

SATURATED PROPORTIONAL DERIVATIVE CONTROL OF A SINGLE-LINK FLEXIBLE-JOINT MANIPULATOR

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ABSTRACT

This paper considers the control of a single-link flexible-joint robot manipulator subject to actuator saturation. Alternative controllers are proposed and compared to one found in literature. In particular, a controller with proportional and derivative components is guaranteed to provide a total torque less than a chosen value, thereby disallowing actuator saturation. It is shown that an equilibrium point of the closed-loop system is asymptotically stable. Additionally, it is shown that the controllers are robust to modelling errors. Finally, this paper presents simulation results demonstrating the performance of the proposed control architecture.

Keywords: proportional-derivative control; saturation avoidance; flexible-joint manipulator.

LE CONTRÔLE PROPORTIONNEL DÉRIVÉ SATURÉ D'UN MANIPULATEUR AVEC DES ARTICULATIONS FLEXIBLES

RÉSUMÉ

Cet article traite sur le contrôle d'un manipulateur robotique avec des articulations flexibles, sujet à une saturation d'actionneur. Des contrôleurs alternatifs sont proposés et comparés à un contrôleur dans la littérature. En particulier, un contrôleur avec une portion proportionnelle et une portion dérivée garantie un moment de torsion total moins qu'une valeur spécifiée. Il est démontré qu'un point d'équilibre du système à circuit fermé est asymptotiquement stable. En plus, il est attesté que les contrôleurs sont robustes aux erreurs de modélisation. Finalement, cet article présente des résultats de simulation qui illustrent l'application des contrôleurs proposés.

Mots-clés : contrôle proportionnel dérivé; l'évitement de saturation; manipulateur avec des articulations flexibles.

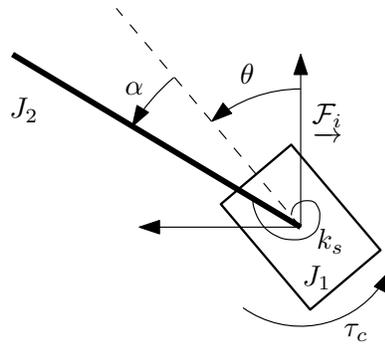


Fig. 1. Schematic of a flexible-joint.

1. INTRODUCTION

Robot manipulators are used in many branches of manufacturing for tasks such as robotic welding and automated assembly. The flexibility of manipulator joints is often left unmodelled and uncontrolled, leading to performance limitations [1, 2]. This flexibility is caused by the gears and belts used to transmit the torque produced by the actuators to the links [2]. Often the natural frequencies of these joints are relatively low (2–3 Hz), which often coincide with the frequency of the trajectory being followed, forcing the operator to wait for any vibrations to decay naturally [3]. Moreover, in large robotic manipulators, such as the Canadarm, even a relatively small joint flexibility can cause significant vibrations at the manipulator tip, which is highly undesirable. Several authors have investigated the modelling and control of flexible-joint robotic manipulators using widely varying techniques [4–8].

Actuator limitations also become a factor when controlling flexible-joint robotic manipulators. Powerful motors are generally large and heavy which is undesirable; the increased mass of the system results in increased power requirements, as well as possible performance limitations [9]. For this reason, somewhat smaller or at least modestly sized motors are used in practice, resulting in limited joint torques. As such, avoiding actuator saturation while simultaneously assuring asymptotic stability of the closed-loop equilibrium point is of great interest. In the context of robotic manipulators, various authors have studied saturation avoidance [10–12]. Spacecraft attitude control accounting for actuator saturation has also been investigated in [13–15]. In particular, in [13] a simple proportional derivative (PD) type control law that explicitly accounts for actuator saturation is presented.

The novel contribution of this paper is adopting and building upon the work of Su and Zheng [13] by designing and analyzing PD control laws for a single-link flexible-joint robotic manipulator. Specifically, two PD controllers will be considered that disallow actuator saturation and simultaneously guarantee asymptotic stability of the equilibrium point of the closed-loop system. Additionally, numerical validation of the proposed controllers will be performed, and their performance will be compared to that of an existing controller found in the literature.

The remainder of this paper is as follows. In Section 2 the dynamics of a single-link flexible robotic manipulator are derived. In Section 3 the proposed controllers are presented and shown to be asymptotically stable, even in the presence of parameter uncertainty. Section 4 presents numerical examples, thus validating the proposed controllers in simulation, and some final remarks are given in Section 5.

2. SYSTEM DYNAMICS

Consider the single-link flexible-joint robotic manipulator shown in Fig. 1. The kinetic and potential energies, as well as the Rayleigh dissipation function, respectively are

$$T = \frac{1}{2} \dot{\mathbf{q}}^T \mathbf{M} \dot{\mathbf{q}}, \quad U = \frac{1}{2} \mathbf{q}^T \mathbf{K} \mathbf{q}, \quad R = \frac{1}{2} \dot{\mathbf{q}}^T \mathbf{D} \dot{\mathbf{q}},$$

where $\mathbf{M} = \mathbf{M}^T > 0$ is the system's mass matrix, $\mathbf{K} = \mathbf{K}^T \geq 0$ is the system's stiffness matrix, $\mathbf{D} = \mathbf{D}^T > 0$ is the system's damping matrix, and $\mathbf{q} = [\theta \quad \alpha]^T$ are the generalized coordinates of the system. The angle θ is the angle of the base hub relative to a fixed inertial frame and α is the angle of the manipulator link relative to the base hub. Using a Lagrangian approach, the equations of motion are

$$\mathbf{M} \ddot{\mathbf{q}} + \mathbf{D} \dot{\mathbf{q}} + \mathbf{K} \mathbf{q} = \hat{\mathbf{b}} \boldsymbol{\tau}_c, \quad (1)$$

where $\hat{\mathbf{b}}$ is the matrix that distributes the applied torque to the system and $\boldsymbol{\tau}_c$ is the torque input to the system. The mass, stiffness, damping, and input matrices of the system are

$$\mathbf{M} = \begin{bmatrix} J_1 + J_2 & J_2 \\ J_2 & J_2 \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} 0 & 0 \\ 0 & k_s \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} d_1 & 0 \\ 0 & d_2 \end{bmatrix}, \quad \hat{\mathbf{b}} = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

where J_1, J_2 are the moment of inertias of the manipulator hub and link respectively, k_s is the torsional spring constant between the hub and link, and d_1, d_2 are the damping coefficients of the hub and link respectively.

3. CONTROL FORMULATION

The issue of actuator saturation will now be addressed by formulating control laws that explicitly avoid actuator saturation. These control laws will be shown to guarantee the asymptotic stability of the closed-loop system.

3.1. Control Law

Consider the following PD control law:

$$\boldsymbol{\tau}_c = u_p + u_d,$$

where u_p is proportional control and u_d is derivative control. Liu et al. [11] propose using $u_p = -k_p \arctan(\theta)$ and $u_d = -k_d \arctan(\dot{\theta})$, where θ is the joint angle, $\dot{\theta}$ is the angular velocity of the joint, the constant k_p is the proportional control gain, and the constant k_d is the derivative control gain. This ensures that $|\boldsymbol{\tau}_c| \leq k_p + k_d$, thereby avoiding actuator saturation. For the attitude control of spacecraft, Su and Zheng [13] propose using proportional and derivative control similar to $u_p = -k_p \frac{p}{\sqrt{1+p^2}}$ and $u_d = -k_d \tanh(\dot{\theta})$, where $p = \tan(\theta/2)$ is a Gibbs parameter. This paper proposes adapting the work of [13] to be used to control a single-link flexible-joint robotic manipulator, as well as introducing alternative control laws.

First, u_p and u_d are specified as

$$u_p = -k_p f(\theta), \quad (2)$$

$$u_d = -k_d \tanh(\dot{\theta}), \quad (3)$$

where $f(x) = \frac{x}{\sqrt{1+x^2}}$. Note that both $f(\theta)$ and $\tanh(\dot{\theta})$ are bounded by ± 1 , which constrains the torque of the actuator to be less than the sum of k_p and k_d , that is $|\boldsymbol{\tau}_c| \leq k_p + k_d$. This property ensures actuator saturation is avoided, as long as k_p and k_d are chosen to add up to less than the maximum torque that the actuator can apply.

An alternative control formulation is

$$u_p = -k_p f(\theta), \quad (4)$$

$$u_d = -k_d f(\dot{\theta}). \quad (5)$$

Eq. (5) is a slight variation of Eq. (3), where $\tanh(\dot{\theta})$ is replaced by $f(\dot{\theta})$.

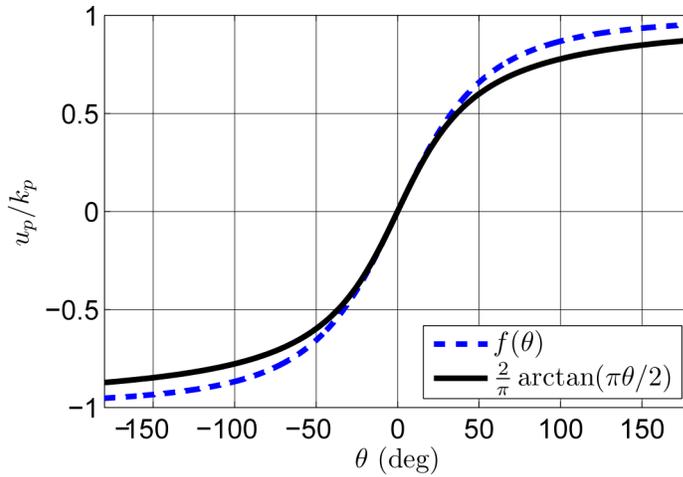


Fig. 2. Relative control effort versus θ .

The two proposed control laws to be investigated are

$$-\tau_c = k_p f(\theta) + k_d f(\dot{\theta}), \quad (6)$$

$$-\tau_c = k_p f(\theta) + k_d \tanh(\dot{\theta}), \quad (7)$$

A comparison of the relative proportional control effort supplied by the function $f(\cdot)$ and the function $\frac{2}{\pi} \arctan(\pi\theta/2)$ is presented in Fig. 2. The function $\frac{2}{\pi} \arctan(\pi\theta/2)$ is included in Fig. 2 to serve as a comparison to a controller found in the literature [11]. The factors $\frac{2}{\pi}$ and $\pi/2$ were added to the arctan function in order to match the linearization of $f(\theta)$ about $\theta = 0$ deg, thus realizing a fair comparison.

Figure 2 serves as motivation to use $f(\theta)$ for proportional control, due to its nonlinearity and relative aggressiveness further away from $\theta = 0$ deg as compared to $2/\pi \arctan(\pi\theta/2)$. This larger relative control effort should allow the controller to drive the system to the desired equilibrium quicker.

3.2. Stability Analysis

The closed-loop stability properties of the manipulator given in Eq. (1), and the two control laws given in Eqs. (6) and (7), each of which disallow the possibility of actuator saturation when the gains k_p and k_d are chosen appropriately, will now be considered. In the following proof, a restriction is applied to the allowable set of initial conditions. Notice that the angle θ could experience unwinding if it does not remain on the domain $-\pi < \theta < \pi$. Unwinding is a phenomenon that occurs when a body is relatively close to a desired set point, yet rotates through one or more full revolutions before settling at the desired set point [16]. For example, consider when the manipulator hub starts at an angle of +359 degrees, which is also -1 degrees, and has zero initial angular velocity. In this situation, the controller would drive the manipulator away from the zero degree position, all the way through +180 degrees, and eventually back to zero degrees. Clearly, this unwinding effect is not desirable. A restriction on the allowable set of initial conditions will guarantee that unwinding does not occur. In the context of spacecraft attitude control, restrictions on initial conditions have been used in [17, 18].

Theorem 1. Consider the dynamics of the single-link flexible-joint manipulator found in Eq. (1) on the domain $-\pi < \theta < \pi$, $-\pi < \alpha < \pi$, $\dot{\theta} \in \mathbb{R}$, $\dot{\alpha} \in \mathbb{R}$ and the control law given by Eq. (6) or (7). If $-\pi < \theta(0) < \pi$ and

$$V(0) = \frac{1}{2} \dot{\mathbf{q}}^T(0) \mathbf{M} \dot{\mathbf{q}}(0) + \frac{1}{2} \mathbf{q}^T(0) \mathbf{K} \mathbf{q}(0) + k_p \left(\sqrt{1 + \theta(0)^2} - 1 \right) < k_p \left(\sqrt{1 + \pi^2} - 1 \right),$$

then the equilibrium point $(\mathbf{q}, \dot{\mathbf{q}}) = (\mathbf{0}, \mathbf{0})$ of the closed-loop system is asymptotically stable.

Proof. First, note that it can be easily shown using Eq. (1) and either Eq. (6) or Eq. (7) that $(\mathbf{q}, \dot{\mathbf{q}}) = (\mathbf{0}, \mathbf{0})$ is in fact an equilibrium point of the closed-loop system. Next, consider the Lyapunov function candidate:

$$V = \frac{1}{2} \dot{\mathbf{q}}^T \mathbf{M} \dot{\mathbf{q}} + \frac{1}{2} \mathbf{q}^T \mathbf{K} \mathbf{q} + k_p \left(\sqrt{1 + \theta^2} - 1 \right).$$

Taking the derivative of V and simplifying using Eq. (1) yields

$$\begin{aligned} \dot{V} &= \frac{1}{2} \left(\ddot{\mathbf{q}}^T \mathbf{M} \dot{\mathbf{q}} + \dot{\mathbf{q}}^T \mathbf{M} \ddot{\mathbf{q}} \right) + \frac{1}{2} \left(\dot{\mathbf{q}}^T \mathbf{K} \mathbf{q} + \mathbf{q}^T \mathbf{K} \dot{\mathbf{q}} \right) + k_p \frac{\theta}{\sqrt{1 + \theta^2}} \dot{\theta} \\ &= \dot{\mathbf{q}}^T (\mathbf{M} \ddot{\mathbf{q}} + \mathbf{K} \mathbf{q}) + k_p \frac{\theta}{\sqrt{1 + \theta^2}} \dot{\theta} \\ &= \dot{\mathbf{q}}^T \hat{\mathbf{b}} \tau_c - \dot{\mathbf{q}}^T \mathbf{D} \dot{\mathbf{q}} + k_p \frac{\theta}{\sqrt{1 + \theta^2}} \dot{\theta} \\ &= -\dot{\mathbf{q}}^T \mathbf{D} \dot{\mathbf{q}} + \dot{\theta} \left(-k_p \frac{\theta}{\sqrt{1 + \theta^2}} - k_d h(\dot{\theta}) \right) + k_p \frac{\theta}{\sqrt{1 + \theta^2}} \dot{\theta} \\ &= -\dot{\mathbf{q}}^T \mathbf{D} \dot{\mathbf{q}} - k_d h(\dot{\theta}) \dot{\theta}, \end{aligned}$$

where $h(\dot{\theta}) \dot{\theta} = \dot{\theta}^2 / \sqrt{1 + \dot{\theta}^2}$ when using Eq. (6), and $h(\dot{\theta}) \dot{\theta} = \dot{\theta} \tanh(\dot{\theta})$ when using Eq. (7). Owing to the fact that \dot{V} is negative semidefinite, the closed-loop system is stable. The restriction on the initial condition ensures that $V(t) \leq V(0) < k_p(\sqrt{1 + \pi^2} - 1)$, which along with $-\pi < \theta(0) < \pi$ guarantees that $-\pi < \theta(t) < \pi$ and unwinding is avoided. To show that the closed-loop system is asymptotically stable, LaSalle's Invariant Set Theorem will be employed [19]. Notice that $\dot{V} = 0$ only if $\dot{\mathbf{q}} = \mathbf{0}$ (because \mathbf{D} is positive definite), which combined with Eqs. (1) and (6) or (7) implies that $\mathbf{q} = \mathbf{0}$ and $\tau_c = 0$. From LaSalle's Invariant Set Theorem it follows that the equilibrium point $(\mathbf{q}, \dot{\mathbf{q}}) = (\mathbf{0}, \mathbf{0})$ is asymptotically stable [19]. \square

Note that this stability analysis holds for any numerical system parameters, provided that $\mathbf{D} = \mathbf{D}^T > 0$ is positive definite, ensuring the controllers are robust to modelling errors. This assumption is reasonable since there will always be some residual friction in the flexible-link system. The restriction on the allowable set of initial conditions is a key part of the stability analysis, as it guarantees that unwinding is avoided.

4. NUMERICAL EXAMPLES

The proposed controllers are now validated by testing them in simulation. The flexible-joint manipulator simulated has numerical values of $J_1 = 2 \times 10^{-3}$ (kg·m²), $J_2 = 3 \times 10^{-3}$ (kg·m²), $k_s = 1$ (N·m/rad), $d_1 = 4 \times 10^{-3}$ (N·m/(rad/s)), and $d_2 = 1 \times 10^{-6}$ (N·m/(rad/s)). The base of the manipulator is fixed, while the hub and link are rotated by an angle θ and α respectively. All control laws are tested on the nominal system and the perturbed system, where the perturbed system has a link inertia that is 20% greater than that of the nominal system.

4.1. Response to Initial Conditions

The controllers were implemented to drive the system with non-zero initial conditions to the equilibrium point of $(\mathbf{q}, \dot{\mathbf{q}}) = (\mathbf{0}, \mathbf{0})$. In these tests $k_p = 0.9$ (N·m) and $k_d = 0.5$ (N·m), which guarantees $|\tau_c| \leq 1.4$ (N·m).

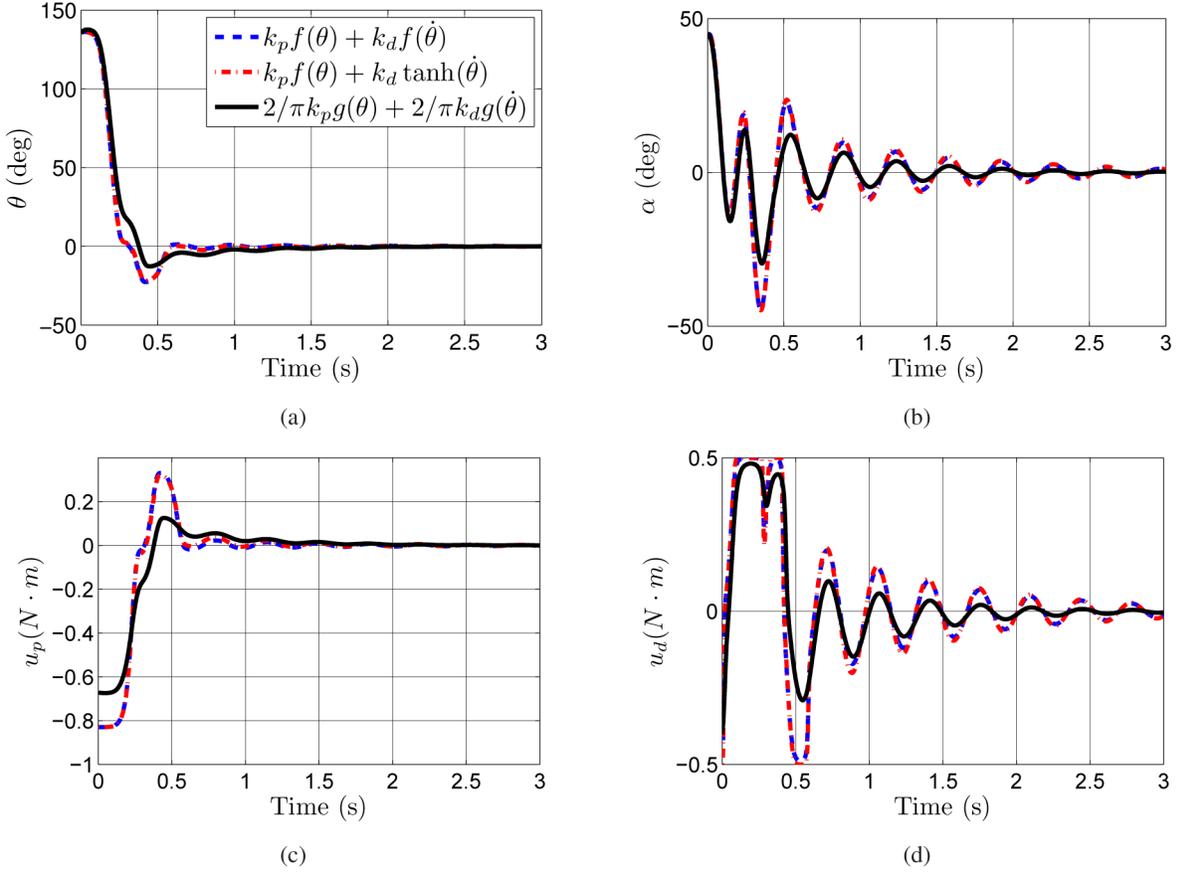


Fig. 3. Simulation of nominal system with initial condition results: (a) θ versus time, (b) α versus time, (c) u_p versus time, and (d) u_d versus time.

The initial conditions of the simulation were $(\mathbf{q}, \dot{\mathbf{q}}) = ([3\pi/4 \ \pi/4]^\top, [\pi \ 0]^\top)$, which satisfies $-\pi < \theta(0) < \pi$. This gives an initial value of

$$\begin{aligned}
 V(0) &= \frac{1}{2} \dot{\mathbf{q}}^\top(0) \mathbf{M} \dot{\mathbf{q}}(0) + \frac{1}{2} \mathbf{q}^\top(0) \mathbf{K} \mathbf{q}(0) + k_p \left(\sqrt{1 + \theta(0)^2} - 1 \right) \\
 &= \frac{1}{2} [\pi/4 \ 0] \begin{bmatrix} 8 \times 10^{-3} & 6 \times 10^{-3} \\ 6 \times 10^{-3} & 6 \times 10^{-3} \end{bmatrix} \begin{bmatrix} \pi/4 \\ 0 \end{bmatrix} + \frac{1}{2} [3\pi/4 \ \pi/4] \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3\pi/4 \\ \pi/4 \end{bmatrix} \\
 &\quad + 0.9 \left(\sqrt{1 + 9\pi^2/16} - 1 \right) \\
 &= 0.25 \times 10^{-3} \pi^2 + \pi^2/32 + 0.9 \left(\sqrt{1 + 9\pi^2/16} - 1 \right) \\
 &< k_p (\sqrt{1 + \pi^2} - 1),
 \end{aligned}$$

which satisfies the initial condition requirement on $V(0)$. In Fig. 3 are the θ and α responses, u_p , the applied proportional torque, and u_d , the applied derivative torque versus time for the proposed controllers and a controller found in [11]. Note that the functions $g(\theta)$ and $g(\dot{\theta})$ in the legend of Fig. 3 represent $\arctan(\pi\theta/2)$ and $\arctan(\pi\dot{\theta}/2)$ respectively. In Fig. 4 are the θ and α responses, u_p versus time, and u_d versus time of the perturbed system.

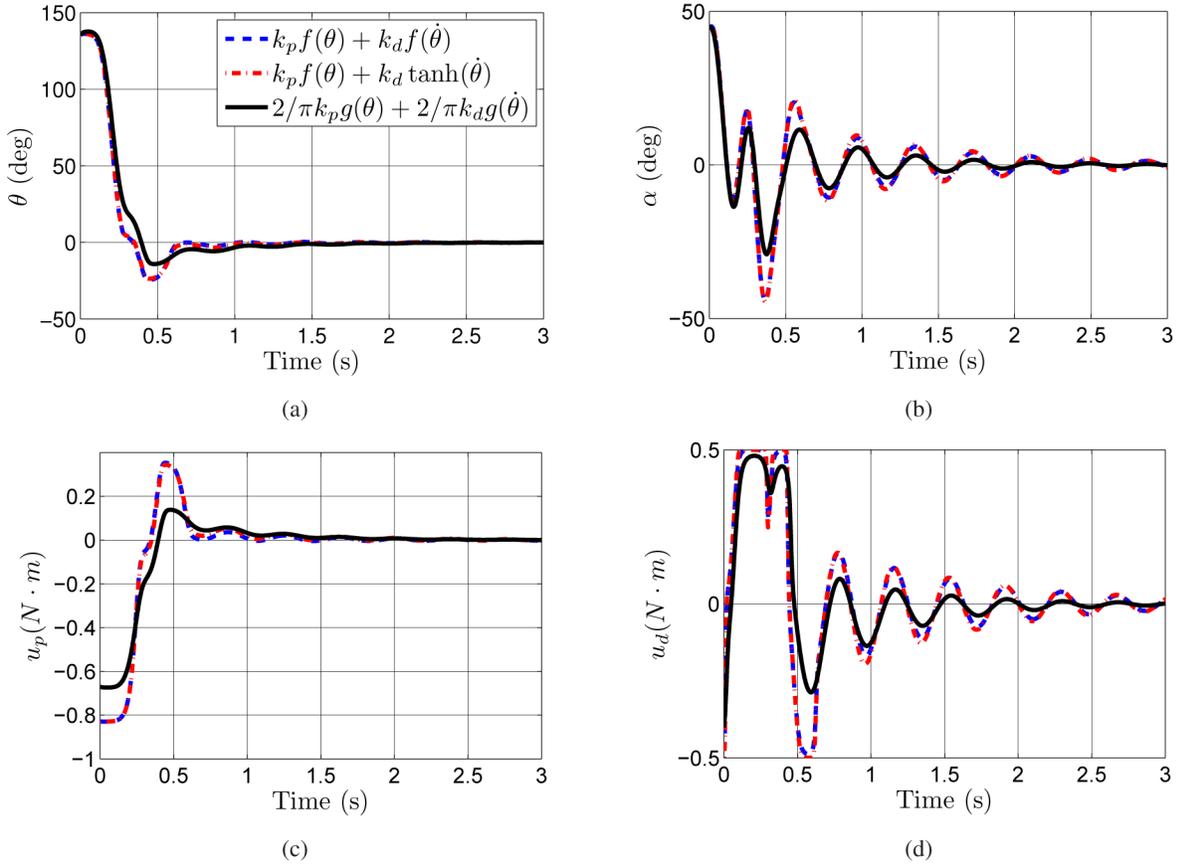


Fig. 4. Simulation of perturbed system with initial condition results: (a) θ versus time, (b) α versus time, (c) u_p versus time, (d) u_d versus time.

4.2. Trajectory Tracking

Trajectory tracking will now be considered. Consider the following desired trajectory, which can be found by interpolating a fifth order polynomial to satisfy a set of boundary conditions:

$$\theta_d(t) = \left[10 \left(\frac{t}{t_f} \right)^3 - 15 \left(\frac{t}{t_f} \right)^4 + 6 \left(\frac{t}{t_f} \right)^5 \right] (\theta_f - \theta_i) + \theta_i, \quad (8)$$

where θ_i is the initial position of θ , θ_f is the final position of θ , and t_f is the time required to move from θ_i to θ_f . This desired trajectory in Eq. (8) was input to the controllers tested as a reference input. Values of $\theta_i = 0$ deg, $\theta_f = 90$ deg and $t_f = 1$ second were used during the simulations. The controllers were tuned to $k_p = 1.7$ (N·m) and $k_d = 0.05$ (N·m), which guarantees $|\tau_c| \leq 1.75$ (N·m). The initial conditions of the simulation were $(\mathbf{q}, \dot{\mathbf{q}}) = (\mathbf{0}, \mathbf{0})$. Note that although trajectory tracking results are presented, the asymptotic stability of the proposed controllers subject to a tracking input has not been proven. In Fig. 5 is the θ and α response of the system, as well as u_p and u_d versus time. Figure 6 shows the same information for the perturbed system.

4.3. Discussion

The results of the simulations show that the proposed controllers behave quite similarly to an existing controller taken from [11], which is also used for saturation avoidance in manipulators. As expected, the

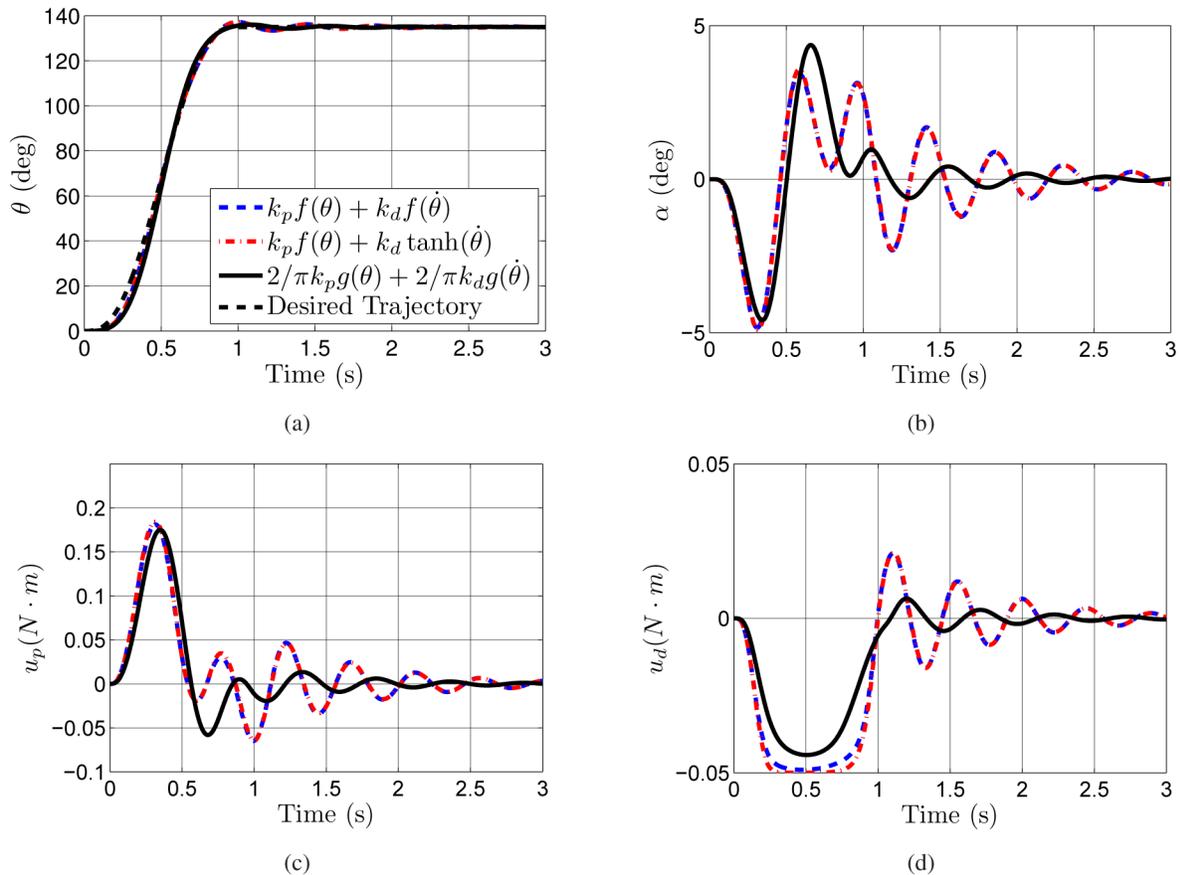


Fig. 5. Simulation of nominal system trajectory tracking results: (a) θ versus time, (b) α versus time, (c) u_p versus time, (d) u_d versus time.

controllers that are shown to be more aggressive in Fig. 2 do result in more overshoot in the initial condition simulation, while having a slightly shorter settling time. In the tracking simulations the controllers gave almost identical system responses. The joint angle was never more than 20 degrees away from the trajectory it was tracking, which meant the saturation functions were within the linear region between $-20 \text{ deg} < \theta < +20 \text{ deg}$ seen in Fig. 2 where they are identical, which explains the almost identical results. Figures 3(d), 4(d), 5(d) and 6(d) show that not all controllers allow the derivative control to fully saturate. It is interesting to note that although the controllers provide relatively different derivative control, the system responses are quite similar.

5. CONCLUSIONS

In this paper we investigated the control of a single-link flexible-joint robotic manipulator using various controllers that ensure actuator saturation avoidance. Moreover, it was shown that the proposed controllers render the desired equilibrium point of the closed-loop system asymptotically stable. The simulation results suggest that the proposed controllers perform similarly to saturation avoidance controllers found in literature, but this may not necessarily be true in all conditions. It is also worth mentioning that certain saturation functions may be computationally simpler than others, which is crucial in robotic applications with limited computational hardware. A large selection of saturation functions is therefore ideal, as the least computationally intensive function can be chosen for a specific application.

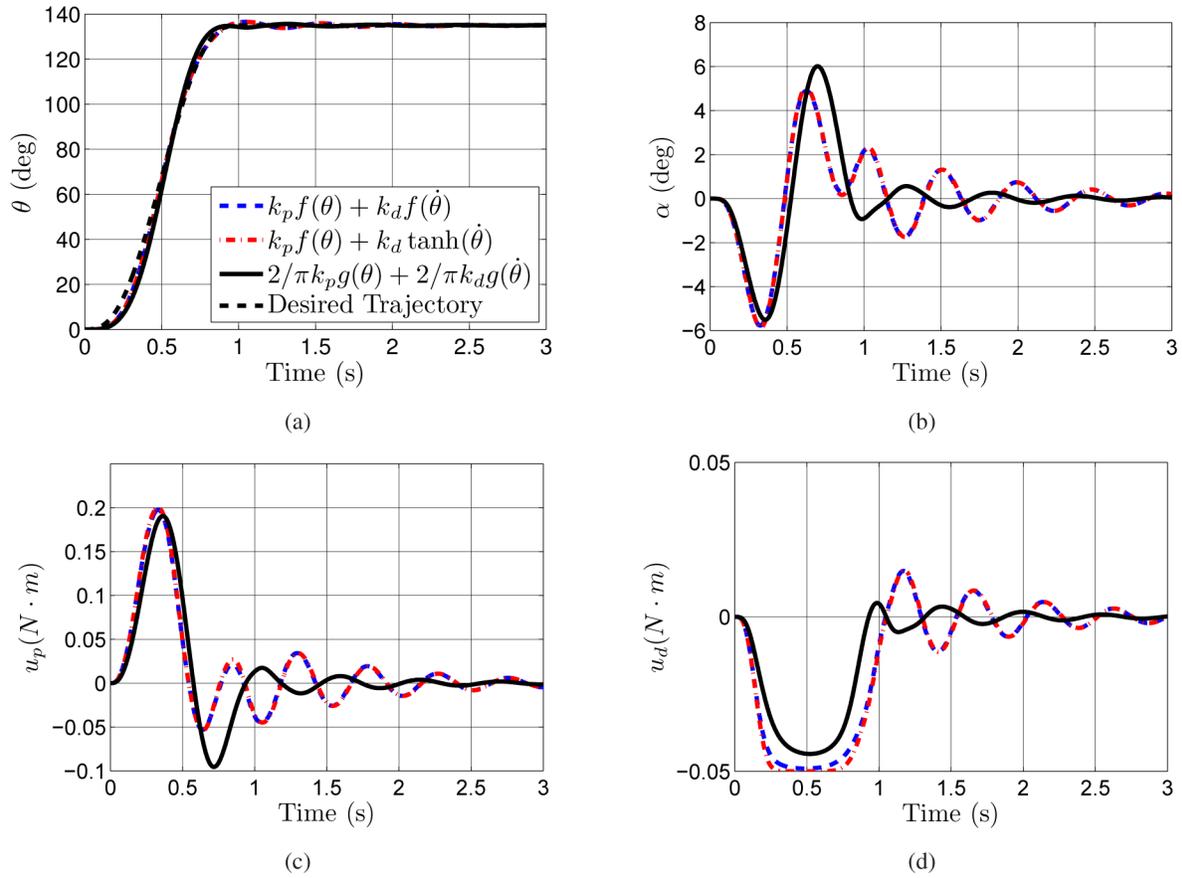


Fig. 6. Simulation of perturbed system trajectory tracking results: (a) θ versus time, (b) α versus time, (c) u_p versus time, (d) u_d versus time.

Although in this paper the control of a one-link flexible-joint robotic manipulator was investigated, this should be extended in future work to include multi-link robotic manipulators. Such an extension would allow the proposed controllers to be implemented on a wide range of robotic manipulators.

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