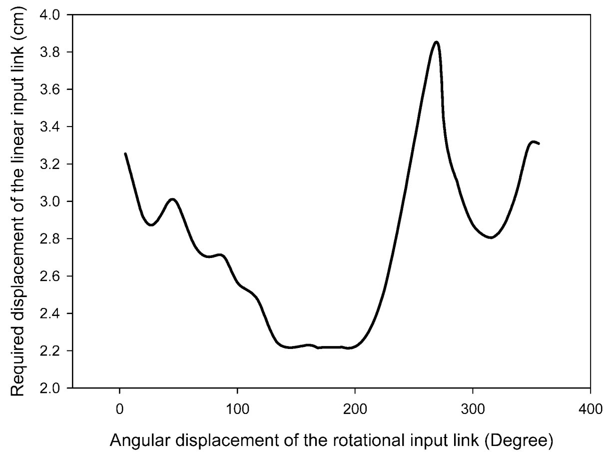

Fig. 4. The required displacements of linear input link corresponding to the current angular displacements of the rotational input link for Example 1.

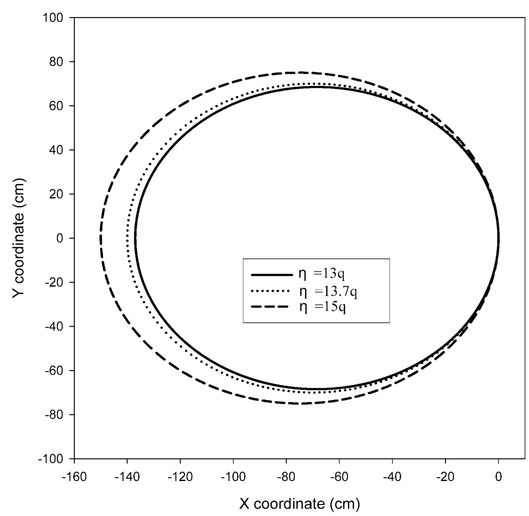

Fig. 5. The required paths in Example 2.

5. EXAMPLES

In the following examples, we demonstrate the proposed five-bar linkage mechanisms as shown in Fig. 2 to generate different kinds of required exact paths by controlling the instantaneous displacement of linear input link while the rotational input link rotates with a constant speed in a cycle.

5.1. Example 1

This example is the same as the one in [14]. A desired path with 25 prescribed precision points, whose coordinates are listed in Table 1, is required to be generated as shown in Fig. 3. The optimal dimensions of the four-bar linkage synthesized in [14] are summarized in Table 2. The required displacements of linear input link corresponding to the current angular displacements of the rotational input link to meet the required exact paths for Example 1 are shown in Fig. 4.

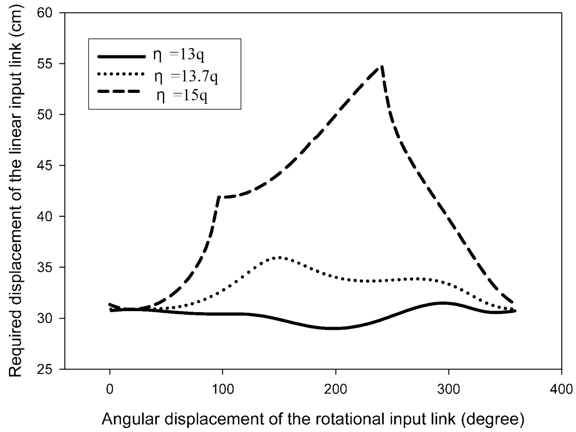

Fig. 6. The required displacements of linear input link corresponding to the current angular displacements of rotational input link for Example 2.

Table 3. The optimal dimensions of the original four-bar linkage according to Ref. [10].

Parameter	L_1 (cm)	L_2 (cm)	L_3 (cm)	L_4 (cm)	L_5 (cm)	$(x_A,\ y_A)$ (cm)	θ_1 (deg)	β (deg)
Value	13.4	32	36	24.9	49	54.98	–43.83	78.99

5.2. Example 2

This example is the same as in [10]. A set of circle paths are required to be generated as shown in Fig. 5. The range of these circle diameters is $13q \leq \eta \leq 15q$, where q is a constant and equal to 10 in this example. The most right point of these circle paths pass through the same point. This point is the origin of the reference frame.

The paths can be expressed as a function of parameter t ($0 \leq t \leq 2\pi$).

$$x_P = \frac{\eta}{2}(\cos t - 1), y_P = \frac{\eta}{2}\sin t \tag{13}$$

The diameters, $\eta = 13q$, $\eta = 14q$ and $\eta = 15q$, are selected for the required paths in this example, respectively. The optimal dimensional synthesis result of the original four-bar linkage is shown in Table 3 [10].

The required instantaneous displacements of linear input link corresponding to the angular displacements of rotational input link to meet the required exact paths for Example 2 are shown in Fig. 6.

The results of these two examples suggest that it is practically feasible to apply this proposed mechanism when the required exact paths including over nine precision points or the several same type required exact paths need to be generated.

6. CONCLUSIONS

A design method of the 2 DOF five-bar linkage mechanisms was presented for exact path generation. A linkage mechanism with minimal structure error corresponding to required exact paths and with a rotational input link and a linear input link was proposed as well. The derivations of instantaneous displacement of the linear input link corresponding to the angular displacement of rotational input link were proposed as well. The required exact paths can be generated by varying the instantaneous displacement of linear input link while the rotational input link rotates with a constant angular speed in a cycle. The examples verified the feasibility and effectiveness of the proposed method. This design approach made the proposed linkage mechanism programmable and adjustable, and increases their flexibility in practical applications.

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