RESEARCH ON FLEXIBLE DRILL STRING VIBRATION INDUCED BY SONIC HARMONIC EXCITATION

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ABSTRACT

The undisturbed sampling of the overburden soil is attracting increased attention due to the rapid increases in the construction of large-scale domestic foundations and environmental protection engineering. To date, systematic theoretical research on sonic drilling technology has rarely been published. In the present paper, the vibration response induced by sonic harmonic excitation is studied by modeling the flexible drill string of a sonic drill; its dynamic theory and design methodology have been developed, which reveal effects of the excitation frequency, the structural parameters on vibration response of the drill string. The study of sonic drill string vibration is beneficial for improving the drilling efficiency and reducing the damage.

Keywords: sonic drill; drill string resonance; modal frequency; dynamic stress; cross-sectional area.

ÉTUDE DE VIBRATION D'EXCITATION HARMONIQUE SIMPLE DE LA COLONNE SOUPLE D'UNE FOREUSE ACOUSTIQUE

RÉSUMÉ

Avec l'accroissement rapide des travaux de génies civils et environnementaux, l'échantillonnage original de sol sans perturbation devient de plus en plus important ; pourtant, c'est rare de trouver dans les publications des études théoriques systématiques de la technique concernée. Le présent article, basé sur la modélisation de la colonne souple d'une foreuse acoustique, a étudié la question de la réflexion de la vibration de la colonne de forage acoustique sous l'action de la force de vibration harmonique simple, et a conçu ainsi sa théorie cinétique et sa méthode de conception. Cette étude favorise l'augmentation de l'efficacité de forage et la réduction des dommages au matériel causés par la vibration d'audiofréquence.

Mots-clés : foreuse acoustique; résonance de la colonne de forage; fréquence modale; contrainte dynamique; surface critique.

NOMENCLATURE

a	wave propagation velocity (m/s)
c	damping of the drill string (Ns/m)
E	elastic modulus of drill string (Pa)
f	excitation frequency (Hz)
F	excitation force (N)
l	length of the drill string (m)
m_e	total static moment of the audio-frequency vibrator eccentric mass (kg)
S	cross-sectional area of drill string (m^2)
t	time (s)
u	displacement field of drill string
x	distance from the vertices at time t (m)
$\mathbf{C}$	global damping matrix
$\mathbf{K}$	global stiffness matrix
$\mathbf{M}$	global mass matrix
Greek symbols	
α	damping coefficients of global mass matrix
β	damping coefficients of global stiffness matrix
δ	filter function
ζ_i	i-th order damping ratio
λ_i	i-th order frequency ratio
ρ	material density of drill string (kg/m^3)
φ_i	i-th order phase angle (rad)
ω	angular frequency of audio-frequency vibrator (rad/s)
ω_i	i-th order modal frequency (rad/s)
Subscripts	
max	maximum value
min	minimum value

1. INTRODUCTION

The essence of sonic drilling is high-frequency vibration drilling. It can combine high-frequency resonance, a low-speed rotary drill string, and static pressure to achieve rotary pressure drilling. At present, this technology is mainly used for environmental drilling and is also widely used in the foundation investigation, sand mineral exploration, blasting holes, water well construction and geotechnical construction, etc. The vibration frequency of the sonic drilling vibrator is typically 50–150 Hz, this frequency range is in a person's typical hearing range, which is why this technology is termed sonic drilling. When the vibrator's drive frequency matches the inherent modal frequency of the drill string, resonance will occur and tremendous energy will be transferred to the drill bit to achieve high-speed drilling. Sonic drilling is characterized by a high drilling rate, good geotechnical fidelity, less environmental pollution, and increased construction safety. It is widely adaptable and has become an indispensable method of environmental drilling [1–4].

Modern sonic drilling technology research began in the 1940s. The first patent related to sonic drilling equipment was awarded to Ray Roussy, a Canadian engineer who later founded the Sonic Drill Corporation. Research has indicated that audio-frequency vibration can significantly improve the effectiveness of rotary drilling. Meanwhile, researchers in Russia reported that sonic drilling was 3 to 20 times faster than common drilling. During the 1980s, the integration of sonic and rotary drilling technology became widely used by companies such as Versa-Drill International, Inc., Acker Drill Co., Inc., and Gus Pech Manufacturing Co. in the United States and the Tone Boring Co., Ltd in Japan. Domestic research on and applications of sonic drilling technology remain in the initial stages. The China University of Geosciences (Beijing) recently developed a sonic drill through support from the Key Program for China national project, and studied a design methodology of the key component such as the sonic top driver [5, 6]. A factory in Wuxi imported sonic vibrator technology and produced the YGL-S100 sonic driller. Its production test was successfully completed in the first half of 2013 [7]. The CRS-V sonic driller, manufactured by the Dutch company SonicSampDrill, was introduced to complete the construction of 34 continuous multichannel groundwater monitoring wells at the groundwater detection base in Tongzhou, Beijing. The CRS-V has proved to be more effective than traditional drills [8].

The method of pile driving, which is similar to sonic drilling, is commonly used in basic investigation and construction. The working frequency of its vibrator, generally between 20 and 50 Hz, is lower than the frequencies used in sonic drilling and cannot reach the low-order modal frequency of the pile itself. Because of the low stiffness of soil, in the pile driving process, the vibrator mainly causes a low-frequency resonance in the pile-soil system, and the range of soil participating in the resonance is relatively large. When modeling, the pile is typically simplified to a rigid body and the soil is simplified to a spring with linear or nonlinear damping. The result is a single-degree-of-freedom spring-mass-damper system that is used to study the dynamic characteristics of the pile driving [9, 10]. Because sonic drilling involves exciting the resonance of a variable-length drill string, which has a flexible body instead of rigid one, the use of pile driving modeling theory to study the dynamic characteristics of the sonic drilling mechanism may adversely affect the fidelity of the model and the accuracy of the research. Theoretical research literature on sonic drilling technology is seldom published at home and abroad.

Jeffrey notes that although the sonic drill string fluidizes the soil and creates a thixotropic transformation of clayey soils, the affected area around the drill rod is minimal. The citation data (Gumenskii and Kamorov, 1961) indicated that the zone of influence of the fluidization around the drill rod and core does not exceed a few millimeters and is typically in the range of 0.8 to 2.0 mm. The transformation is reversible once the vibration ceases [1]. When a sonic drill is working in a saturated stratum or with little drilling fluid, vibration deformation causes the pore water pressure in the soil close to the drill string to increase rapidly, which reduces the effective stress between the particles. The soil particles will suspend in the water when the increase in pore water pressure causes the effective stress between the soil particles to decrease to zero. At that point, the soil is considered to be in the flow condition, and its shear strength and stiffness decrease to approximately zero. The fluidized soil is a weak constraint on the flexible vibration of the drill string and the vibration energy of the drill string is mainly consumed by the viscous liquid. So the modeling of the drill string is different from the modeling of the low-frequency pile driving. Using dynamic modeling of the flexible drill string, systematic research can be performed to study the effects of the technical parameters of the sonic drill on the flexible drill string displacement field, velocity field, strain field, and stress field. The results of this research can improve the efficiency of sonic drilling and reduce the dangers of the audio-frequency vibration on the machinery.

2. MODELING AND SOLVING FOR THE VIBRATION OF THE SONIC DRILL STRING

As shown on the left side of Fig. 1, for the sonic drilling system, the upper end of the flexible drill string can translate along the mast guide rail, regarding it as a mobile hinge. The drill string and drill bit are weakly constrained by the viscous flow boundary of the liquefied soil. Thus, they may be regarded as a free end or translational pair. The right side of Fig. 1 presents the vibration model of the flexible sonic drill string. The movement of the drill string consists of the small drill string sinking speed (bulk motion) and the elastic vibration of the flexible drill string itself (relative motion). The modeling mainly focuses on the relative motion of the drillstring induced by the vibrator.

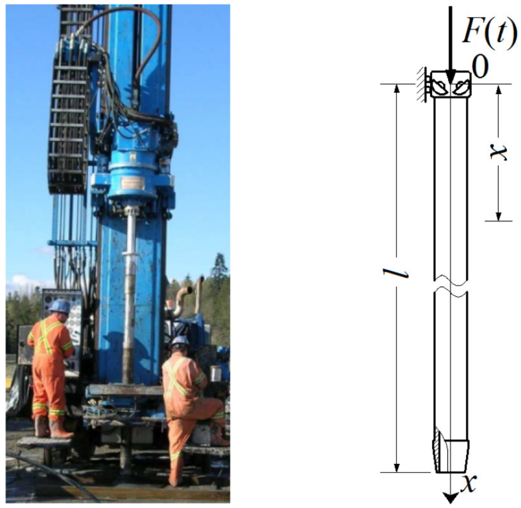

Fig. 1. Sonic drill rig and simplified model of the flexible drill string.

Assume that $u(x,t)$ is the longitudinal displacement of the rod's section that is a distance x from the vertices at time t, the wave propagation velocity in the medium is $a=\sqrt{E/\rho}$. The mass of the vibrator and the influence of the horizontal displacement on the longitudinal vibration are ignored [11]. The effects of gravity are also ignored because the excitation force is considerably greater than the weight of the drill string. Then, the longitudinal vibration differential equation of the uniform section sonic drill string under the longitudinally distributed load $f(x,t)$ is

$$\rho S\frac{\partial^2 u}{\partial t^2}+c\frac{\partial u}{\partial t}-ES\frac{\partial^2 u}{\partial x^2}=f(x,t). \tag{1}$$

Applying the separation of variables method, the solution of the homogeneous differential equation (1) can be obtained as

$$u_i(x,t)=X_i(x)q_i(t)=\left(A_i\sin\frac{\omega_i}{a}x+B_i\cos\frac{\omega_i}{a}x\right)q_i(t). \tag{2}$$

The principal mode equation (2), which corresponds to each modal frequency ω_i, satisfies the homogeneous equation (1). The general solution of the forced longitudinal vibration of a drill string can be expressed as the superposition of an infinite number of linearly independent particular solutions (principal modes)

$$u(x,t)=\sum_{i=1}^{\infty}X_i(x)q_i(t). \tag{3}$$

Substituting Eq. (3) into Eq. (1) yields

$$\rho S\sum_{i=1}^{\infty}X_i\ddot{q}_i+c\sum_{i=1}^{\infty}X_i\dot{q}_i-ES\sum_{i=1}^{\infty}X_i''\,q_i=f(x,t). \tag{4}$$

The jth-order regular coordinate equation can be derived by multiplying both sides of Eq. (4) by X_j and integrating this result with respect to l:

$$m_j\ddot{q}_j(t)+c_j\dot{q}(t)+k_jq_j(t)=Q_j(t). \tag{5}$$

Due to the vibration mode equation of an elastic body's multi-degree-of-freedom system which is orthogonal, when $i = j$, it can be shown that

$$\begin{cases} m_j = \int_0^l \rho S X_i X_j \, dx = \int_0^l \rho S X_j^2 \, dx \\ k_j = -\int_0^l ESX_i'' X_j \, dx = \int_0^l \omega_j^2 \rho S X_j^2 \, dx \\ c_j = c \int_0^l X_j^2(x) \, dx \end{cases} \tag{6a}$$

Then, the generalized force of the jth-order regular coordinate is

$$Q_j(t) = \int_0^l f(x,t) X_j(x) \, dx. \tag{6b}$$

By introducing a filter function δ, the concentrated load $F(t) = m_e \omega^2 \sin \omega t$ of the sonic vibrator on the top of drill string can be transformed into a distributed force as follows

$$f(x,t) = m_e \omega^2 \sin \omega t \, \delta(x) = \begin{cases} m_e \omega^2 \sin \omega t & x = 0, \\ 0 & x \neq 0. \end{cases} \tag{7}$$

Equation (7) is the distributed force form of the concentrated load on the top driver. It does not change the stress of the drill string. The regular generalized force is obtained as

$$Q_j(t) = \int_0^l m_e \omega^2 \sin \omega t X_j(x) \delta(x) dx = m_e \omega^2 \sin \omega t X_j(0). \tag{8}$$

The above equation is the regular generalized force form of $F(t) = m_e \omega^2 \sin \omega t$.

For the sonic drill system, the boundary conditions can be defined by Eq. (9) under the following conditions: first, there is a movable hinge on both ends; second, there is a free or linear motion pair on both ends; and third, one end is a movable hinge, whereas the other end is free or exhibits linear motion.

$$\frac{\partial u(0,t)}{\partial x} = \frac{\partial u(l,t)}{\partial x} = 0. \tag{9}$$

$A = 0$ can be obtained by substituting Eq. (9) into Eq. (2) because $\omega_i \neq 0$. Solving the drill string modal frequency equation $\sin \omega_i / al = 0$ yields

$$\omega_i = \frac{i\pi}{l} \sqrt{\frac{E}{\rho}}, \quad i = 0, 1, 2, \ldots \tag{10}$$

The coefficient of the vibration mode equation is

$$B_i = \sqrt{\frac{2m_i}{\rho S l}}, \quad i = 0, 1, 2, \ldots$$

The vibration mode equations of each mode are

$$X_i(x) = B_i \cos \frac{i\pi x}{l} = \sqrt{\frac{2m_i}{\rho S l}} \cos \frac{i\pi x}{l}, \quad i = 0, 1, 2, \ldots \tag{11}$$

By substituting Eq. (11) into Eq. (8), the regular generalized force is obtained as

$$Q_i(t) = \sqrt{\frac{2m_i}{\rho S l}} m_e \omega^2 \sin \omega t. \tag{12}$$

The above equation is substituted into Eq. (5) and let $\lambda_i = \omega/\omega_i$ and $\zeta_i = c_i/2m_i\omega_i$ be the i-th order frequency ratio and the i-th order damping ratio respectively to obtain the steady forced vibration response of the drill string under the simple harmonic excitation force as follows:

$$q_i(t) = \sqrt{\frac{2}{\rho Slm_i}} \frac{m_e\lambda_i^2}{\sqrt{(1-\lambda_i^2)^2 + (2\zeta_i\lambda_i)^2}} \sin(\omega t - \varphi_i), \tag{13}$$

where

$$\varphi_i = \arctan\frac{2\zeta_i\lambda_i}{1-\lambda_i^2}.$$

Thus, under the effect of the simple harmonic excitation force $F(t) = m_e\omega^2 \sin\omega t$, the steady-state solution of the forced vibration of the drill string is

$$u(x,t) = \frac{2m_e}{\rho Sl}\sum_{i=0}^{\infty} \frac{\lambda_i^2 \sin(\omega t - \varphi_i)}{\sqrt{(1-\lambda_i^2)^2 + (2\zeta_i\lambda_i)^2}} \cos\frac{i\pi x}{l}. \tag{14}$$

The above equation demonstrates that the continuous drill string excited vibration induced by the excitation force is the superposition of the forced vibration responses of all modes. The output energy of the sonic vibration of the drill string is in the form of damping.

The velocity field $v(x,t) = \partial u(x,t)/\partial t$ of the drill string and the acceleration field $a(x,t) = \partial^2 u(x,t)/\partial t^2$ of the inertial mass are directly proportional to the first and second powers of the excitation frequency, respectively.

3. ANALYSIS OF THE DRILL STRING SONIC VIBRATION RESPONSE

3.1. Displacement Response and Frequency Control of the Drill String

In the sonic drilling process, the drill bit amplitude is an important factor that affects the drilling rate. By letting $x = l$ in Eq. (14), the equation of the drill bit vibration response can be obtained as

$$u(l,t) = \frac{2m_e}{\rho Sl}\sum_{i=0}^{\infty} \frac{\lambda_i^2 \sin(\omega t - \varphi_i)}{\sqrt{(1-\lambda_i^2)^2 + (2\zeta_i\lambda_i)^2}} \cos i\pi \tag{15}$$

Rewrite (15)

$$\frac{\rho S}{m_e}u(l,t) = \frac{2}{l}\sum_{i=0}^{\infty} \frac{\lambda_i^2 \sin(\omega t - \varphi_i)}{\sqrt{(1-\lambda_i^2)^2 + (2\zeta_i\lambda_i)^2}} \cos i\pi. \tag{16}$$

The amplification coefficient $\lambda_i^2/\sqrt{(1-\lambda_i^2)^2 + (2\zeta_i\lambda_i)^2}$ is closely related to frequency ratio and damping ratio.

Damping is a key factor that influences the vibration of the drill string. The damping is not of a single form when the drill string vibrates; external viscous damping and internal damping are also present [12]. In the analysis of dynamics, the commonly used damping model is Rayleigh damping $\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K}$. In Rayleigh damping model, the high order damping ratio is much larger than the low ones, but according to Wen's experiment, the modal damping ratios of each order vibration have the same order of magnitude, and the value of a higher-order modal damping ratio is slightly larger. For a small damping $(0 \le \zeta_i \le 0.2)$ system, the modal damping ratios of each order vibration are typically set identically [13].

The drill string of sonic drill works in viscous liquid, and its modal damping ratio is taken to be $\zeta_i = \zeta = 0.01$.

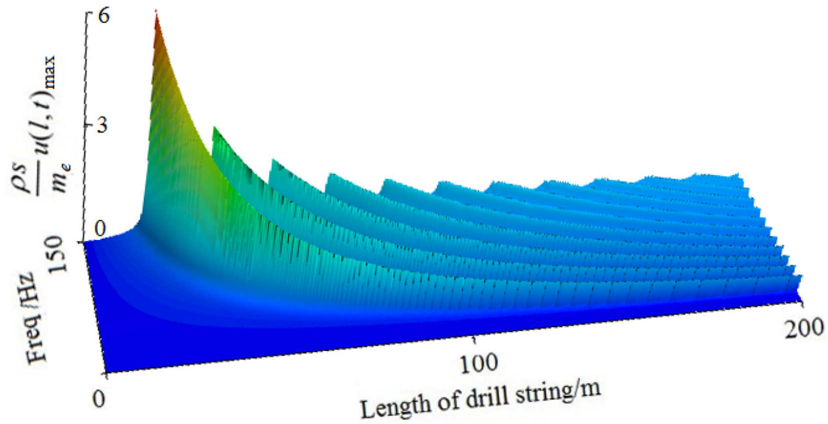

Fig. 2. Relation curve between the modal frequency and the length of the drill string.

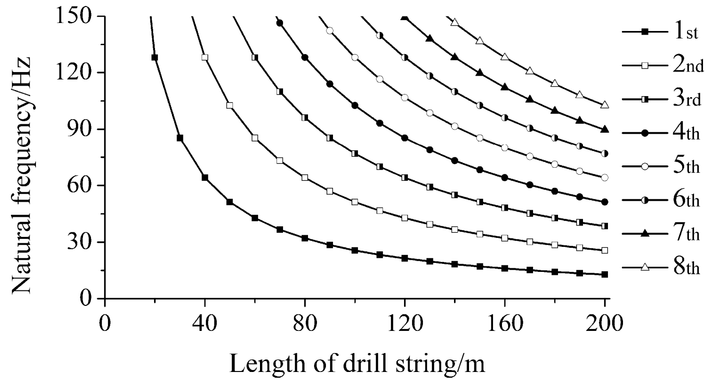

Fig. 3. Relation between the modal frequency of the drill string and its length.

The changing relationship of

$$\frac{\rho S}{m_e}u(l,t)_{\max}$$

(the maximum value of $\rho S/m_e u(l,t)$) *versus* the drill string length l and the vibration frequency ω is obtained using Eq. (16). In the plane determined by the drill string length and its vibration frequency, a curved surface can be drawn by calculating the values of $\rho S/m_e u(l,t)_{\max}$ corresponding to different values of l and ω in the vibration period, as shown in Fig. 2. The region of the maximum value peak indicates that the drill string is in a resonance state. When the drill string achieves resonance, the amplitude of the drill bit is considerably larger than in the non-resonant regions. The amplitude decreases with increasing drill string length or quality $(M = \rho Sl)$ and with decreasing modal frequency.

Figure 3 presents the relation curve between the modal frequency of the drill string and the drill string length according to Eq. (10). Each "wave peak" in Fig. 2 is consistent with the natural frequencies of the drill string determined by Eq. (10). The amplitude of the "wave peak" is primarily determined by the resonant mode shape of the drill string induced by the excitation force.

When the length of the drill string is 48 m and the excitation frequency f of the sonic vibrator is 53.36 Hz, which is the first natural frequency of the drill string determined by Equation (10), the drill string will be in a resonance state under the vibration force. When adding a 3 m drill rod to the 48 m drill string, the length of the drill string changes to 51 m. Assume that the frequency of the excitation force is still 53.36 Hz. Using

Eq. (15), the specific ratio of the drill bit displacement responses of two drill strings with different lengths can be calculated as $u(48,t)_{\max}/u(51,t)_{\max} \approx 5.6$. The difference in the drill string length is only 3 m, but if the vibration frequency does not change, the vibration amplitude of the drill bit will decrease sharply compared to the amplitude of the 48 m drill string that is in its resonance state. Under the single-frequency excitation force, the excitation frequency must be adapted to the changing drill string length to keep the drill string resonant. During the sonic drilling process, the driving frequency of the sonic top driver should be in accordance with Eq. (10) as the drill string length varies to realize efficient drilling. Therefore, Eq. (10) can be regarded as the frequency control equation of the vibrator in the sonic drilling process.

3.2. Dynamic Stress of the Drill String

Some regions inside the drill string will be subject to considerable alternating stress when the drill string reaches a resonance state. This stress may easily cause the drilling pipe to rupture. By taking the partial derivative of Eq. (14) with respect to x, we can obtain the strain field $\varepsilon = \partial u(x,t)/\partial x$ of the drill string and then use it to obtain the dynamic stress field $\sigma(x,t)$ of each point inside the drill string

$$\sigma(x,t) = E\varepsilon = E\partial u(x,t)/\partial x = -\frac{2Em_e\pi}{\rho S l^2}\sum_{i=0}^{\infty}\frac{i\lambda_i^2\sin(\omega t-\varphi_i)}{\sqrt{(1-\lambda_i^2)^2+(2\zeta\lambda_i)^2}}\sin\frac{i\pi x}{l}. \tag{17}$$

Equation (17) demonstrates that the internal stress of the drill string is inversely proportional to the square of the drill string's length (l^2) and its cross-sectional area S, which means that a shorter and thinner drill string will generate greater dynamic stress; $\lambda_{i_r} = 1$ when the drill string reaches the i_r-th order resonance, a higher resonance modal order will generate greater dynamic stress.

When the length of the drill string with yield strength σ_s is given as l, the resonant angular frequency ω_i of the i-th-order mode can be determined. When the strength safety requirements of the drill string are satisfied, the critical cross-sectional area of the sonic drill string where the string does not undergo dynamic stress buckling can be obtained as follows:

$$S_c = -\frac{2Em_e\pi}{\rho\sigma_s l^2}\left[\sum_{i=0}^{\infty}\frac{i\lambda_i^2\sin(\omega t-\varphi_i)}{\sqrt{(1-\lambda_i^2)^2+(2\zeta\lambda_i)^2}}\sin\frac{i\pi x}{l}\right]_{\max}. \tag{18}$$

In the resonance state, the maximum dynamic stress is $\sigma(x,t)_{\max}$, which is close to

$$\frac{\pi Em_e}{\rho s l^2}\,\frac{i_r}{\zeta}$$

and the critical cross-sectional area is S_c, which is close to

$$\frac{\pi Em_e}{\rho\sigma_s l^2}\,\frac{i_r}{\zeta}.$$

The maximum dynamic stress and critical cross-sectional area are highly sensitive to damping, and they are nearly inversely proportional to the damping ratio ζ. Equation (17) can be rewritten as follows:

$$\frac{\rho S}{Em_e}\sigma(x,t) = -\frac{2\pi}{l^2}\sum_{i=0}^{\infty}\frac{i\lambda_i^2\sin(\omega t-\varphi_i)}{\sqrt{(1-\lambda_i^2)^2+(2\zeta\lambda_i)^2}}\sin\frac{i\pi x}{l}. \tag{19}$$

Again taking the length of the drill string as 48 m, Eq (10) can be used to obtain the first three natural frequencies of the drill string as approximately 53.36, 106.72, and 160.08 Hz. When the excitation frequency

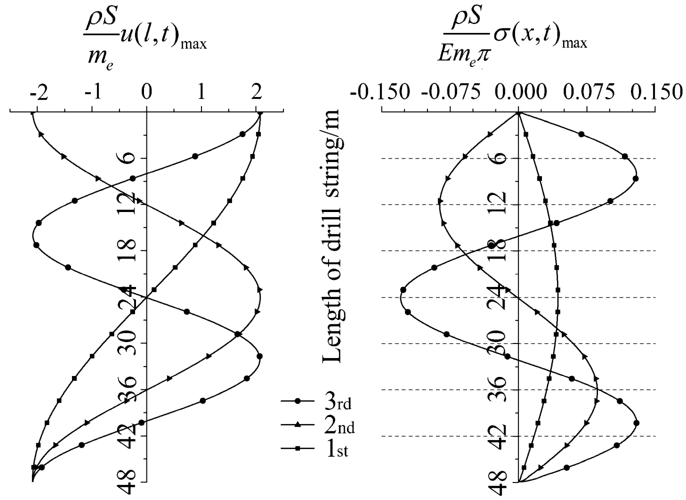

Fig. 4. Maximum displacement field and corresponding stress field of a 48 m drill string.

of the sonic vibrator is equal to these first three natural frequencies, the changes in the displacement field $\rho S/m_e u(x,t)_{\max}$ and the stress field $\rho S/Em_e\sigma(x,t)_{\max}$ in the 1st, 2nd, and 3rd-order resonance states of the drill string can be obtained. These fields are shown in Fig. 4.

For a given damping ratio, because

$$u(x,t)_{\max} \to \frac{m_e}{\rho Sl}\,\frac{1}{\zeta},$$

the order number of the resonance has a smaller effect on the displacement field

$$\frac{\rho S}{m_e}u(x,t)_{\max};$$

as the maximum stress inside the drill string

$$\sigma(x,t)_{\max} \to \frac{\pi E m_e}{\rho s l^2}\,\frac{i_r}{\zeta},$$

it is nearly proportional to the order number of the resonance i_r.

In Fig. 4, when the drill string is in its first-order resonance state, the maximum stress point inside the drill string appears at the 24 m position, which is the middle of the 48 m drill string. The largest stress points appear at 12 and 36 m of the 48 m drill string when it is in its second-order resonance state. They appear at 8, 24, and 40 m in the third-order resonance state. Suppose that each drill rod is 3 m long. Then, the screwed joint is on the maximum dynamic stress point when the drill string length is 12, 24, or 36 m, which are integer multiples of 3 m. The screw joints of the drill rods are the weak link of the drill string because stress concentrations are easily caused in those locations. Normally, when the drill string is in the i_rth-order resonance state, the drill rod screw joints will appear at certain maximum stress points as long as the product of m (the number of drill rods) and l_{rod} (the length of a single drill rod) satisfy the following equation:

$$ml_{\text{rod}} = \frac{(2n-1)}{4f}\sqrt{\frac{E}{\rho}} \quad n = 1,2,\ldots,i_r, \tag{20}$$

When faced with a hard formation that is difficult to penetrate, the screw joint will be under large alternating stress for a long period of time, particularly when the sonic drill works at short-hole drilling. In this situation, the drill rod joint may easily produce a fatigue crack or even break due to the effect of the alternating sonic stress.

4. CONCLUSIONS

The following conclusions can be drawn from the modeling study of the sonic drill:

1. During the sonic drilling process, when the length of the drill string changes, the driving frequency of the vibrator should be adjusted according to the frequency control equation

$$\omega_i = \frac{i\pi}{l}\sqrt{\frac{E}{\rho}} \text{ to keep the drill string in resonance.}$$

2. The vibration of the continuous drill string induced by the excitation force $F(t)$ is the superposition of the forced vibration of the *n*th order $(n = 0, 1, 2, \ldots)$ modes. When the drill string remains in a low-order resonance, the amplitude of the drill bit decreases with the extension of the drill string, and its value is mainly determined by the amplitude of the resonant order mode.

3. During the drilling, the order of the resonant mode shape has only a slight influence on the displacement field of the drill string; however, it has a significant influence on the dynamic stress inside the drill string, the maximum stress is nearly proportional to the order number of the resonance.

4. When

$$ml_{\text{rod}} = \frac{(2n-1)}{4f}\sqrt{\frac{E}{\rho}}$$

is met, the joints of the drill rods will experience great alternating stress. Because the dynamic stress of the drill string is inversely proportional to its cross-sectional area, the critical area of the drill string should be verified in the design process. Otherwise, it will affect the strength of the drill string and its fatigue life.

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