STUDY OF A NEW AND LOW-COST MEASUREMENT METHOD OF VOLUMETRIC ERRORS FOR CNC FIVE-AXIS MACHINE TOOLS

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ICETI 2012-A1017_SCI
No. 13-CSME-18, E.I.C. Accession 3476

ABSTRACT

An effective and inexpensive volumetric error measurement method is an essential of the software-based error compensation method that can improve the machining accuracy of a CNC machine tool without increasing hardware manufacturing cost. In this paper, a new volumetric-error measurement method incorporating of three derived error models, two-step measurement procedure, and use of telescoping ball-bar was proposed for three major types of five-axis machine tools. Comparing to the methods currently used in industry, the proposed method provides the advantages of low cost, easy setup, and high efficiency. The simulation and experimental results have shown the feasibility and effectiveness of the method.

Keywords: error measurement; volumetric error; degree-of-freedom; five-axis machine tool.

ÉTUDE D'UNE NOUVELLE MÉTHODE DE MESURE D'ERREURS VOLUMÉTRIQUES PEU COÛTEUSE POUR UNE MACHINE-OUTILÀ CINQ AXES

RÉSUMÉ

Une méthode efficace et peu coûteuse de mesure d'erreurs volumétriques est essentielle pour un logiciel de compensation d'erreurs qui peut améliorer l'efficacité d'une machine-outil CNC sans accroître le coût du matériel de production. Dans cet article, une nouvelle méthode de mesure d'erreurs volumétriques incorporant trois modèles dérivés, une procédure de mesure en deux étapes, et l'utilisation d'une barre à bille télescopique, a été proposé pour trois types principaux de machine-outil à cinq axes. En comparaison avec les méthodes actuellement utilisées dans l'industrie, la machine proposée a l'avantage d'être peu coûteuse, facile à installer, et de grande efficacité. Les résultats de simulation et d'expériences ont démontrés la faisabilité et l'efficacité de la méthode.

Mots-clés : mesure d'erreur ; erreur volumétrique ; degré-de-liberté ; machine-outil à cinq axes.

1. INTRODUCTION

Due to the development of modern industry, in-use precision five-axes machine tools have increased over triple, and the application for industrial circles have significantly increased. Because of the tight tolerance of products, in addition to the degrees of freedom for machining, the machining accuracy is the key factor to show the capability and possible application of a five-axis machine tool. Volumetric error compensation technique has been recognized as an effective way to further improve the accuracy of multi-axis machine tools. In error compensation process, knowing the values of errors prior to compensation is necessary. Thus, a method that can effectively determine volumetric errors of a five-axis machine tool is an essential of the error compensation technique. The existing measurement method requires an expensive instrument and has complex measurement process. In this study, a volumetric error measurement method with characteristics of low cost, easy setup, and high efficiency was developed for five-axis CNC machine tools.

Many researches had been carried out for machine accuracy tests. Bryan [1, 2] developed the magnetic ball-bar to obtain the total position error of a machine at various points. Although the test was not complete for all types of errors, it was quick and easy to perform, and gives good estimates for some of the error components. Two versions of the Magnetic Ball Bar [MBB] and a simple method for testing coordinate measuring machines and machine tools have been developed at the Lawrence Livermore National Laboratory. The method was intended to replace the circular comparison standard of the circular test for machine tools. Compared with the standard discs used in the circular tests, the MBB is more cost effective, easier to use, and more accurate. Based on the assumption that machines were rigid bodies, Ehmann [3] used Homogeneous Transformation Matrix to develop geometry error models for multi-axis machines. Kiridena and Ferreira [4] used HTM to build individual mathematics model of geometry errors for common five-axis mechanism. In order to enhance the accuracy of high speed machining, Cao et al. [5] investigated the variations of interference fit and bearing preload condition induced by centrifugal expansion deformations at high speed. With consideration of the centrifugal expansion deformation, a dynamic model of high-speed rolling ball bearings was presented with experimental validation. On the basis of the statistics of the relation of selection and theoretical variances, Hajiyev [6] proposed a new structure and algorithmic provision of continuously operated measurement system with the error self-correction which allows to detect the changes in the statistical characteristics of errors. Erkan et al. [7] presented a cluster approach to the analysis of volumetric error for five-axis machine tools. Lei et al. [8] proposed a reduced error model which describes the influence of each unknown and not measurable link error on the overall position errors of a five-axis machine tool, and developed a probe-ball device which can measure the overall position errors of five-axis machine tools directly. Based on the reduced model and the overall position errors, the link errors can be estimated accurately with the least square estimation method. Sakamoto [9] used double ball-bar (DBB) to measure geometry errors for five-axis machine, and found the values of individual error by the inference of mathematics error model of measurement.

Most of the published measurement methods were aimed at measuring error components of an individual axis of motion that are subsequently used in association with the machine error models to evaluate the volumetric errors. However, these measurements cannot be directly used for error compensation. Based on the assumption that points in machine's workspace located close to each other exhibit the same total position errors, Wang [10, 11] developed the single socket method (SSM). By using ball-bar, the method provides the capability of directly measuring the total position errors of a machine at discrete points in its workspace. The measurements can be directly used for cutting trajectory error compensation. However, the method is still unable to identify the orientation errors of a five-axis machine tool. To perform the error compensation for a five-axis machine, both of the total position errors and orientation errors of the machine need to be determined. Most current measurement techniques for five-axis machines are only for single axis calibration, and usually require expensive measurement instruments, such as laser measurement equipment etc. In addition, it is usually very time-consuming for instrument setup for continuously measuring the machine errors when the machine moves with 5-DOF motion. In this study, with extending the previous research outcome-SSM, a new volumetric errors measurement method incorporating with error models and the two-step measurement procedures, was developed for the three major types of five-axis machine tools. Errors measured with this method can be directly used for error compensation [12]. The instrument used in this method is a telescoping ball bar system which is much cheaper than other precision measurement instrument. Thus, the proposed method offers the advantages of low cost, easy setup, and high efficiency for implementation in industry.

In this paper, the measurement principle and the error models for the three types of five-axis machines were presented in Section 2. The algorithm and measurement procedure of the method were addressed in Section 3. In Section 4, both the simulation analysis and experimental results were discussed. Finally, conclusions were given in Section 5.

2. MEASUREMENT PRINCIPLE

Five-axis machine tools compose of three translation axes (T) and two rotating axes (R). According to the configuration of machine, three major types of five-axis machine tools (Fig. 1) are widely used for industry: (a) RRTTT type – two rotational axes attached to the machine spindle and three translational axes for the movements of table and spindle housing; (b) TTTRR type – three translational axes (x, y, and z axis) for the movements of table and spindle housing, and two rotational axes attached to the table; (c) RTTTR type – one rotational axis attached to machine spindle, three translational axes (x, y, and z axis) for the movements of table and spindle housing, and one rotational axis attached to the table. The volumetric errors of five-axis machine tools include three total position errors and two rotation errors.

To measure the volumetric errors of a five-axis machine tool, the 5-DOF movement of the machine is divided into two consecutive movements: a 3-DOF translation and a 2-DOF rotation. Firstly, the machine moves for the translation movement, and SSM is used to measure the total position errors at this location. Then, the machine moves for the rotation movement, and SSM is again used to measure the total position errors at this location. The errors measured at this location are the total position errors caused by the 5-DOF movements. Finally, the orientation errors of the machine can be determined by substituting the two sets of total position errors into the derived orientation error models.

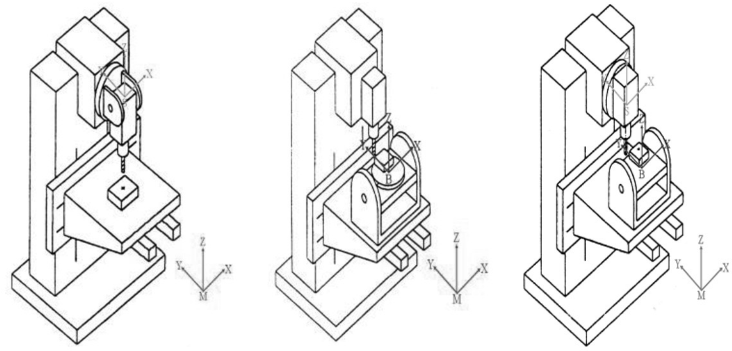
2.1. Error Models

2.1.1. Total position error models

Figure 2 shows the fundamental concept of SSM [6, 7]. When a reference point (x_0, y_0, z_0) is selected, total position errors $(\Delta x, \Delta y, \Delta z)$ at point (x_{P1}, y_{P1}, z_{P1}) can be determined by

$$\begin{bmatrix} \Delta x \\ \Delta y \\ \Delta \end{bmatrix} = \begin{bmatrix} x_{P1} - x_0 & y_{P1} - y_0 & z_{P1} - z_0 \\ x_{P2} - x_0 & y_{P2} - y_0 & z_{P2} - z_0 \\ x_{P3} - x_0 & y_{P3} - y_0 & z_{P3} - z_0 \end{bmatrix}^{-1} \begin{bmatrix} l_{n,1} \cdot \Delta l_1 \\ l_{n,2} \cdot \Delta l_2 \\ l_{n,3} \cdot \Delta l_3 \end{bmatrix}, \tag{1}$$

where (x_{P2}, y_{P2}, z_{P2}) and (x_{P3}, y_{P3}, z_{P3}) are the coordinates of the two neighboring points. $l_{n,i}$ and $\Delta l_i (i = 1, 2, 3)$ respectively represent the nominal distance and distance error between (x_0, y_0, z_0) and the three points, (x_{P1}, y_{P1}, z_{P1}), (x_{P2}, y_{P2}, z_{P2}), and (x_{P3}, y_{P3}, z_{P3}). The distance error $\Delta l_i (i = 1, 2, 3)$ can be directly measured with use of ball-bar. Equation (1) is the total position error model. It can be used to determine the total position errors of a multi-axis machine tool with the advantages of low cost, easy setup, and quick measurement.



(a) RRTTT-type (b) TTTRR-type (c) RTTTR-type

Fig. 1. Schematic of three types of five-axis machine tools.

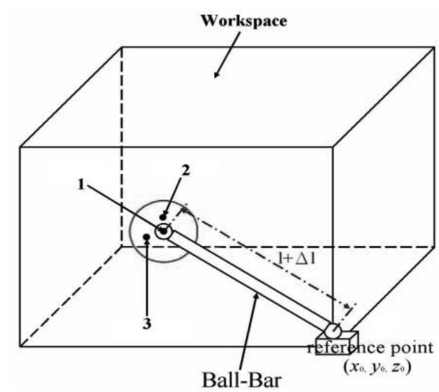


Fig. 2. The fundamental concept of single socket method.

2.1.2. Orientation error models

With use of differential homogeneous transformation matrix method, the orientation error models that can fast determine the rotation errors of a machine were derived for the three types of five-axis machine tools.

2.1.2.1. *Error model for a RRTTT-type machine tool.* With respect to the global machine coordinate frame $(x,y,z)_m$, the RRTTT machine can move along the x-, y- and z-axis, and the machine spindle can simultaneously rotate about the x-axis and y-axis. When the machine moves along x-, y- and z-axis for x_m, y_m, and z_m, and the machine spindle rotates about x-axis for an angle of α and about y-axis for an angle of β, the nominal coordinate of the tool tip, $[P]_1^{M,\text{Nominal}}$, with respect to the global machine coordinate frame, $(x,y,z)_m$, can be expressed with homogeneous transformation matrix as

$$[P]_1^{M,\text{Nominal}} = T_{xyz} T_\alpha T_\beta \; [P]^S, \tag{2}$$

where $[P]^S = [0 \;\; 0 \;\; z_r \;\; 1]^T$ represents the coordinates of tool tip with respect to the coordinate frame of the spindle, $(x,y,z)_s$. T_{xyz} represents the homogeneous transformation matrix of $(x,y,z)_s$ w.r.t. $(x,y,z)_m$ and can be expressed as

$$T_{xyz} = \text{Tran}(x_m, x_s)\text{Tran}(y_m, y_s)\text{Tran}(z_m, z_s). \tag{3}$$

T_α and T_β represent the homogeneous transformation matrix of the two rotation movements of the spindle w.r.t. $(x,y,z)_m$. When errors exist, the homogeneous transformation matrix becomes

$$
\begin{aligned}
[P]_1^{M,\text{Actual}} &= \begin{bmatrix} 1 & 0 & 0 & x_1+\Delta x_{V1} \\ 0 & 1 & 0 & y_1+\Delta y_{V1} \\ 0 & 0 & 1 & z_1+\Delta z_{V1} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\alpha+\Delta\alpha) & -\sin(\alpha+\Delta\alpha) & 0 \\ 0 & \sin(\alpha+\Delta\alpha)\cos(\alpha+\Delta\alpha) & 0 & \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&\times \begin{bmatrix} \cos(\beta+\Delta\beta) & 0 & \sin(\beta+\Delta\beta) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\beta+\Delta\beta) & 0 & \cos(\beta+\Delta\beta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ z_r \\ 1 \end{bmatrix} \\
&= \begin{bmatrix} x_1+\Delta x_{V1}+z_r\sin(\beta+\Delta\beta) \\ y_1+\Delta y_{V1}-z_r\cos(\beta+\Delta\beta)\sin(\alpha+\Delta\alpha) \\ z_1+\Delta z_{V1}+z_r\cos(\alpha+\Delta\alpha)\cos(\beta+\Delta\beta) \\ 1 \end{bmatrix}.
\end{aligned} \tag{4}
$$

Equation (4) can also be expressed as

$$
[P]_1^{M,\text{Actual}} = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ 1 \end{bmatrix} + \begin{bmatrix} \Delta x_{V2} \\ \Delta y_{V2} \\ \Delta z_{V2} \\ 1 \end{bmatrix} = \begin{bmatrix} x_2+\Delta x_{V2} \\ y_2+\Delta y_{V2} \\ z_2+\Delta z_{V2} \\ 1 \end{bmatrix}. \tag{5}
$$

In Eqs. (4) and (5), x_2, y_2, and z_2 are the coordinates of the nominal position of the tool tip w.r.t. $(x,y,z)_m$ after 5-DOF movements is made. $\Delta x_{V1}, \Delta y_{V1}$, and Δz_{V1} represent the total position errors of the tool tip caused by the three translational movements w.r.t. $(x,y,z)_s$. $(\Delta x_{V2}, \Delta y_{V2}, \Delta z_{V2})$ and $(\Delta\alpha, \Delta\Delta)$ are respectively the total position and orientation errors of the machine at the final position (x_2, y_2, z_2).

By converting the cosine and sine terms in Eqs. (4) and (5) to Taylor extension series, and neglecting the higher order terms, the orientation errors can be obtained as

$$
\Delta\beta = -\frac{\sec(\beta)(x_1-x_2+\Delta x_{V1}-\Delta x_{V2}+z_r\sin(\beta))}{z_r}, \tag{6}
$$

$$
\Delta\alpha = \frac{(-y_1+y_2-\Delta y_{V1}+\Delta y_{V2})\cos(\alpha)+(-z_1+z_2-\Delta z_{V1}+\Delta z_{V2})\sin(\alpha)}{(z_1-z_2+\Delta z_{V1}-\Delta z_{V2})\cos(\alpha)+(-y_1+y_2-\Delta y_{V1}+\Delta y_{V2})\sin(\alpha)}. \tag{7}
$$

Using Eqs. (6) and (7), when $\Delta x_{V1}, \Delta y_{V1}, \Delta z_{V1}, \Delta x_{V2}, \Delta y_{V2}$, and Δz_{V2} are measured by SSM (Eq. (1)), the orientation errors of the cutter can be determined.

2.1.2.2. *Error model for TTTRR-type machine tools*. For a TTTRR machine tool, the machine table can simultaneously rotate about x-axis and z-axis. The two rotation centers locate at different locations, and there is a fixed z-direction distance between the two centers. With the similar derivative process described in previous section, the nominal coordinates of a reference point set on the machine table with respect to the coordinate frame, $(x,y,z)_m$, can be expressed as

$$
\begin{aligned}
[P]_2^{M,\text{Nominal}} &= T_{xyz}\, T_\alpha\, T_{z'}\, T_\gamma [P]^{Rz} \\
&= \begin{bmatrix} 1 & 0 & 0 & x_1 \\ 0 & 1 & 0 & y_1 \\ 0 & 0 & 1 & z_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) & 0 \\ 0 & \sin(alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&\times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & z_{\alpha\gamma} \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} \cos(\gamma) & -\sin(\gamma) & 0 & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} x_r \\ y_r \\ z_r \\ 1 \end{bmatrix},
\end{aligned} \tag{8}
$$

where $[P]^{Rz} = [x_r \;\; y_r \;\; z_r \;\; 1]^T$ represents the coordinates of the reference point with respect to the coordinate frame set at the rotation center on z-axis. $T_{xyz}(=\text{Tran}(x_m,x_b)\text{Tran}(y_m,y_b)\text{Tran}(z_m,z_b))$ is the homogeneous transformation matrix of $(x,y,z)_b$ w.r.t. $(x,y,z)_m$, and the coordinate frame of machine table is set at the rotation center on x-axis. T_z, represents the homogeneous transformation matrix between the frames at the rotation center on z-axis and at the rotation center x-axis. T_α and T_γ are the homogeneous transformation matrices for the two rotational movements. α and γ are the rotational angles about the x-axis and z-axis, respectively.

Based on Eq. (8), when errors exist, the actual coordinates of reference point w.r.t. $(x,y,z)_m$ become

$$
\begin{aligned}
[P]_2^{M,\text{Actual}} &= \begin{bmatrix} 1 & 0 & 0 & x_1+\Delta x_{V1} \\ 0 & 1 & 0 & y_1+\Delta y_{V1} \\ 0 & 0 & 1 & z_1+\Delta z_{V1} \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\alpha+\Delta\alpha) & -\sin(\alpha+\Delta\alpha) & 0 \\ 0 & \sin(\alpha+\Delta\alpha) & \cos(\alpha+\Delta\alpha) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&\times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & z_{\alpha\gamma} \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} \cos(\gamma+\Delta\gamma) & -\sin(\gamma+\Delta\gamma) & 0 & 0 \\ \sin(\gamma+\Delta\gamma) & \cos(\gamma+\Delta\gamma) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} x_r \\ y_r \\ z_r \\ 1 \end{bmatrix} \\
&= \begin{bmatrix} x_1+\Delta x_{V1}+x_r\cos(\gamma+\Delta\gamma)-y_r\sin(\gamma+\Delta\gamma) \\ y_1+\Delta y_{V1}+x_r\cos(\alpha+\Delta\alpha)\sin(\gamma+\Delta\gamma)+y_r\cos(\alpha+\Delta\alpha)\cos(\gamma+\Delta\gamma)-(z_r+z_{\alpha\gamma}\sin(\alpha+\Delta\alpha) \\ z_1+\Delta z_{V1}+x_r\sin(\alpha+\Delta\alpha)\sin(\gamma+\Delta\gamma)+y_r\cos(\gamma+\Delta\gamma)\sin(\alpha+\Delta\alpha)+(z_r+z_{\alpha\gamma})\cos(\alpha+\Delta\alpha) \\ 1 \end{bmatrix}
\end{aligned} \tag{9}
$$

Assume that

$$
[P]_2^{M,\text{Actual}} = \begin{bmatrix} x_2+\Delta x_{V2} \\ y_2+\Delta y_{V2} \\ z_2+\Delta z_{V2} \\ 1 \end{bmatrix}. \tag{10}
$$

In Eqs. (9) and (10), (x_2,y_2,z_2) are the nominal coordinates of the reference point w.r.t. $(x,y,z)_m$ after the 5-DOF movements are made. $(\Delta x_{V2},\Delta y_{V2},\Delta z_{V2})$ and $(\Delta\alpha,\Delta\gamma)$ are the total position and orientation errors of the machine w.r.t. $(x,y,z)_m$ caused by the 5-DOF movements. $(\Delta x_{V1}, \Delta y_{V1}, \Delta z_{V1})$ are the total position errors of the machine w.r.t. $(x,y,z)_b$. They are caused by the three translational movements, and can be directly measured by SSM by setting the reference point on the machine table. Converting the cosine and sine terms in Eqs. (9) and (10) to Taylor extension series and neglecting the higher order terms, the orientation errors can be expressed as

$$\Delta\gamma = \frac{x_1 + x_2 + \Delta x_{V1} - \Delta x_{V2} + x_r\cos(\gamma) - y_r\sin(\gamma)}{y_r\cos(\gamma) + x_r\sin(\gamma)}, \tag{11}$$

$$\Delta\alpha = \frac{(y_1 - y_2 + \Delta y_{V1} - \Delta y_{V2} - (z_r + z_{ay})\sin(\alpha))(y_r\cos(\gamma) + x_r\sin(\gamma)) + \cos(\alpha)(x_r^2 + y_r^2 + (x_1 - x_2 + \Delta x_{V2})(x_r\cos(\gamma) - y_r\sin(\gamma)))}{(z_r + z_{\alpha\gamma})\cos(\alpha)(y_r cos(\gamma) + x_r\sin(\gamma)) + \sin(\alpha)(x_r^2 + y_r^2 + (x_1 - x_2 + \Delta x_{V2})(x_r\cos(\gamma) - y_r\sin(\gamma)))}. \tag{12}$$

Using Eqs. (11) and (12), when the total position errors $\Delta x_{V1}, \Delta y_{V1}, \Delta z_{V1}, \Delta x_{V2}, \Delta y_{V2}$, and Δz_{V2} are determined by SSM, the orientation errors can be calculated.

2.1.2.3 *Error Model for a RTTTR-type machine tool.* For a RTTTR five-axis machine tool, the machine spindle and the machine table can rotate about *y*-axis and *x*-axis, respectively. Using the homogeneous transformation matrix, when the machine spindle makes 4-DOF movements, the nominal coordinate of the tool tip w.r.t. $(x,y,z)_m$ can be expressed as

$$[P]_3^{M,\text{Nominal}} = T_{xyz}\, T_\beta [P]^S = \text{Tran}(x_m, x_s)\text{Tran}(y_m, y_s)\text{Tran}(z_m, z_s)\text{Rot}(y_m, \beta)[P]^S, \tag{13}$$

where $[P]^S = [0\,0\,z_r\,1]^T$ represents the coordinates of tool tip w.r.t. $(x,y,z)_s$. T_{xyz} represents the homogeneous transformation matrix between the coordinate frame $(x,y,z)_s$ and $(x,y,z)_m$, and T_β is the homogeneous transformation matrix representing the rotation about *y*-axis w.r.t. $(x,y,z)_m$. β is the rotation angle at machine spindle.

After making 4-DOF movements, the nominal coordinate of the reference point set on the machine table w.r.t. $(x,y,z)_m$ can be expressed as

$$[P]_3^{M,\text{Nominal}} = T_{xyz} T_\alpha [P]^B = \text{Tran}(x_m, x_b)\text{Tran}(y_m, y_b)\text{Tran}(z_m, z_b)\text{Rot}(x_m, \alpha)[P]^B, \tag{14}$$

where $[P]^B = [x_{tr} y_{tr} z_{tr} 1]^T$ represents the nominal coordinate of the reference point w.r.t. $(x,y,z)_b$ after three translation movements are made. T_{xyz} is the homogeneous transformation matrix between the coordinate frames $(x,y,z)_b$ and $(x,y,z)_m$. T_α is the homogeneous transformation matrix for the rotation movement of the machine table.

With the similar derivation addressed in the previous sections, when errors exist, Eq. (13) becomes

$$[P]_3^{M,\text{Actual}} = \begin{bmatrix} x_1 + \Delta x_{V1} + z_r\sin(\beta + \Delta\beta) \\ y_1 + \Delta y y_{V1} \\ z_1 + \Delta z_{V1} + z_r\cos(\beta + \Delta\beta) \\ 1 \end{bmatrix} = \begin{bmatrix} x_2 + \Delta x_{V2} \\ y_2 + \Delta y_{V2} \\ z_2 + \Delta z_{V2} \\ 1 \end{bmatrix}, \tag{15}$$

where $[P]_3^{M,\text{Actual}}$ is the actual coordinate of the tool tip. (x_2, y_2, z_2) and $(\Delta x_{V2}, \Delta y_{V2}, \Delta z_{V2})$ are respectively the nominal coordinates and the total position errors of the tool tip w.r.t. $(x,y,z)_m$ after 4-DOF movements are made. $(\Delta x_{V1}, \Delta y_{V1}, \Delta z_{V1})$ are the total position errors of the tool tip w.r.t. $(x,y,z)_s$ after the three translational movements are made. $\Delta\beta$ is the orientation error of the machine spindle. When errors exist, Eq. (14)

becomes

$$[P]_3^{M,\text{Actual}} = \begin{bmatrix} x_{t1} + \Delta x_{tV1} + x_{tr} \\ y_{t1} + \Delta y_{tV1} + y_{tr}\cos(\alpha+\Delta\alpha) - z_{tr}\sin(\alpha+\Delta\alpha) \\ z_{t1} + \Delta z_{tV1} + y_{tr}\sin(\alpha+\Delta\alpha) + z_{tr}\cos(\alpha+Delta\alpha) \\ 1 \end{bmatrix} = \begin{bmatrix} x_{t2} + \Delta x_{tV2} \\ y_{t2} + \Delta y_{tV2} \\ z_{t2} + \Delta z_{tV2} \\ 1 \end{bmatrix}, \quad (16)$$

where $[P]_{t3}^{M,\text{Actual}}$ is the actual coordinate of reference point. (x_{t2}, y_{t2}, z_{t2}) and $(\Delta x_{t2}, \Delta y_{t2}, \Delta z_{t2})$ are respectively the nominal coordinate and the total position errors of the reference point w.r.t. $(x,y,z)_m$ after the 4-DOF movements are made. $(\Delta x_{tV1}, \Delta y_{tV1}, \Delta z_{tV1})$ are the total position errors of the reference point w.r.t. $(x,y,z)_b$ after the three translation movements are made. $(\Delta x_{t2}, \Delta y_{t2}, \Delta z_{t2})$ and $\Delta\alpha$ are respectively the translation and orientation errors of the machine table.

Converting the cosine and sine terms to Taylor extension series and neglecting the higher order terms, the orientation error of the machine spindle ($\Delta\beta$) and the orientation error of reference point on the machine table ($\Delta\alpha$) can be expressed as

$$\Delta\beta = -\frac{\sec(\beta)(x_1 - x_2 + \Delta x_{V1} - \Delta x_{V2} + z_r\sin(\beta))}{z_r}, \quad (17)$$

$$\Delta\alpha = \frac{y_{t1} - y_{t2} + \Delta y_{tV1} - \Delta y_{tV2} + y_{tr}\cos(\alpha) - z_{tr}\sin(\alpha)}{z_{tr}\cos(\alpha) + y_{tr}\sin(\alpha)}. \quad (18)$$

The variables, $(x_1,y_1,z_1), (x_2,y_2,z_2), (x_{t1},y_{t1},z_{t1}), (x_{t2},y_{t2},z_{t2})$, z_r, z_{tr}, α and β, are known value when the machine is driven. With use of the single socket method, total position errors $(\Delta x_{V1}, \Delta y_{V1}, \Delta z_{V1})$, $(\Delta x_{V2}, \Delta y_{V2}, \Delta z_{V2})$ of the tool tip can be directly measured. $(\Delta x_{tV1}, \Delta y_{tV1}, \Delta z_{tV1})$ and $(\Delta x_{tV2}, \Delta y_{tV2}, \Delta z_{tV2})$ can also be directly measured at the reference point. After substituting all the errors into Eqs. (17) and (18), the orientation errors ($\Delta\alpha$ and $\Delta\beta$) of the machine can be determined.

3. THE TWO-STEP MEASUREMENT PROCEDURE

According to the derived error models, it is noted that the orientation errors of a five-axis machine tool are functions of the two sets of total position errors: (1) the total position errors measured after three translational movements are made; (2) the total position errors measured after the 2-DOF rotation are made. Thus, a two-step measurement procedure incorporating with SSM was developed for determination of the two sets of total position errors. For a RRTTT-type five-axis machine tool, the measurement procedure is:

1. Set the magnetic centre mount on machine bed, and install ball-bar to the machine. The end of ball-bar attached to machine table is the reference point.

2. Move machine spindle to the first location with three translational movements.

3. Use SSM to determine the total position errors occurring at the first location, i.e. $(\Delta x_{V1}, \Delta y_{V1}, \Delta z_{V1})$ in Eq. (6).

4. Move machine spindle to the second location with two rotational movements.

5. Use SSM to determine the total position errors occurring at the second location, i.e. $(\Delta x_{V2}, \Delta y_{V2}, \Delta z_{V2})$ in Eq. (7).

6. Determine the orientation errors $\Delta\alpha$ and $\Delta\beta$ with substituting the two sets of total position errors into Eqs. (8) and (9).

For a TTTRR-type five-axis machine tool, the measurement procedure is similar to the procedure for the RRTTT-type machine except that the end of the ball-bar mounted on the machine spindle should be the reference point. Since the two rotation movements are made by the turning table, the orientation errors occur at the turning table. By substituting the measured total position errors, $(\Delta x_{V1}, \Delta y_{V1}, \Delta z_{V1})$ and $(\Delta x_{V2}, \Delta y_{V2}, \Delta z_{V2})$, into Eqs. (13) and (14), orientation errors, $\Delta\alpha$ and $\Delta\gamma$ can be determined.

For a RTTTR-type five-axis machine tool, because the machine spindle and machine table can separately make 1-DOF rotation, the two orientation errors should be determined separately. The measurement procedure for orientation error $\Delta\beta$ is:

1. Set the magnetic center mount on machine bed, and install ball-bar on the machine as regular setup.
2. Move machine spindle to the first location with three translational movements.
3. Use SSM to measure the total position errors at the first location, i.e. $(\Delta x_{V1}, \Delta y_{V1}, \Delta z_{V1})$ in Eq. (17).
4. Rotate machine spindle to the second location.
5. Set reference point on the turning table, and use SSM to measure the total errors at the second location, i.e. $(\Delta x_{V2}, \Delta y_{V2}, \Delta z_{V2})$ in Eq. (17).
6. Determine the orientation errors $\Delta\beta$ by substituting $(\Delta x_{V1}, \Delta y_{V1}, \Delta z_{V1}), (\Delta x_{V2}, \Delta y_{V2}, \Delta z_{V2})$ into Eq. (19).

The measurement procedure for orientation error $\Delta\alpha$ is:

1. Set the magnetic center mount on machine bed, and install ball-bar on the machine as regular setup.
2. Move the turning table to the first location with three translational movements.
3. Use SSM to determine the total errors occurring at the first location, i.e. $(\Delta x_{tV1}, \Delta y_{tV1}, \Delta z_{tV1})$ in Eq. (18).
4. Rotate the turning table to the third location.
5. Set the reference point on the machine spindle, and use SSM to measure the total errors $(\Delta x_{tV2}, \Delta y_{tV2}, \Delta z_{tV2})$ at the third location in Eq. (18).
6. Determine the orientation errors $\Delta\alpha$ by substituting $(\Delta x_{tV1}, \Delta y_{tV1}, \Delta z_{tV1})$, $(\Delta x_{tV2}, \Delta y_{tV2}, \Delta z_{tV2})$ into Eq. (20).

To enhance the convenience of use of the proposed measurement method for industry, a computer-aided measurement system integrated with a ball-bar system was developed in Matlab language based on the proposed error models and measurement procedures.

4. SIMULATION AND EXPERIMENTAL ILLUSTRATIONS

To verify the proposed measurement method, both computer simulation and experiments were conducted.

In the simulation, a RRTTT-type five-axis machine tool was selected. The total length of spindle and cutter was 250 mm (i.e. the radius of rotation). Several 5-DOF movements were planned for verification. Each movement was composed of three translations in x, y, and z direction and two rotations about the x- and y-axis (rotation angles: α, β). All movements have the same translations as x-travel: –50 mm, y-travel: –15 mm, and z-travel: 250 mm. Different rotation combinations and nominal orientation errors were selected:

Table 1. The simulation results of measuring volumetric errors for a RRTTT-type five-axis machine tool.

		1	2	3	4	5
Rotat. Angles	α/β	30°/0°	0°/40°	45°/60°	60°/50°	30°/60°
assumed Angles.	Δα/Δβ	0.1°/0°	0°/0.08°	0.12°/0.1°	0.15°/0.09°	0.1°/0.1°
Nomin. Coord.B/f Rotat. (x_1, y_1, z_1) (mm)					(-50,-15, 500)	
Total position errors B/f Rotat. $(\Delta x_{V1}, \Delta y_{V1}, \Delta z_{V1})$ (mm)					(0, 0, 0)	
Nomin Coord. After Rotat.(mm)	x_2	-50	-210.697	-266.506	-241.511	-266.506
	y_2	110	-15	73.388	124.168	47.5
	z_2	283.494	308.489	411.612	419.652	391.747
Total position errors After Rota.(mm)	Δx_{V2}	0	-0.267	-0.218	-0.252	-0.218
	Δy_{V2}	0.378	0	-0.083	-0.051	-0.001
	Δz_{V2}	0.218	0.225	0.452	0.514	0.436
Calculated Error	$\Delta\alpha'$	0.1°	0°	0.12°	0.15°	0.1°
	$\Delta\beta'$	0°	0.07995°	0.09985°	0.08992°	0.09985°

1. $\alpha = 30°$, $\beta = 0°$, $\Delta\alpha = 0.1°$ and $\Delta\beta = 0°$;
2. $\alpha = 0°$, $\beta = 40°$, $\Delta\alpha = 0°$ and $\Delta\beta = 0.08°$;
3. $\alpha = 45°$, $\beta = 60°$, $\Delta\alpha = 0.12°$ and $\Delta\beta = 0.1°$;
4. $\alpha = 60°$, $\beta = 50°$, $\Delta\alpha = 0.15°$ and $\Delta\beta = 0.09°$;
5. $\alpha = 30°$, $\beta = 60°$, $\Delta\alpha = 0.1°$ and $\Delta\beta = 0.1°$.

The nominal location after three translations was $(-50, -15, 500)$. According to the calculation of homogeneous transformation matrix and the D–H rule, the nominal locations of the machine spindle ($\Delta x2$, $\Delta y2$, $\Delta z2$) after rotations are $(-50, 110, 283.494)$, $(-210.697, -15, 308.489)$, $(-266.506, 73.388, 411.612)$, $(-241.511, 124.168, 419.652)$, and $(-266.506, 47.5, 391.747)$, respectively. For simplicity, the total position errors caused by translational movements were neglected in this simulation. The total position errors of the machine after rotations were calculated and shown in Table 1. After substituting the total position errors into Eqs. (8) and (9), the orientation errors, $\Delta\alpha$ and $\Delta\beta$, were obtained (Table 1). It can be seen that the orientation errors were all very close to the nominal orientation errors. The average difference between the nominal orientation errors and calculated orientation errors were about 0.0001°. It proves that the derived error models are quite accurate.

In the experiments, a TTTRR-type five-axis machining center produced by Dah-Lih Machinery Co. was used. Four 5-DOF test movements were conducted. Each test movement included a 3-DOF translation (–49.275 mm along the *x*-axis; –14.312 mm along the *y*-axis; and –177.777 mm along the *z*-axis) and a 2-DOF rotation. Four sets of 2-DOF rotation were set for the experiment: (1) $\alpha = 0°$, $\gamma = 30°$; (2) $\alpha = 15°$, $\gamma = 0°$; (3) $\alpha = 10°$, $\gamma = 30°$; (4) $\alpha = 10°$, $\gamma = 45°$. SSM was used to measure the total position errors of the machine at different locations. After substituting the measured errors into Eqs. (13) and (14), the orientation errors were determined. Both of the total position errors and orientation errors were shown in Table 2. The maximum orientation error, $\Delta\alpha = -0.0426°$ and $\Delta\gamma = 0.0684°$ occurred at $\alpha = 0°$, $\gamma = 30°$. It was noted that $\Delta\gamma$ is larger than $\Delta\alpha$. This is because the driving mechanism of the turning table was damaged before. The experimental results reflect the facts.

Table 2. The experimental results of measuring volumetric errors for a TTTRR-type five-axis machine tool.

		1	2	3	4
Rotat. Angle	α/γ	0°/30°	15°/0°	10°/30°	10°/45°
Nomin. Coord.B/f rotation (x,y,z) (mm)				(-49.275,-14.312,-177.777)	
Nomin. Coord.B/f rotation (mm)	x	-35.517	-49.275	-35.517	24.723
	y	-37.032	32.188	-5.599	-13.409
	z	-177.777	-175.424	-181.507	-182.884
Total position errors B/f Rotation (mm)	Δx	0.0067	-0.0104	-0.1740	-0.1740
	Δy	0.0373	0.2396	-0.1721	-0.1721
	Δz	0.0519	0.0297	-0.0619	-0.0619
Total position errors After Rotation (mm)	Δx	0.0313	0.1020	-0.2206	-0.1993
	Δy	0.1255	-0.0861	-0.0555	-0.0180
	Δz	0.0259	-0.0448	-0.1307	-0.1116
Calculated Orientation error	$\Delta\alpha$	-0.0426°	-0.0161°	-0.0128°	-0.0396°
	Δγ	0.0684°	0.4514°	-0.2403°	-0.2513°

5. CONCLUSIONS

A new volumetric error measurement method composing of error models, a two-step measurement procedure, and use of SSM was developed for the three major types of five-axis machine tools in this study. The error models for RRTTT-type, TTTRR-type, and RTTTR-type five-axis machine tools were newly derived. The two-step measurement procedures were developed for standard operation. Simulation and experiments were conducted to verify the proposed measurement method. The results have shown good feasibility and reliability of the proposed method. Without using expensive measurement instrument and complex measurement procedures, the proposed method offers the advantages of low cost, easy set up, and high efficiency for determining the volumetric errors of five-axis machine tools.

ACKNOWLEDGEMENT

This study was supported by the National Science Council of the Republic of China under the grant number NSC-98-2622-033-014-CC3.

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