STRUCTURAL SYNTHESIS OF NOVEL BASIC TWO-DOF DIFFERENTIALS

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ABSTRACT
Differentials are very important in many crucial applications, but almost all differentials are geared differential mechanisms. Therefore, the main purpose of this paper is to synthesize non-gear or novel differentials. All kinds of basic differentials can be contained in the results by the method. First, a modified graph representation is provided for differential mechanisms with different type. Second, the design concept of composition and decomposition the geometric constraint is presented. Then, ten fundamental entities and seven properties of differentials are collated. Finally, eight feasible results with two degrees-of-freedom and two basic loops are obtained. About the eight results, there are two are existing designs and the other six are novel.

Keywords: differential mechanisms; structural design; mechanism design.

SYNTHÈSE STRUCTURALE DE NOUVEAUX DIFFÉRENTIELS DE BASE À DEUX DEGRÉS DE LIBERTÉ

RÉSUMÉ
Les différentiels sont très importants dans plusieurs applications capitales, mais presque tous sont des mécanismes de différentiels à engrenages. Par conséquent, le but de cet article, est de faire une synthèse de différents types innovateurs de différentiels sans engrenages. Toutes sortes de différentiels de base peuvent être contenues dans les résultats par cette méthode. Premièrement, une représentation graphique modifiée est fourni pour des mécanismes de différentiels de différents types. Deuxièmement, le concept de création de composition et décomposition des contraintes géométriques est proposé. Ensuite, dix entités fondamentales et sept propriétés des différentiels sont compilées. Finalement, huit résultats possibles avec 2 degrés de liberté et deux boucles de base sont obtenus. Des huit résultats, deux sont des modèles existants et les six autres sont nouveaux.

Mots-clés : mécanismes des différentiels ; synthèse structurale ; conception mécanique.

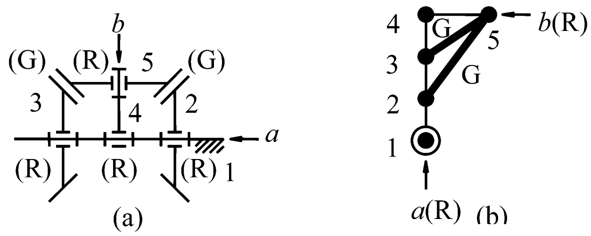

Fig. 1. A typical differential mechanism.

1. INTRODUCTION

Differential mechanisms are well-known for a long time and are widely used in many applications, such as automotive drive systems, planetary differential transmissions, planetary-type continuously variable transmissions, robot grippers, hybrid machines, south-pointing chariots, and power-assisted wheelchairs, etc. As early as in 1906, the first geared differential mechanism was invented [1]. In 1918, a cam-follower differential mechanism was designed [2]. Based on this mechanism, a similar differential mechanism was developed in 1961 [3]. There are different kinds of differential mechanisms. However, up to now, only geared differential mechanisms are commonly used. In Fig. 1a the functional schematics of a typical differential mechanism is shown. It is a spherical mechanism with two degrees-of-freedom (DOF) and with three terminals which are links 2–4 for inputs/outputs.

There are many literatures regarding geared differential mechanisms. Hirose [4] introduced a general form and derived the kinematic constraint equation among input/output links of geared differential mechanisms with 1 input and 2 outputs. In 1990, Morozumi and Kishi [5] studied the mechanisms of a new type south-pointing chariot with an external spur-gear differential gear train. In 1994, Yan and Hsieh [6] proposed a methodology to synthesize the atlas of 2-DOF gear differentials for automotive vehicles. By using a standard differential mechanism and simple epicyclic gear trains as differential building blocks, Kota and Bidare [7] synthesized geared differential mechanisms with DOF greater than two. Based on 1-DOF geared kinematic chains, Hsu and Wu [8] identified the atlas of 2-DOF geared differential mechanisms. By choosing inputs and main components as well as selecting admissible connecting links from the existing atlas of non-fractionated geared kinematic chains, Chen and Yao [9] synthesized fractionated geared differential mechanisms with three and four input/output links. However, gearless differential mechanisms are not considered in the above investigations.

Conceptual design is the most creative and important step in the design of mechanisms. However, some mechanisms with the same functions but distinct structures, such as the geared differential mechanism [1] and gearless differential mechanism [2, 3], could not be synthesized with available design methodology. The purpose of this paper is to propose a systematic method for synthesizing mechanisms with the same functions but distinct structures simultaneously. All possible designs of 2-DOF differential mechanisms with two basic loops will be synthesized based on the concepts of modified graph representation and virtual axes.

2. MODIFIED GRAPH REPRESENTATION

2.1. Rigid Links and Joints

Based on the geometric constraints, the kinematic pairs of a mechanism can be classified into Type-I and Type-II joints. The incident links of a Type-I joint are constrained by the same geometric constraints. For example, a revolute pair is a Type-I joint because its incident links are constrained by the same axis from the viewpoints of each link. And, the incident links of a Type-II joint are constrained by different geometric

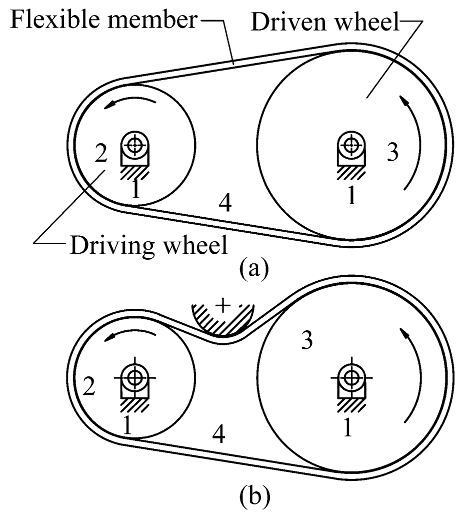

Fig. 2. A flexible connecting mechanism.

constraints. For example, a gear pair is a Type-II joint because its incident links are constrained by different gear shapes from the viewpoints of each gear. For frequently used kinematic pairs of planar and spherical mechanisms, revolute pairs (*R*) and prismatic pairs (*P*) are Type-I joints, and gear pairs (*G*) and cam pairs (*A*) are Type-II joints. For the typical differential mechanism shown in Fig. 1a, there are four Type-I joints (*R* pairs) and two Type-II joints (*G* pairs). The geometric constraints of Type-I joints are the key to the design method, and a suitable representation is needed for expressing them clearly.

In graph representations, thin edges can be represented as the geometric constraints of Type-I joints, and thick edges can be represented as the geometric constraints of Type-II joints. The geometric constraints formed by multiple joints also must be expressed clearly. For example, the modified graph representation for the differential mechanism in Fig. 1a is shown in Fig. 1b. The vertexes represent links in which solid dot is the rigid link and circle is the ground link. Edges represent geometric constraints of joints in which thin edges and thick edges represent the geometric constrains of revolute pairs and gear pairs, respectively. The collinear edges represent the same geometric constraint, e.g., the geometric constraints formed by multiple joints, like coaxial revolute pairs of links 1, 2, 3, and 4 in Fig. 1, are represented by a collinear thin edge. The mechanism in Fig. 1a has two axes, axis a and axis b, so Fig. 1b has two collinear thin edges, edge *a* and edge *b*, respectively.

2.2. Flexible Links and Virtual Axes

Links of mechanisms can be rigid members, flexible members, or compression members. Mechanisms with flexible members or compression members, such as belt drives or hydraulic drives, are also considered in this work. Based on the geometric constraints of axes by R pairs, we propose the concept that the flexible/compression members have virtual axes. For the flexible connecting mechanisms shown in Fig. 2a, a constraint is added on the flexible member, Fig. 2b. However, in the process of conceptual design, the structure and function of these two cases are the same, and it is not certain that whether the constraint is existed and where the location is. Here, we call such a constraint a “virtual axis”. Flexible/compression members have virtual axes and the properties of virtual axes are similar to the axes of *R* pairs. Figure 3 shows the representations of mechanisms with flexible/compression members, where gray dot represents flexible/compression members, and dotted edges represent virtual axes incident to flexible/compression members, which are labeled as VR_f and VR_c, respectively.

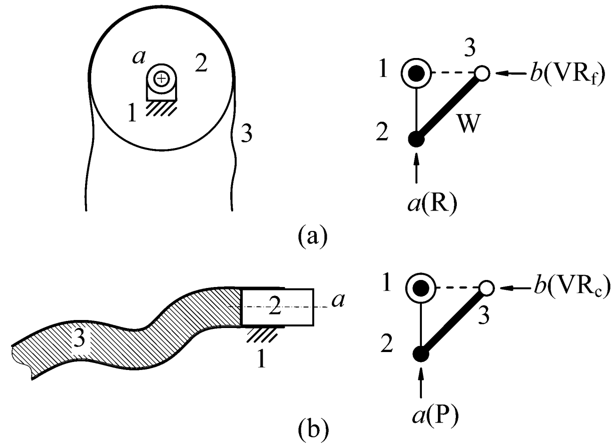

Fig. 3. Mechanisms with flexible or compression members.

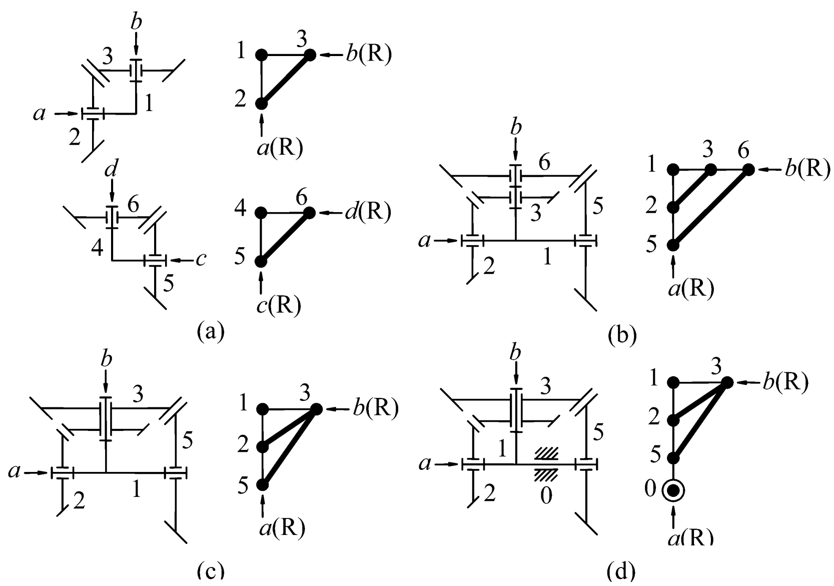

Fig. 4. Composition and decomposition of mechanisms.

3. DESIGN CONCEPTS AND REQUIREMENTS

Since the incident links of a Type-I joint has identical geometric constraint, the geometric constraint of Type-I joints can be taken for composition or decomposition and the incident links on the same geometric constraint of Type-I joints can also be taken for composition or decomposition.

For example, Fig. 4a shows two bevel-gear drives. There are four axes (geometric constraints) due to R pairs (Type-I joints): axes *a*, *b*, *c*, and *d*. It can be composed by axis *a* with axis *c*, axis *b* with axis *d* (Fig. 4b). Then the links on axis *b* in Fig. 4b, link 3 and link 6, can be composed as in Fig. 4c. And the geared differential mechanism shown in Fig. 4d can be obtained by assigning axis *a* in Fig. 4c as a fixed constraint. For the geared differential mechanisms shown in Figs. 2 and 4d, their functional schematics are different, but their graph representations and functions are the same. Therefore, two bevel-gear drives can be composed into one geared differential mechanism. Similarly, a geared differential mechanism also can be decomposed into two bevel-gear trains reversely based on the same logic. Furthermore, the bevel-gear trains can be treated as the fundamental entities of geared differential mechanism as shown in Fig. 2. Based on this concept, novel mechanisms can be synthesized by composing some fundamental entities.

To determine the fundamental entities and the design requirements of differential mechanisms, the following seven properties should be taken into account:

1. The fixed constraint must be assigned on one of the geometric constraints of R pairs for rotated inputs or outputs.
2. There are at least three members on the fixed axis as inputs or outputs.
3. The three members on the fixed axis must be rotatable for continually inputs or outputs.
4. The three members on the fixed axis must rotate with constant speed ratio.
5. The three members on the fixed axis must have differential rotated motion with each other.
6. There are at least two geometric constraints of Type-I joints to be composed or decomposed.
7. The typical differential mechanism is composed of two similar fundamental entities with 1-DOF and one loop.

4. DESIGN PROCESS

Based on the above design concepts, the design process for the conceptual design of differential mechanisms with 2-DOF is as follows:

Step 1: Fundamental entities decision.

Step 2: Geometric constraints composition.

Step 3: Links composition.

Step 4: Fixed constraints assignation.

Step 5: Particularization.

The idea of the design process begins by determining suitable fundamental entities hereby graphs. The second step is to compose the same kinds of geometric constraints, R, P, VR_f, and VR_c. The third step is to compose the links that are incident to the same geometric constraints by design requirements. The fourth step is to assign the fixed constraint to an axis as input/output. The last step is to obtain the atlas of 2-DOF differential mechanisms by particularization.

In the following, the atlas of novel differential mechanisms is synthesized to illustrate this design process in detail.

4.1. Fundamental Entities Decision

The first step of the design process is to decide suitable fundamental entities by the graph representations. Based on the properties of the existing mechanisms, there are four requirements for the fundamental entities: 1-DOF, one loop, at least two geometric constraints of Type-I joints to be composed, and at least one R pair for the input/output axis.

Table 1. Feasible fundamental entities.

n	j	$\sum_{i=1}^{j} f_i$	f_i	Joint assortments
3	3	4	1, 1, 2	R, R, G
				R, R, A
				R, P, G
				R, P, A
4	4	4	1, 1, 1, 1	R, R, R, R
				R, R, R, P
				R, R, P, P
				R, P, R, P

Fundamental entities can be decided by joint assortments, and it starts from the Grübler–Kutzbach criterion:

$$F = \lambda(n - j - 1) + \sum_{i=1}^{j} f_i \tag{1}$$

where F is the number of degrees-of-freedom of a mechanism, λ is the degrees-of-freedom of the space in which a mechanism is intended to function, n is the number of links, j is the number of joints, and f_i is the degrees of relative motion permitted by joint i.

After considering frequently used kinematic pairs and requirements of the fundamental entities, eight feasible fundamental entities can be obtained as listed in Table 1. Their graphs and schematics of the decided fundamental entities are shown in Figs. 5a–h.

Furthermore, the flexible/compression members are also considered as the fundamental entities in this work. The mechanism with flexible members as shown in Fig. 3a satisfies the requirements of the fundamental entities, and can be used for fundamental entities as shown in Fig. 5i. The mechanism with compression members as shown in Fig. 3b has no R pairs and does not satisfy the fourth requirement. So one *R* pair should be added to become a feasible fundamental entity as shown in Fig. 5j. Consequently, there are ten suitable fundamental entities for the design process (Fig. 5).

4.2. Geometric Constraints Composition

The second step of the design process is to compose geometric constraints. Based on the examination of the existing mechanisms, the fundamental entities with identical or similar properties can be composed. By the results of Step 1, two items can be taken from each fundamental entity in Fig. 5 for composing geometric constraints as shown in Figs. 6a–h, j, and k. Besides, the fundamental entities with similar properties also can be composed. Therefore, fundamental entities in Figs. 5a and e can also be taken for composing when the two gears in Fig. 5a are with the same rotated speed and the four-bar linkage in Fig. 5e is a parallelogram mechanism as shown in Fig. 6i. There are 11 possible groups for composing as shown in Fig. 6.

The rule of geometric constraints composition is that only the same kinds of geometric constraints of Type-I joints can be composed with each other, such as *R* pairs can only be composed with *R* pairs, and *P* pairs can only be composed with *P* pairs.

The design requirement of this step is that each group of Fig. 6 must be composed two geometric constraints, and at least one of composed geometric constraints is *R* pair for input/output. Hence, 14 possible compositions (Fig. 7) can be obtained in this step.

4.3. Links Composition

The third step of the design process is to compose the links in the results of the previous step based on the rules of links composition and design requirements.

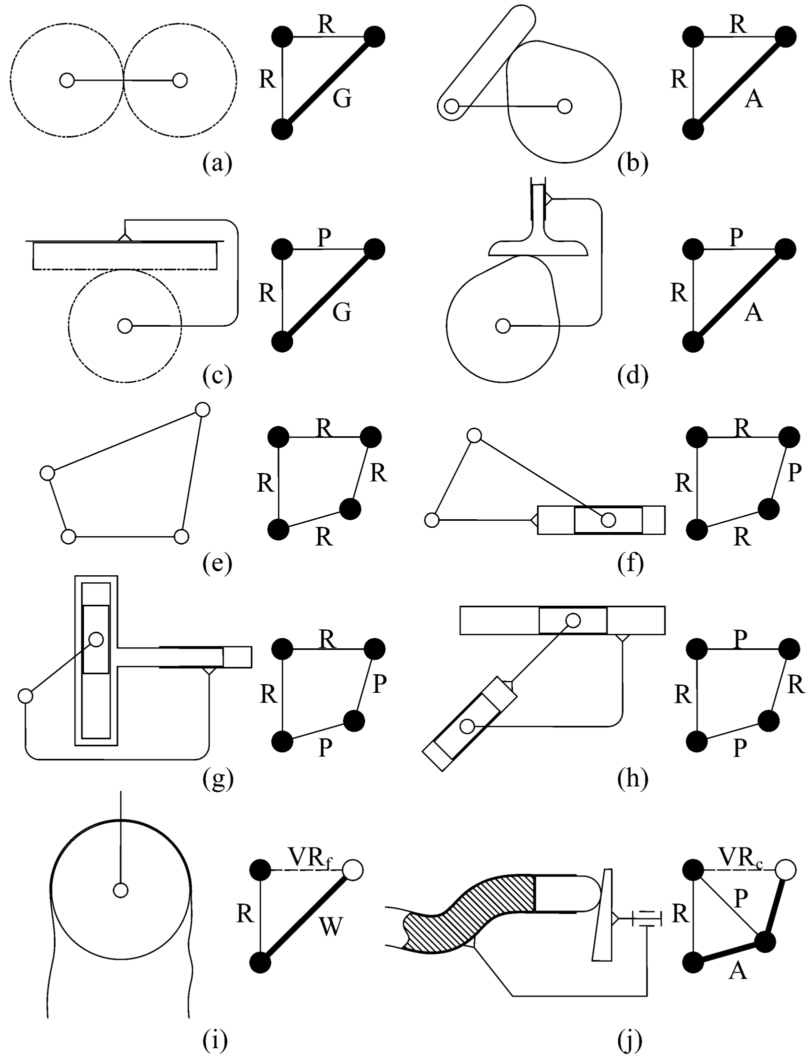

Fig. 5. Results of fundamental entities decision.

The first rule of links composition is that only the links on the same geometric constraints of Type-I joints, such as the same axes or virtual axes, can be composed with each other. The second rule of links composition is that it cannot form multigraphs after composing, such as the links of the same fundamental entity cannot be composed. And, 15 results can be obtained from this step.

4.4. Fixed Constraints Assignation

The fourth step of the design process is to assign the fixed constraint to an axis for inputs or outputs. The rule of fixed constraints assignation is that only geometric constraints of Type-I joints can be assigned, and the assigned geometric constraints must add one fixed link on.

Based on the properties of the existing mechanisms, the design requirements of this step are that the fixed constraint must be assigned on one geometric constraint of *R* pairs with three links. After assigning the fixed constraints, 15 results can be obtained from this step, shown in Fig. 8.

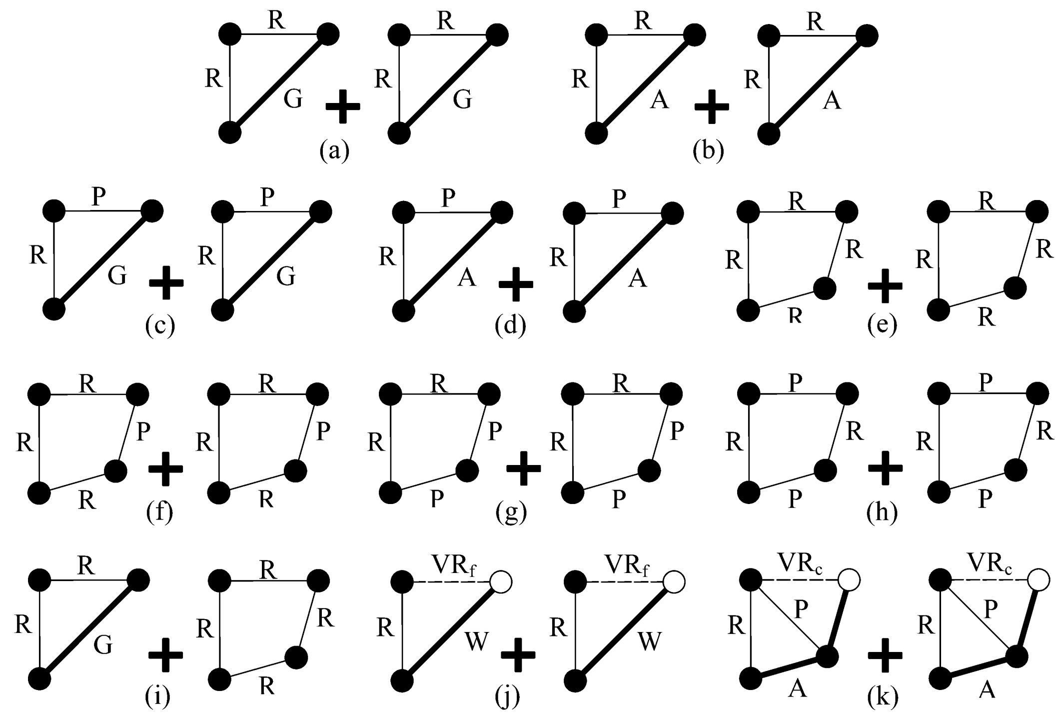

Fig. 6. Groups for composing.

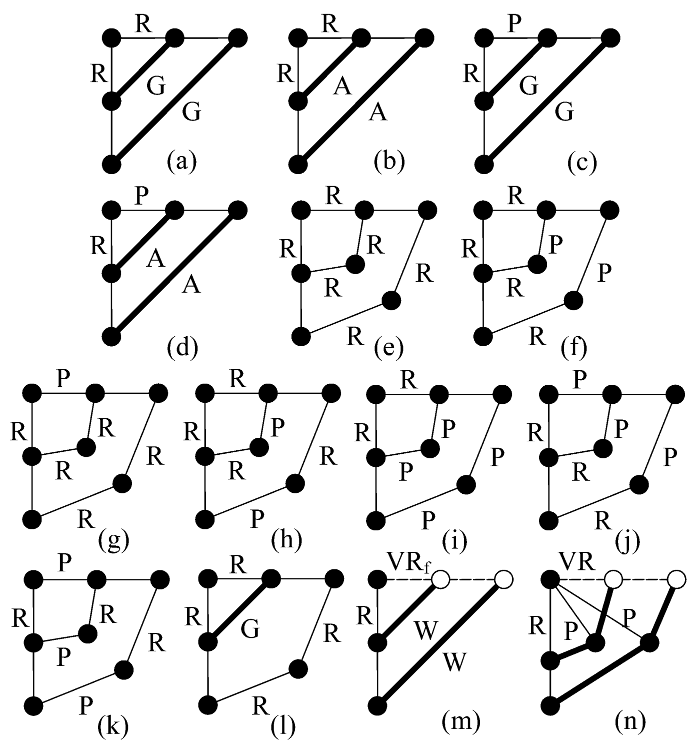

Fig. 7. Results of geometric constraints composition.

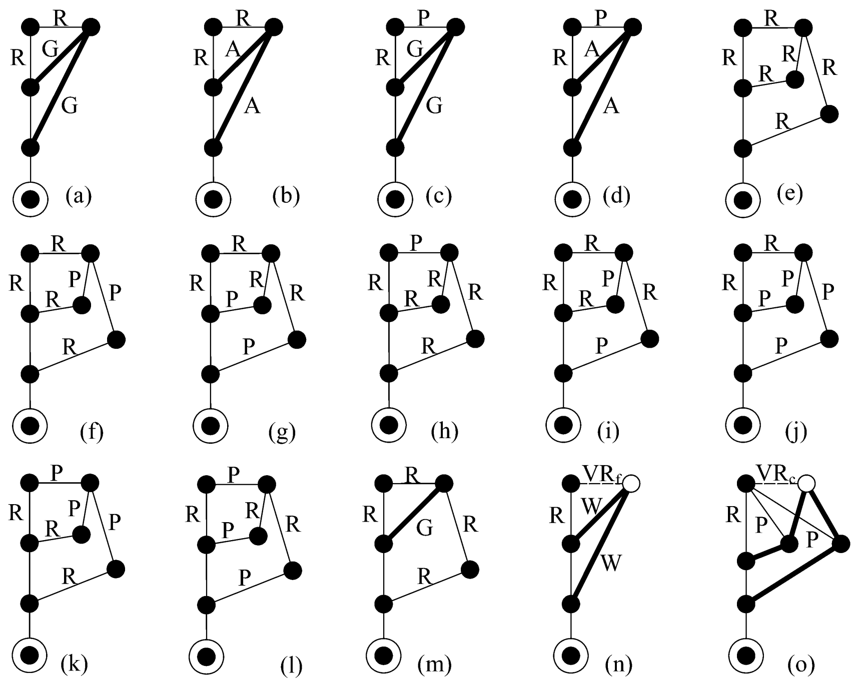

Fig. 8. Results of fixed constraints assignation.

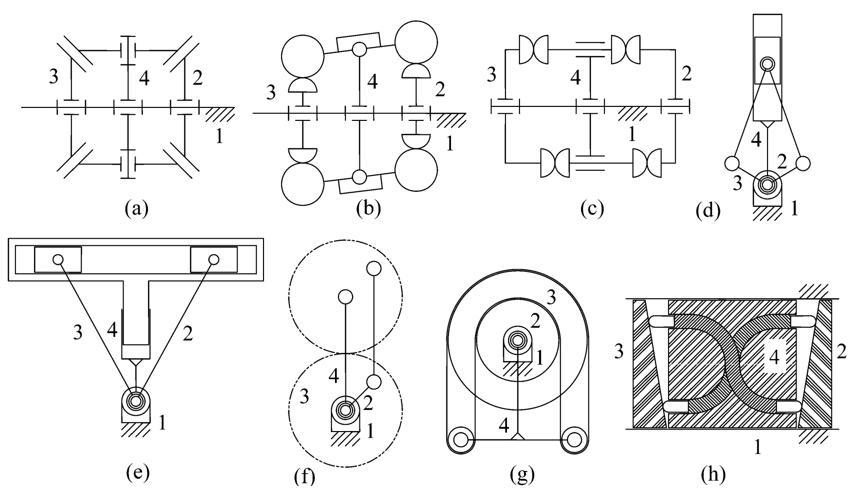

Fig. 9. Results of particularization.

4.5. Particularization

The last step of the design process is to obtain the atlas of 2-DOF differential mechanisms by particularization.

They must be checked by the seven requirements and drew their functional schematics. Particularization is processed by submitting the functional schematics of fundamental entities in Fig. 5 to the 15 results in Fig. 8. The results in Figs. 8c, e–g, i, j, and l are not suitable for design requirements.

Finally, there are eight feasible results for 2-DOF differential mechanisms, which are Figs. 8a, b, d, h, k, and m–o, and are shown in Fig. 9a–h, respectively.

Although their structures and numbers of links are different, they have the same behavior and functions for input/output. Figures 9a and c are existing mechanisms, and the other six results, Fig. 9b, d–h, should be novel.

5. CONCLUSIONS

In summary, this paper proposes a design process for the conceptual design of all possible differential mechanisms with 2-DOF including mechanisms with the same functions but distinct structures based on the modified graph representation and the concept of virtual axes. And eight feasible results are obtained by the design process.

Abstraction for systematic design method is needed, just like graph representations are used to represent mechanisms. However, abstraction usually makes particularization difficult. Based on the proposed concept of fundamental entities, particularization is easy to process by submitting the functional schematics of fundamental entities to the results of graph representations.

Furthermore, this approach is not only useful for the conceptual design of differential mechanisms with 2-DOF and two basic loops but also for other types of mechanisms.

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