

# MULTI-OBJECTIVE OPTIMAL MECHANICAL POWER FOR TURBINE OF RIVER CURRENT POWER GENERATION USING FUZZY DOMINANT DIRECTED-GRAPH METHOD

Jian-Long Kuo and Kai-Lun Chao  
*Department of Mechanical and Automation Engineering,  
National Kaohsiung First University of Science and Technology,  
Nantze, Kaohsiung 811, Taiwan  
E-mail: jlkuo@nkfust.edu.tw*

ICETI 2012-J1131\_SCI  
No. 13-CSME-66, E.I.C. Accession 3524

---

## ABSTRACT

The fuzzy dominant directed graph (fuzzy DDG) method is proposed in this paper to realize the multi-objective optimal mechanical turbine power of a river current (RC) power generation system. The testing case illustrates the problem with vertex S/N ratio values included. Fuzzy potential values are derived. The experimental results are provided to verify the validity of the fuzzy-DDG method. It is clear that this method is applicable; it easily and quickly finds the optimal solution.

**Keywords:** river current (RC); power generation; Multiple Performance Characteristics Index (MPCI); fuzzy dominant directed graph (fuzzy DDG); multi-objective optimization.

---

## OPTIMISATION MULTI-OBJECTIF DÉNERGIE MÉCANIQUE POUR TURBINE DE PRODUCTION D'ÉNERGIE ÉLECTRIQUE PAR COURANT DE RIVIÈRE

### RÉSUMÉ

La méthode de graphe à ensemble dominant à logique floue (fuzzy DDG) est proposée dans cet article pour réaliser l'optimisation multi-objectif d'énergie mécanique d'une turbine d'un système de production d'énergie électrique produite par courant de rivière. Les tests d'essai ont mis en évidence le problème avec les valeurs des moyennes de sommets S/N inclus. Les valeurs floues potentielles sont dérivées. Les résultats sont fournis pour vérifier la validité de la méthode fuzzy DDG. Il se dégage que cette méthode est applicable ; elle trouve facilement et rapidement la solution optimale.

**Mots-clés :** courant de rivière ; production d'énergie ; performance multiple ; index caractéristique ; ensemble dominant à logique floue (fuzzy DDG) ; optimisation multi-objectif.

## 1. INTRODUCTION

Recently, renewable energy such as wind, solar and ocean power, has become more and more important in regard to future energy supplies [1–3]. Like ocean current power, river current (RC) power generation systems also have great potential for providing energy in renewable energy applications. River currents have rapid water speed. An AC generator with turbine can harvest the river current power into the AC generator. In Taiwan, there are many rivers stemming from the Central Mountain.

The output power of an AC generator is related to power density, gear transmission, radius of turbine, and water current speed. The Taguchi method is used to find the optimal solution for the output power and efficiency. In this paper, the optimization of output power and the optimization of efficiency of AC generator are discussed. The multi-objective optimization problems concerning both output power and efficiency are examined. A fuzzy MPCI is used to find the optimal solution via the Taguchi method. Two optimal methods are illustrated: the first one is conventional fuzzy inference process, and the second one is the proposed fuzzy DDG approach. The fuzzy DDG does not require the conventional fuzzy inference; only the matrix operation is required. The fuzzy potential value is defined to attest to the strength of the experimental data.

The optimal output power for an RC power generation system is studied herein. The optimization of output power is analyzed by using the Taguchi method. The overall study cases are classified in Fig. 1. Five cases are studied. The first is to derive the optimal output power of the AC generator. The second case is to derive the optimal efficiency of the AC generator. The third case is to simultaneously optimize both output power and efficiency. MPCI is used to perform the multiple objective functions, and conventional fuzzy inference is performed. The fourth and fifth cases are to find the MPCI by using the Fuzzy DDG method. The results show that the fourth and fifth cases have easier arithmetic matrix operation; no tedious fuzzy inference process is required.

The major problem of the power generation system concerns output power and power efficiency. Therefore, the two objective functions are selected. The multi-objective optimization problem for the two objective functions is studied.

## 2. STRUCTURE OF RIVER CURRENT POWER GENERATION

### 2.1. River Current Power Generation

The river current power generation pre-study MSIPSB project aims to study the possibility of establishing a river current power generation system near the Cross Sea Bridge at Peng-Hu Island in Taiwan. Due to the cost and limited budget, the pre-study project seeks to verify the possibility of setting up a small-scale river current power generation system.

There are two types of ocean current power generation: the first one is tidal current power, and the second one is ocean current power. The developed AC generator is suitable for both types. The turbine of the AC generator is used to take advantage of the river current flowing through the turbine. The tidal current generator has to be set up in a specific place where the tidal change is large. The river current generator is usually set up deep under the water. The depth of the generator depends on where the river current speed has maximum value. The river current power generation has great potential in renewable energy and has the following advantages compared with wind power:

1. It is noiseless since it is set up under the sea. The noise above the sea is eliminated.
2. Smaller radius turbine: for the same output power, the turbine for river current power is smaller than the turbine for wind power.

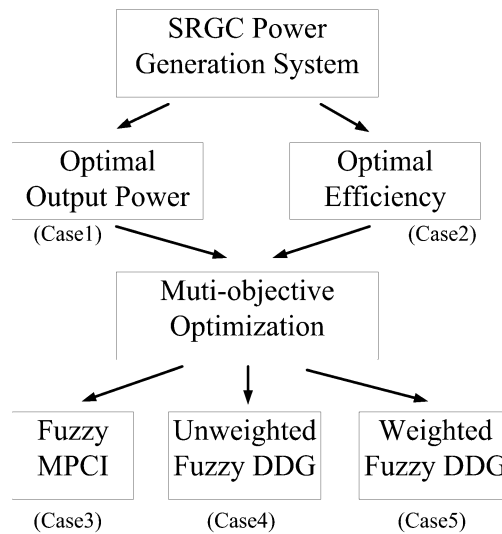


Fig. 1. Overall view of optimization problem on RC power generation system.

## 2.2. River Current Power Pre-study MSIPSB Project

In order to observe and easily control the variables in river current power generation, an experimental dynamometer with a 4-ton water tank was built in our laboratory, as shown in Fig. 2. It is very useful to emulate the river current in an indoor environment. The turbine of the AC generator was coupled with another turbine driven by a prime mover on the opposite side to emulate generated river current. The location is near the Cross Sea Bridge at Penghu Island in Taiwan. The pre-study of a practical system is assessed in this project. A dynamometer with a 4-ton water tank was used to assess all of the possible operating conditions.

Three ocean currents gather here and the tidal current and ocean wave are also significant: they are China's coastal current, Kuroshio tributary current and South China Sea warm Current. The clear seawater, pristine beaches and beautiful skies make Penghu a favorite ocean resort area. In summer the Kuroshio tributary current, and in winter China's coastal current, pass through the Penghu sea territory, contributing to a rich and diverse marine life around the island. The maximum ocean current speed range is 3 m/s. The ocean wave height is about 1.5 m in summer and 1.7 m in winter. Not only the ocean current but also the air current (wind power) is powerful in this area. The river current power generation system was set up near the Cross Sea Bridge. In this area, there are many ocean currents due to the special geographic position. A narrow water canal makes the ocean current speed up in this area. Pon-Hu Island is located in the centre of Taiwan Strait. Ho-Men Canal in PON-HU Island is named due to its rapid ocean current speed.

At the first stage of this project, a laboratory-scale was built first to evaluate the future river current power generation. A small turbine was used to receive the emulated river current power. In this paper, a dynamometer with water tank is developed for the associated study and evaluation. The AC generator is used to harvest the river current energy. The river current testing system is built up as a dynamometer with a water tank to emulate the practical river current behavior. For outdoor testing of mequipment, it is difficult to control the experimental factors. It is not easy to control the system variables for experimentation. However, for indoor testing of equipment, the testing conditions are easy to control. The required measurement data are easy to obtain in order to to analyze the problem.

## 3. DOMINANT DIRECTED GRAPH FORMULATION

### 3.1. Fuzzy Directed Graphs

A directed graph or digraph is a pair  $G = (V_g, A_g)$  with the following definitions [10–20]:

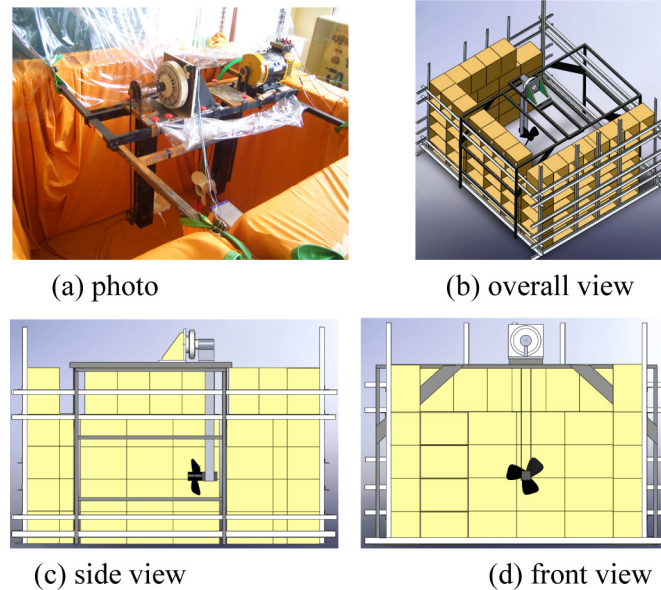


Fig. 2. The experimental dynamometer with water tank for the river current power generation system.

- (a) a set  $V_g$ , whose elements are called vertices or nodes,
- (b) a set  $A_g$  of ordered pairs of vertices, called arcs, directed edges, or arrows.

It differs from an ordinary or undirected graph in that the latter is defined in terms of edges, which are unordered pairs of vertices.

First, a finite set of elements is defined as  $\{P_1, P_2, \dots, P_n\}$ . A finite collection of ordered pairs  $(P_i, P_j)$  of distinct elements of this set is defined together. No ordered pair is repeated. The elements of the set are called vertices. The ordered pairs are called directed edges in the directed graph  $G$ . The notation  $P_i \rightarrow P_j$  indicates that the directed edge  $(P_i, P_j)$  belongs to the directed graph  $G$ . For a directed graph, the vertices are represented as points in the plane.

The directed edge  $P_i \rightarrow P_j$  is represented by drawing a line or arc from vertex  $P_i$  to vertex  $P_j$ , with an arrow pointing from  $P_i$  to  $P_j$ ;  $P_i \leftrightarrow P_j$  means that both  $P_i \rightarrow P_j$  and  $P_j \rightarrow P_i$  hold at the same time. A bidirectional single line with two oppositely pointing arrows is defined between  $P_i$  with  $P_j$ .

A directed graph may have separate “components” of vertices that are connected to any other vertex. Also,  $P_i \rightarrow P_j$  is not permitted in a directed graph. A vertex cannot be connected to itself by a single arc that does not pass through any other vertex. With a directed graph having  $n$  vertices, an  $n \times n$  matrix  $M = (m_{ij})$  is defined in the following, called the crisp vertex matrix of the graph. Its elements are specified with the following crisp relation:

$$m_{ij} = \left\{ \begin{array}{ll} d, & \text{where } d = 1, \text{ if } P_i \rightarrow P_j \\ 0, & \text{otherwise} \end{array} \right\} \quad (1)$$

where  $i, j = 1, 2, \dots, n$ .

Furthermore, with a directed graph having  $n$  vertices, an  $n \times n$  matrix  $K = (k_{ij})$  is defined in the following, called the fuzzy vertex matrix of the graph. Its elements are specified with the following fuzzy relation:

$$k_{ij} = \left\{ \begin{array}{ll} d, & \text{where } d \in [0, 1], \text{ if } P_i \rightarrow P_j \\ 0, & \text{otherwise} \end{array} \right\} \quad (2)$$

where  $i, j = 1, 2, \dots, n$ .

According to the above definition, vertex matrices have the following two properties:

- (i) All entries belong to  $d \in [0, 1]$ . When  $d = 1$ , the relation is the crisp value.
- (ii) All diagonal entries are 0.

Conversely, any matrix with these two properties determines a unique directed graph. It can be a vertex matrix. To find the number of all possible  $r$ -step connections ( $r = 1, 2, \dots$ ) from one vertex  $P_i$  to another vertex  $P_j$  of an arbitrary directed graph, the following technique is developed. The case for  $i = j$  is also included in which  $P_i$  and  $P_j$  are the same vertex. The number of 1-step connection from  $P_i$  to  $P_j$  is simply defined as  $m_{ij}$ . That is to say, depending on whether  $m_{ij}^{(2)}$  is zero or one, either zero or one 1-step connection from  $P_i$  to  $P_j$  can be specified. For the case of the number of 2-step connections, the square of the vertex matrix is considered. The  $(i, j)$ -th element of  $M^2$  can be expressed as

$$m_{ij}^{(2)} = m_{i1}m_{1j} + m_{i2}m_{2j} + \dots + m_{in}m_{nj} \quad (3)$$

If  $m_{i1} = m_{1j} = 1$ , a 2-step connection  $P_i \rightarrow P_1 \rightarrow P_j$  from  $P_i$  to  $P_j$  can be found. If either  $m_{i1}$  or  $m_{1j}$  is zero, such a 2-step connection cannot be found in a graph. Then, we can conclude that  $P_i \rightarrow P_1 \rightarrow P_j$  is a 2-step if, and only if,  $m_{i1}m_{1j} = 1$ . In a generalized form,  $P_i \rightarrow P_k \rightarrow P_j$  is a 2-step connection from  $P_i$  to  $P_j$  for any  $k = 1, 2, \dots, n$ , if, and only if, the term  $m_{ik}m_{kj}$  is one. Otherwise, the term is zero. Thus, the  $(i, j)$ -th element of  $M^2$  is the total number of 2-step connections from  $P_i$  to  $P_j$ .

### 3.2. Fuzzy Dominant Directed Graphs

In many groups of individuals, there is a definite dominance relation between any two members of the group. For any two given individuals  $A_d$  and  $B_d$ , either  $A_d$  dominates  $B_d$  or vice versa, but not both. The directed graph can describe the same fact in the same way. The relation  $P_i \rightarrow P_j$  means  $P_i$  dominates  $P_j$ ; this means that for all different pairs, either  $P_i \rightarrow P_j$  or  $P_j \rightarrow P_i$ , but not both. In general, the following fact is defined.

**Definition 1.** A fuzzy dominant-directed graph is a fuzzy directed graph such that for any different pair of vertices  $P_i$  and  $P_j$ , either  $P_i \rightarrow P_j$  or  $P_j \rightarrow P_i$ , but not both. The directed graph can be applied to a league with  $n$  sports teams. They play against each other exactly one time, as in one round of a round-robin tournament in which no ties are allowed. The fuzzy relation  $P_i \rightarrow P_j$  means that team  $P_i$  beat team  $P_j$  in their single match. The definition of a dominant-directed group is satisfied. Therefore, dominant directed graphs are sometime called tournaments.

**Theorem.** In any dominance-directed graph, there is at least one vertex from which there is a 1- step or 2-step connection to any other vertex.

A vertex with the largest total number of 1-step and 2-step connections to other vertices has the above property stated in the theorem. There is a simple way to find such vertices using the vertex matrix  $M$  and its square matrix  $M^2$ . The sum of the entries in the  $i$ -th row of  $M$  is the total number of 1-step connections from  $P_i$  to other vertices. Also, the sum of the entries of the  $i$ -th row of  $M^2$  is the total number of 2-step connections from  $P_i$  to other vertices. Therefore, the sum of the entries of the  $i$ -th row of the matrix  $A_M = M + M^2$  is the total number of 1-step and 2-step connections from  $P_i$  to other vertices. This means that a row of  $A_M = M + M^2$  with the largest row sum specifies a vertex having the property stated in the above theorem. Therefore, a vertex with the largest number of 1-step and 2-step connections to other

vertices can be further called a “powerful” vertex. This concept can be concluded as the following definition.

**Definition 2.** The un-weighted crisp potential value of the vertex of a dominance-directed graph is the total number of 1-step and 2-step connections from it to other vertices. Alternatively, the crisp potential value of a vertex  $P_i$  is the sum of the entries of the  $i$ -th row of the matrix  $A_M = M + M^2$ , where  $M$  is the crisp vertex matrix of the crisp directed graph.

**Definition 3.** The weighted fuzzy potential value of the vertex of a dominance-directed graph is the total number of 1-step and 2-step connections from it to other vertices. Alternatively, the weighted fuzzy potential value of a vertex  $Q$  is the sum of the entries of the  $i$ -th row of the matrix  $B_K = K + K^2$ , where  $K$  is the fuzzy vertex matrix of the fuzzy directed graph.

### 3.3. Multi-objective Problem Solving by Fuzzy DDG Method

When the DDG method is applied together with the Taguchi method, the S/N ratio should be considered. Therefore, the fuzzy vertex matrix is modified into a weighted fuzzy vertex matrix. The weighted fuzzy vertex matrix  $K = (k_{ij})$  is defined as follows:

$$k_{ij} = \eta_i m_{ij} = d, \quad d \in [0, 1] \quad (4)$$

to describe the fuzzy relation for  $m_{ij}$  where  $m_{ij} = 1$  or  $0$ .

The following two objective functions in the RC power generation system are considered:

- Objective function #1: to derive the optimal output power of the AC generator.
- Objective function #2: to derive the optimal efficiency of the AC generator.

For the objective function #1, the un-weighted crisp potential value derived by considering the vertex order only is defined as

$$A_{M1} = M_1 + M_1^2 \quad (5)$$

The weighted fuzzy potential value derived by considering the S/N ratio values is defined as

$$B_{K1} = K_1 + K_1^2 \quad (6)$$

For the objective function #2, the un-weighted crisp potential value derived by considering the vertex order only is defined as

$$A_{M2} = M_2 + M_2^2 \quad (7)$$

The weighted fuzzy potential value derived by considering the S/N ratio values is defined as

$$B_{K2} = K_2 + K_2^2 \quad (8)$$

Therefore, for the multiple objective problems, the composite index can be defined as: The un-weighted crisp potential value derived by considering the vertex order only, is defined as

$$A_{\text{tot}} = \sum_j A_j = A_1 + A_2 \quad (9)$$

The weighted fuzzy potential value derived by considering the S/N ratio values is defined as

$$B_{\text{tot}} = \sum_j B_j = B_1 + B_2 \quad (10)$$

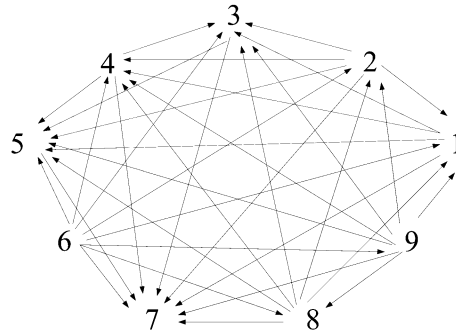


Fig. 3. Directed graph for objective function #1.

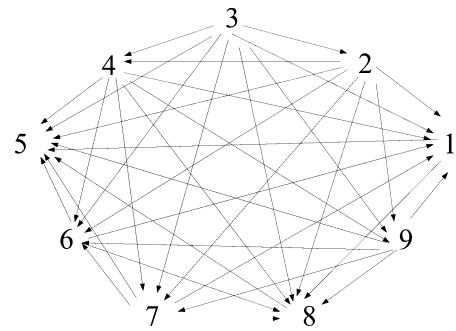


Fig. 4. Directed graph for objective function #2.

The fuzzy potential values can be obtained with the above defined matrix operation. No conventional fuzzy inference is required. The fuzzification and defuzzification processes are not required in this approach. The graphic representations for the two matrices are plotted in Figs. 3 and 4.

The formulation can be very useful to apply onto multi-objective optimization. The proposed two matrices are mapped to two objectives: output power and efficiency respectively. The derived matrices and results are derived as follows:

$$M_1 = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \end{bmatrix} \quad (11)$$

$$A_{M1} = M_1 + M_1^2$$

$$A_{M1} = \begin{bmatrix} 0 & 0 & 1 & 1 & 3 & 0 & 4 & 0 & 0 \\ 1 & 0 & 3 & 2 & 4 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 2 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 4 & 3 & 6 & 5 & 7 & 0 & 8 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 4 & 3 & 5 & 0 & 6 & 0 & 0 \\ 3 & 2 & 5 & 4 & 6 & 0 & 7 & 1 & 0 \end{bmatrix} \quad (12)$$

$$M_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix} \quad (13)$$

$$A_{M2} = M_2 + M_2^2$$

$$A_{M2} = \begin{bmatrix} 0 & 0 & 0 & 0 & 2 & 0 & 0 & 1 & 0 \\ 5 & 0 & 0 & 1 & 7 & 4 & 3 & 6 & 2 \\ 6 & 1 & 0 & 2 & 8 & 5 & 4 & 7 & 3 \\ 4 & 0 & 0 & 0 & 6 & 3 & 2 & 5 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 3 & 0 & 0 & 2 & 0 \\ 2 & 0 & 0 & 0 & 4 & 1 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 5 & 2 & 1 & 4 & 0 \end{bmatrix} \quad (14)$$

#### 4. CONCLUSION

In this paper, the optimization problems of output power and the efficiency for a turbine of an RC power generation system are discussed. It is easier to find the optimal solution quickly by way of proposed formulation. It is easy and convenient to apply proposed optimization technique to analyze the power generation problem.

#### ACKNOWLEDGEMENTS

The experimental equipments from the T.S. Inc. are deeply appreciated. The authors would like thank the National Science Council for its financial support. Thanks to Tzeng Tseng for typing the article.

#### REFERENCES

1. Jones, A.T. and Westwood, A., "Recent progress in offshore renewable energy technology development", *IEEE on Power Engineering Society General Meeting*, Vol. 2, pp. 2017–2022, June 2005.
2. Elghali, S.E., Benbouzid, M.E.H. and Charpentier, J.F., "Marine tidal current electric power generation technology: state of the art and current status", *IEEE International on Electric Machines & Drives Conference*, Vol. 2, pp. 1407–1412, May 2007.



3. Lang, C., "Harnessing tidal energy takes new turn", *IEEE Spectrum*, Vol. 40, No. 9, pp. 13, Sep. 2003.
4. Drouen, L., Charpentier, J.F., Semail, E. and Clenet, S., "Study of an innovative electrical machine fitted to marine current turbines", in *Proceedings OCEANS 2007 in Europe*, pp. 18–21, June 2007.
5. Spooner, E. and Williamson, A.C., "Direct coupled, permanent magnet generators for wind turbine applications", *IEE Proceedings Electric Power Applications*, Vol. 143, No. 1, pp. 1–8, January 1996.
6. Yuen, K., Thomas, K., Grabbe, M., Deglaire, P., Bouquerel, M., Osterberg, D. and Leijon, M., "Matching a permanent magnet synchronous generator to a fixed pitch vertical axis turbine for marine current energy conversion", *IEEE Journal of Oceanic Engineering*, Vol. 34, No. 1, pp. 24–31, June 2009.
7. Jangamshetti, S.H. and Rau, V.G., "Site matching of wind turbine generators: A case study", *IEEE Transactions on Energy Conversion*, Vol. 14, No. 4, pp. 1537–1543, Dec. 1999.
8. Musgrove, P.J., "Wind energy conversion – An introduction", *IEE Physical Science, Measurement and Instrumentation, Management and Education, Reviews*, Vol. 130, No. 9, pp. 506–516, Dec. 1983.
9. Lin, K.H., Sullivan, L.P. and Taguchi, G., "Using Taguchi methods in quality engineering", *Quality Progress*, pp. 55–59, 1990.
10. Kumar, A., Motwani, J. and Otero, L., "An application of Taguchi's robust experimental design technique to improve service performance", *International Journal of Quality and Reliability Management*, Vol.13, pp. 85–98, 1996.
11. Alberto, A.B., "Numerical evaluation of the performance of a compression ignition Cng engine for heavy duty trucks with an optimum speed power turbine", *International Journal of Engineering and Technology Innovation*, Vol. 1, No. 1, pp. 12–26, 2011.
12. Soliman, A.M.A., Kaldas, M.M.S., Barton, D.C. and Brooks, P.C., "Fuzzy-skyhook control for active suspension systems applied to a full vehicle model", *International Journal of Engineering and Technology Innovation*, Vol. 2, No. 2, pp. 85–96, 2012.
13. Lin, Y.C., Le, Q.K., Lai, L.W., Liao, R.M., Jeng, M.S. and Liu, D.S., "Optimizing the organic/inorganic barrier structure for flexible plastic substrate encapsulation", *International Journal of Engineering and Technology Innovation*, Vol. 2, No. 3, pp. 184–194, 2012.