ADAPTIVE SLIDING-MODE CONTROL: SIMPLEX-TYPE METHOD

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ABSTRACT
The simplex method is easy and brief for designing the sliding mode, but it also has some disadvantages. Since the control vectors are constant, the chattering phenomenon also occurs when switching control takes place in simplex-type SMC scheme. Hence, we make few modifications to the simplex method that form an irregular simplex such that it improves the choice of simplex control vector and chattering phenomenon. The irregular simplex is obtained by an adaptive control law. The stabilization of a nonlinear multi-input system by using adaptive control based on simplex-type sliding-mode control philosophy is examined in this paper. The adaptive law and stabilization theorem are proposed and proved. The simulation results demonstrate that the simplex-type adaptive sliding-mode control proposed in this paper is a good solution to the chattering problem in the simplex sliding-mode control.

Keywords: simplex, adaptive control, sliding mode.

COMMANDE ADAPTIVE PAR MODE-GLISSANT: MÉTHODE DE TYPE SIMPLEXE

RÉSUMÉ
La méthode de type simplexe est facile et concise pour la conception du mode-glissant, mais elle a aussi des désavantages. Étant donné que les vecteurs de commande sont constants, le phénomène de broutage survient aussi lors du passage de commande dans le schéma SMC de type simplexe. Par conséquent, nous avons fait quelques modifications à la méthode simplexe qui forme un simplexe irrégulier de sorte qu'il améliore l'option des vecteurs de commande simplexe et le phénomène de broutage. Le simplexe irrégulier est obtenu par une loi de commande adaptive. La stabilisation du système linéaire à multiples entrées en utilisant la commande adaptive basée sur la commande adaptive par mode-glissant de type simplexe est examinée. La loi adaptive et le théorème de stabilisation sont proposés et testés. Les résultats de simulation démontrent que le commande adaptive par mode-glissant de type simplexe proposée dans cet article est bonne solution au problème de broutage dans la méthode de commande par mode-glissant de type simplexe.

Mots-clés : méthode simplexe; commande adaptive; mode-glissant.

1. INTRODUCTION

The simplex sliding-mode control was first proposed by Baida and Izosimovfor [1] for the control of multi-input continuous systems. Diong [2–4] extended it to linear multivariable continuous systems. Others [5–9] extended the research from multi-input linear continuous systems to multi-input nonlinear continuous systems and demonstrated the feasibility of these systems. In this paper, we will develop the simplex-type adaptive sliding-mode control (STASMC) in nonlinear multi-input system. Basically, the simplex SMC control design procedure is similar to that of the conventional sliding-mode control (SMC). It includes sliding and reaching conditions. In the first step, we determine the sliding surface such that the closed-loop systems are stable in the sliding mode. Next, a set of simple hitting control vectors is chosen so that the states will move toward the sliding surface as soon as possible.

The simplex-type SMC is different from the conventional SMC in some fundamental aspects. The choice of control vectors is easier than that in the SMC. Suppose in an m dimensional vector space, the control vectors must satisfy the simplex definition, and the simplex dependent set contains only $m+1$ vectors. A basic feature of simplex SMC for $m=2$ is shown in Fig. 1, where σ is the switch function, u is the designed control vector, and $\sum$ is the corresponding region. The simplex method is easy and brief to design the sliding mode, but it also has some disadvantages. Since the control vectors are constant, the chattering phenomenon also occurs when switching control takes place in simplex-type SMC scheme. In order to ensure conditions are satisfied, sometime we must select sufficiently large magnitude of control vector. However, increasing the magnitude of the control vector aggravates the chattering problem. Hence, we make few modifications to the simplex method that form an irregular simplex such that it improves the choice of simplex control vectors and chattering phenomenon. The irregular simplex is obtained by an adaptive control law. The approaches and the proof are introduced in the subsequent section. In this paper, the adaptive control law and irregular simplex are proposed and an example will examine the proposed method.

The remainder of this paper is organized as follows: In Section 2, the Simplex-Type Sliding-Mode Control is described. In Section 3, the Adaptive Simplex Sliding-Mode Control-based simplex control is addressed. Section 4 presents an example to demonstrate the validity of the propounded of adaptive simplex sliding-mode control. Finally, we conclude with Section 5.

2. SIMPLEX-TYPE SLIDING-MODE CONTROL

Considering a class of nonlinear affine systems

$$\dot{x} = f(x) + g(x)u, \tag{1}$$

where $U \in R^m$ is the control input, $x \in R^n$ is system states, and $m < n$. We want to control the state variables $x = x(t)$ of the system to fulfill the condition

$$||x(t)|| \leq L, \quad t \geq 0 \tag{2}$$

such that the prescribed sliding function

$$\sigma(x) = Sx = 0, \tag{3}$$

where $S \in R^{m \times n}$, the sliding manifold is defined by the mapping $\sigma : R^n \to R^m$.

2.1. The Sliding-Mode Dynamics Design

By means of linearization with respective to the equilibrium points of the nominal system (1), one can get a linear multi-input system (4)

$$\dot{x} = Ax + Bu, \tag{4}$$

where $x \in R^n$ and $u \in R^m$ represent the state and control vectors of the system, respectively, and all the matrices have appropriate dimensions. System (4) is assumed to be controllable. Several sliding surface design methods have been developed. In this paper, we adopt the design approaches of [2–4] as the method of determining the sliding function and sliding-mode dynamics. Suppose $H \in R^{(n-m)\times n}$ and $S \in R^{m\times n}$ is selected such that both of them have full rank, and H satisfies

$$HB = 0. \tag{5}$$

The plant is transformed by

$$\begin{bmatrix} x_r \\ \sigma \end{bmatrix} = Tx = \begin{bmatrix} H \\ S \end{bmatrix} x, \tag{6}$$

where $T \in R^{n\times n}$ is invertible, and

$$T^{-1} = [\bar{H} \quad \bar{S}]. \tag{7}$$

Applying the transformation T defined by (6) to (7), we obtain

$$\begin{bmatrix} \dot{x}_r \\ \dot{\sigma} \end{bmatrix} = \begin{bmatrix} HA\bar{H} & HA\bar{S} \\ SA\bar{H} & SA\bar{S} \end{bmatrix} \begin{bmatrix} x_r \\ \sigma \end{bmatrix} + \begin{bmatrix} 0_{(n-m)\times m} \\ I_m \end{bmatrix} u, \tag{8}$$

where $x_r = Hx$ and $\sigma = Sx$. In the sliding mode, $\sigma = \dot{\sigma} = 0$, the sliding-mode dynamics from (8) are then obtained by

$$x_r = HA\bar{H}x_r. \tag{9}$$

Let $M \in R^{m\times(n-m)}$, then matrix S can be parameterized by

$$S = B^* + MH. \tag{10}$$

This in turn gives

$$\bar{H} = H^* - BM, \tag{11}$$

where B^* and H^* are the pseudo inverse of B and H. The sliding-mode dynamic (9) can now be described in terms of M by

$$\dot{x}_r = (A_r - B_r M)x_r, \tag{12}$$

where

$$A_r = HAH^*, \quad B_r = HAB. \tag{13}$$

2.2. Simplex Control Strategy

Definition 1. The set $U = \{u_1, u_2, \dots, u_{m+1}\}$, where $u_i \in R^m$ are nonzero and distinct vectors, is said to form a simplex in R^m if any m of the vectors are linearly independent and there exist $m+1$ real positive constants $c_1, c_2, \dots,$ and c_{m+1} such that $\sum_{i=1}^{m+1} c_i u_i = 0$ and $\sum_{i=1}^{m+1} c_i = 1$.

This definition means that a simplex is a set of $m+1$ affinely independent vectors in R^m such that o^m is in the interior of the convex hull of those vectors. For simplex-type nonlinear sliding-mode control, let us effect a coordinate transformation, the following transformation is performed [1, 8, 9]

$$\varepsilon = (Sg)^{-1}\sigma \tag{14}$$

the system of equations describe the variable ε will be

$$\dot{\varepsilon} = (Sg)^{-1}\,\dot{\sigma} = (Sg)^{-1}\,Sf + u \tag{15}$$

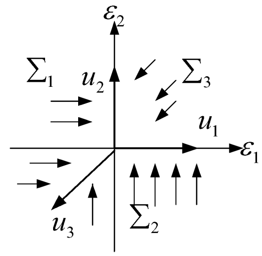

Fig. 1. The regular simplex of control vectors u_1, u_2 and u_3.

the ε space is divided into $m+1$ regions, which are described by

$$\sum_i = \{\varepsilon : \varepsilon = \sum_{j \neq i, j=1}^{m+1} \lambda_j u_j, \lambda_j > 0\}, \quad i = 1, \ldots, m+1 \tag{16}$$

thus, each $\sum_i$ is an infinite cone, with vertex at 0^m, situated on the side of R^m opposite to u_i.

Definition 2. If the set $U = \{u_1, u_2, \ldots, u_{m+1}\}$ forms a simplex set in ε space, then the control strategy for simplex method is defined as

$$u = f(\varepsilon) = \begin{cases} u_j, \quad \varepsilon \in \sum_j \\ u_s = (Sg)^{-1} Sf - cu_i, \quad \varepsilon \in u_i, s \neq i \end{cases}, \tag{17}$$

where $c > 0$, the significance of (17) is that when ε is in $\sum_j$, then the control u is selected as $u_j; u = u_s$ when $\varepsilon \in u_i, s \neq i$, i.e. when ε is exactly at the u_i, then u_s can tackle the stabilization problem at the intersection. The basic configuration of Eqs. (14) and (17) for two-input case is depicted in Fig. 1, where ε is decomposed into three regions determined by the selection of the simplex set U. Figure 1 shows the regular simplex control.

In order to get the reaching condition, the following assumption is needed.

Assumption 1 [14]. There exist real numbers α_i and ξ such that the evolution of x determined by (1) and (17) satisfy

$$(Sg)^{-1} Sf = -\sum_{i=1}^{m+1} \alpha_i u_i \quad \alpha_i \geq 0 \tag{18}$$

$$\sum_{i=1}^{m+1} \alpha_i = \xi \quad 0 \leq \xi < 1 \tag{19}$$

The reaching condition for regular simplex is now represented by the following theorem:

Theorem 1. If the regular simplex set $U = \{u_1, u_2, \ldots, u_{m+1}\}$ is chosen and the ε state space is partitioned into $m+1$ non-overlapping regions as (16), the control is defined by (17), and Assumption 1 is satisfied, then the origin $\varepsilon = 0$ is asymptotically stable and the convergence takes place in finite time.

Proof. From the definition of (17), two cases will be discussed in this proof.

Case 1: Suppose the control $u \in R^m$ and the states of the plant locate at $\sum_j$, i.e. ε in the $\sum_j$ region, $j \in [1, m+1]$. From Definition 1, the control vector u_j can be represented by

$$u_j = -\sum_{i=1, j\neq j}^{m+1} \frac{c_i}{c_j} u_i = -\sum_{i=1, j\neq j}^{m+1} \gamma_i u_i, \quad \gamma_i = c_i/c_j > 0. \tag{20}$$

In (16), the ε in the $\sum_j$ is represented as $\varepsilon = \sum_{i=1,i\neq j}^{m+1} \lambda_i u_i$, where $\sum_{i=1,i\neq j}^{m+1} \lambda_i = \beta$ and $\lambda_i \geq 0$. In fact, for any $\varepsilon \neq 0, \beta(\varepsilon) = \sum_{i=1}^{m+1} \lambda_i > 0$ and $\beta(\varepsilon) = 0$ as $\varepsilon = 0$. Thus, $\beta(\varepsilon)$ can be treated as a Lyapunov function. According to (17), the control law is represented as $u = u_j$ for $\varepsilon \in \sum_j$. We define

$$U_j = [u_1 \ldots, u_{j-1}\ u_{j+1} \ldots, u_{m+1}] \tag{21}$$

which is nonsingular and define the following m-dimensional vectors

$$\begin{aligned} \Lambda_j(k) &= [\lambda_1^T \quad \cdots \quad \lambda_{j-1}^T \lambda_{j+1}^T \quad \cdots \quad \lambda_{m+1}^T]^T, \\ G_j &= [\gamma_1^T \quad \cdots \quad \gamma_{j-1}^T \gamma_{j+1}^T \quad \cdots \quad \gamma_{m+1}^T]^T, \\ K_j &= [\alpha_1^T \quad \cdots \quad \alpha_{j-1}^T \alpha_{j+1}^T \quad \cdots \quad \alpha_{m+1}^T]^T, \\ b &= [1 \quad \cdots \quad 1\ 1 \quad \cdots \quad 1]^T. \end{aligned} \tag{22}$$

Then, we can obtain

$$\varepsilon = \sum_{i=1,i\neq j}^{m+1} \lambda_i u_i = U_j \Lambda_j \tag{23}$$

$$\beta = \sum_{i=1,i\neq j}^{m+1} \lambda_i = b^T \Lambda_j \tag{24}$$

$$u_j = \sum_{i=1,jneqj}^{m+1} \gamma_i u_i = -U_j G_j, \tag{25}$$

because ε is in the $\sum_j$ region, we obtain (26) from (17) and (25)

$$u = u_j = -U_j G_j. \tag{26}$$

Thus, Eq. (18) can be represented as

$$(Sg)^{-1}\, Sf = -\sum_{i=1}^{m+1} \alpha_i u_i = -U_j K_j + \alpha_j U_j G_j / \tag{27}$$

From (15), the derivative of ε is $\dot{\varepsilon} = U_j \dot{\Lambda}$, and then we can get

$$\dot{\Lambda}_j = U_j^{-1} \dot{\varepsilon} = U_j^{-1}((Sg)^{-1}\, Sf + u_j) = -K_j + \alpha_j G_j - G_j \tag{28}$$

which can be expanded as

$$\begin{aligned} [\lambda_1^T \cdots \lambda_{j-1}^T \lambda_{j+1}^T \cdots \lambda_{m+1}^T]^T = -[\alpha_1^T \cdots \alpha_{j-1}^T \alpha_{j+1}^T \cdots \alpha_{m+1}]^T + \alpha_j [\gamma_1^T \cdots \gamma_{j-1}^T \gamma_{j+1}^T \cdots \gamma_{m+1}^T]^T \\ - [\gamma_1^T \cdots \gamma_{j-1}^T \gamma_{j+1}^T \cdots \gamma_{m+1}^T]^T. \end{aligned} \tag{29}$$

Now, we want to prove the rate of change of the function $\beta(\varepsilon)$ should be negative. Differentiation of β yields

$$\dot{\beta} = b^T \dot{\Lambda}_j = \dot{\lambda}_1 + \cdots + \dot{\lambda}_{j-1} + \dot{\lambda}_{j+1} + \cdots + \dot{\lambda}_{m+1}. \tag{30}$$

The above equation is equal to the row-by-row summation of (29), that is,

$$\dot{\beta} = -\sum_{i=1,jneqj}^{m+1} \alpha_i + \alpha_j \sum_{i=1,jneqj}^{m+1} \gamma_i - \sum_{i=1,ineqj}^{m+1} \gamma_i = -\sum_{i=1}^{m+1} \alpha_i - (1-\alpha_j) \sum_{i=1,ineqj}^{m+1} \gamma_i. \tag{31}$$

According to Assumption 2, $\alpha_i \geq 0$ and $0 < \sum_{i=1}^{m+1} \alpha_i = \xi < 1$, thus $\sum_{i=1,ineqj}^{m+1} > 0$ and $(1-\alpha_j) > 0$. Moreover, $\gamma_i > 0$ and $\sum_{i=1,\neq j}^{m+1} \gamma_i > 0$, we can obtain

$$\dot{\beta} = -\sum_{i=1}^{m+1} \alpha_i - (1-\alpha_j) \sum_{i=1,ineqj}^{m+1} \gamma_i < 0. \tag{32}$$

Case 2: Suppose the states of the plant are at u_i, i.e. $\varepsilon \in u_i, ineqj$. Similarly, we have the following results:

$$\varepsilon = \lambda_i u_i, \tag{33}$$

$$V(\varepsilon) = \lambda_i, \tag{34}$$

$$u_s = -\sum_{l=1}^{m+1} \alpha_l u_l - c u_i, \tag{35}$$

where $c > 0$. Since $\varepsilon \in u_i$, we can obtain (36) from (17) and (35)

$$u = u_s - -\sum_{l=1}^{m+1} \alpha_l u_l - c u_i. \tag{36}$$

Concluding (18), (33), (34) and (36), one can obtain the differences of $V(\varepsilon)$ as

$$\begin{aligned} \dot{V}(\varepsilon) &= \dot{\lambda}_i = \dot{\varepsilon} u_i^T (u_i u_i^T)^{-1} = ((sg)^{-1}\, sf + u) u_i^T (u_i u_i^T)^{-1} \\ &= \left(\sum_{l=1}^{m+1} \alpha_l u_l - \sum_{l=1}^{m+1} \alpha_l u_l - c u_i \right) u_i^T (u_i u_i^T)^{-1} = -c < 0. \end{aligned} \tag{37}$$

This completes the proof.

3. ADAPTIVE SIMPLEX SLIDING-MODE CONTROL

No matter in which region ε is, when using adaptive rule we adjust the control vectors u, where the new control vectors u no longer satisfy the simplex Definition 1, so the following theorem is addressed in this problem.

Theorem 2. The ε space is divided into $m+1$ regions by the regular simplex set $U = \{u_1, u_2, \ldots, u_{m+1}\}$, when $\varepsilon \in \sum_j$, $j = 1, \ldots, m+1$, the adaptive control change quantity $du = -u_j - (Sg)^{-1}\, Sf - \eta\varepsilon$, then the irregular simplex control, $u_j' = u_j + du = (Sg)^{-1}\, Sf - \eta\varepsilon$, cause the origin of ε domain asymptotically stable in finite time, and then results in σ domain being asymptotically stable in finite time.

Proof. Suppose the control $u \in R^m$ and the states of the plant locate at $\sum_j$, i.e. ε in the $\sum_j$ region, $j \in [1, m+1]$. In order to obtain the adaptive control change quantity du, the control u in Eq. (15) is modified as

$$\dot{\varepsilon} = (Sg)^{-1}\, Sf + u + du. \tag{38}$$

From Definition 1, the control vector u_j can be represented by

$$u_j = -\sum_{i=1, ineqj}^{m+1} \frac{c_i}{c_j} u_i = -\sum_{i=1, ineqj}^{m+1} \gamma_i u_i, \tag{39}$$

where $\gamma_i = c_i/c_j > 0$. According to (17), the control law can be represented as $u = u_j$ for $\varepsilon \in \sum_j$. Thus, the control law is repressed as

$$u = u_j = -\sum_{i=1, ineqj}^{m+1} \gamma_i u_i. \tag{40}$$

Furthermore, from Assumption 1, $(Sg)^{-1}\, Sf = \sum_{i=1}^{m+1} \alpha_i u_i, \alpha_i \geq 0$, then we can get

$$\dot{\varepsilon} = (Sg)^{-1}\, Sf + u_j + du = \sum_{i=1}^{m+1} \alpha_i u_i - \sum_{i=1, ineqj}^{m+1} \gamma_i u_i + du. \tag{41}$$

The update law is selected by considering the Lyapunov function

$$\nu(\varepsilon) = \frac{1}{2} \varepsilon^T\, \varepsilon. \tag{42}$$

According to the Lyapunov stability criterion, if the time derivative of the function $\dot{\nu}(\varepsilon)$ is negative, then (1) is asymptotically stable

$$\dot{\nu}(\varepsilon) = \varepsilon^T\, \dot{\varepsilon}. \tag{43}$$

Substituting (41) into (43), we have

$$\dot{\nu} = \varepsilon^T\, \dot{\varepsilon} = \varepsilon^T ((Sg)^{-1}\, Sf - \sum_{i=1, ineqj}^{m+1} \gamma_i u_i + du). \tag{44}$$

If

$$du = \sum_{i=1, ineqj}^{m+1} \gamma_i u_i - (Sg)^{-1}\, Sf - \eta \varepsilon,$$

substituting this du into (44) yields

$$\dot{\nu} = \varepsilon^T \dot{\varepsilon} < \eta ||\varepsilon||/ \tag{45}$$

The derivative of the Lyapunov function $\nu(\varepsilon)$ is negative definite, so system (1) is asymptotically stable in the finite time, it means $\varepsilon \to 0$. From (14), $\sigma = (Sg)\varepsilon$ yields $\sigma \to 0$. In (45), η is the adaptive rate that changes the ε_j convergence time, and $|\bullet|$ is Euclidean norm.

For any partition of the ε space into $m+1$ regions by using an irregular simplex, it is possible to associate with each of such regions a suitably oriented control vector that belongs to irregular simplex which causes the origin of the ε space to be asymptotically stable in finite time. The control strategy is defined as $u = u'_j = (Sg)^{-1}\, Sf - \eta \varepsilon, \varepsilon \in \sum_j$, or $\varepsilon \in u_i, s \neq i$ where u'_j is the new irregular simplex control vector; η is adaptive rate.

Table 1. Five different cases for simulation.

Case 1	$x_0 = [10 \quad 0 \quad 10 \quad 0]^T$	$u_1 = [5 \quad 0]^T, u_2 = [0 \quad 5]^T, u_3 = [-5 \ -5]^T$	simplex only $\eta = 0$
Case 2	$x_0 = [10 \quad 0 \quad 10 \quad 0]^T$	$u_1 = [10 \quad 0]^T, u_2 = [0 \quad 10]^T, u_3 = [-10 \ -10]^T$	simplex only $\eta = 0$
Case 3	$x_0 = [10 \quad 0 \quad 10 \quad 0]^T$	$u_1 = [10 \quad 0]^T, u_2 = [0 \quad 10]^T, u_3 = [-10 \ -10]^T$	With update law $\eta = 35$
Case 4	$x_0 = [-10 \quad 0 \quad 10 \quad 0]^T$	$u_1 = [10 \quad 0]^T, u_2 = [0 \quad 10]^T, u_3 = [-10 \ -10]^T$	With update law $\eta = 35$
Case 5	$x_0 = [10 \quad 0 \quad -10 \quad 0]^T$	$u_1 = [10 \quad 0]^T, u_2 = [0 \quad 10]^T, u_3 = [-10 \ -10]^T$	With update law $\eta = 35$

4. EXAMPLE

Consider a nonlinear system described as follows:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = 9.62\sin(x_1) + 5 + 2u_1 + 6.25\sin(x_3)$$
$$\dot{x}_3 = x_4$$
$$\dot{x}_2 = 9.62\sin(x_3) + 1.5625 + 1.6u_2 + 10\sin(x_2).$$

The first step is the selection of H, which must satisfy $HB = 0$. Thus, after some computation one can get

$$H = \begin{bmatrix} 1\,0\,1\,0 \\ 1\,0\,0\,0 \end{bmatrix}, \quad M = \begin{bmatrix} 0 & 1.5 \\ 1.25 & -1.8750 \end{bmatrix}, \quad S = \begin{bmatrix} 1.5 & 0.5 & 0 & 0 \\ -0.625 & 0 & 1.25 & 0.625 \end{bmatrix}.$$

The eigenvalues of $A_S - B_S M$ are designed as $\{-2 \quad -3\}$. The choice of the simplex control vector must be big enough to satisfy the simplex definition and fulfill Assumption 1, then Theorem 1 will be met. There are five cases for simulations in this nonlinear system example as shown in Table 1. Cases 1 and 2 are to be used to explain affection between Theorem 1 and Assumption 1. For the sake of comparison between simplex control and irregular simplex adaptive control law, throughout cases 2 to 5 we use the same simplex control vector to proceed with the simulation. So the control vectors are chosen as $u_1 = [10 \quad 0]^T$, $u_2 = [0 \quad 10]^T$, $u_3 = [-10 \quad -10]^T$, and adaptive rate is chosen as $\eta = 35$. The simulation results of case 1 for time and phase trajectories are depicted in Fig. 2.

The choice of the simplex control vector should not only be to satisfy Theorem 1 but also be big enough to fulfill Assumption 1. According to Fig. 2, it is shown that the small magnitudes of the simplex control vectors, $u_1 = [5\ 0]^T, u_2 = [0\ 5]^T, u_3 = [-5\ \ -5]^T$, cannot meet Assumption 1 that results in an unstable response. Therefore, if we do not compensate the regular simplex u_j the system is always unstable. The simulation results of case 2 for time and phase trajectories are depicted in Fig. 3. The control vectors $u_1 = [10 \quad 0]^T$, $u_2 = [0 \quad 10]^T$, $u_3 = [-10 \quad -10]^T$, are appropriately large, hence it is big enough to satisfy the simplex definition and fulfill Assumption 1 such that it can stabilize the system. The time and phase trajectories of cases 3 to 5 for three different initial values are depicted in Figs. 4–6, respectively. From the figures of time response, one can see the proposed irregular simplex adaptive control law can achieve a smoother response than a non-adaptive one. In case 2, the states of the system will repeatedly cross the switching surfaces and slide to the boundary of origin. In this situation, since the control vectors are constant and have no strategy to reduce the control quantity, the chattering phenomenon occurs in the boundary of origin. However, the proposed irregular simplex adaptive control law can achieve a smooth response, where the adaptive law can drive the states towards the origin with lower chattering, because the control change

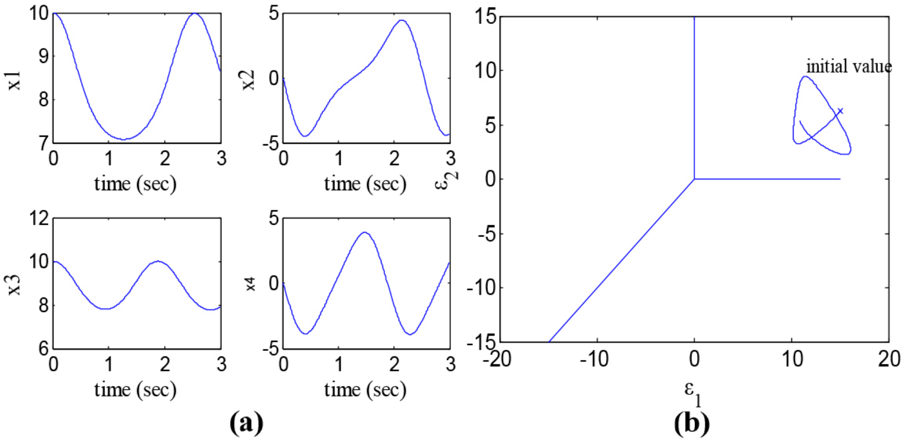

Fig. 2. The system response plots of case 1 without update law under $x_0 = [10\ 0\ 10\ 0]^T$ (a) The time response of case 1. (b) The phase response of case 1.

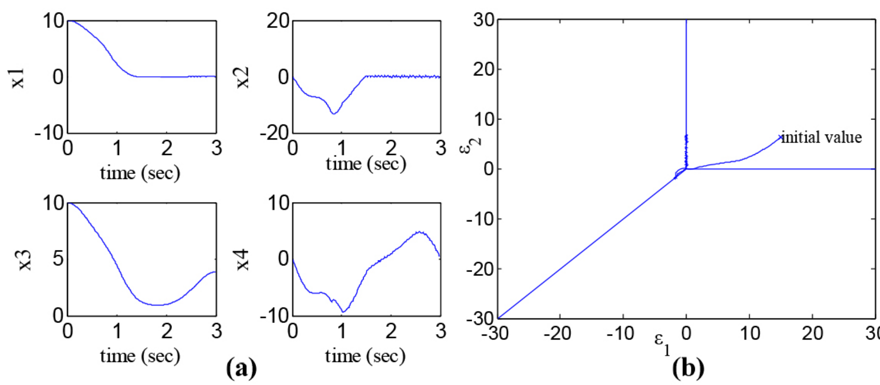

Fig. 3. The system response plots of case 2 without update law under $x_0 = [10\ 0\ 10\ 0]^T$ (a) The time response of case 2. (b) The phase response of case 2.

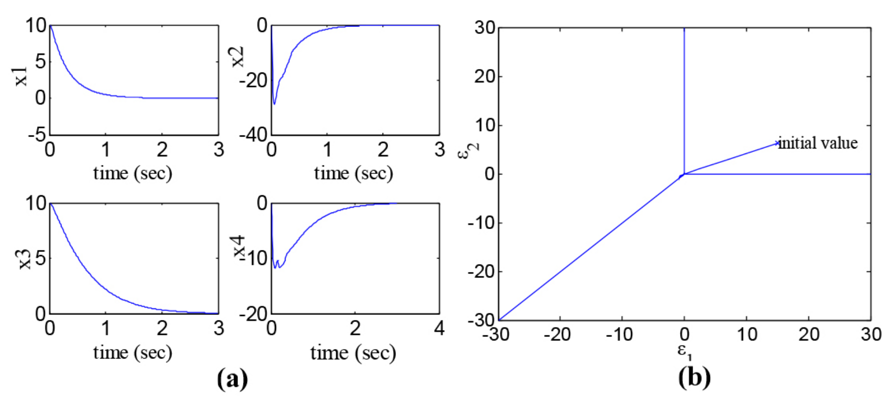

Fig. 4. The system response plots of case 3 without update law under $x_0 = [10\ 0\ 10\ 0]^T$ (a) The time response of case 3. (b) The phase response of case 3.

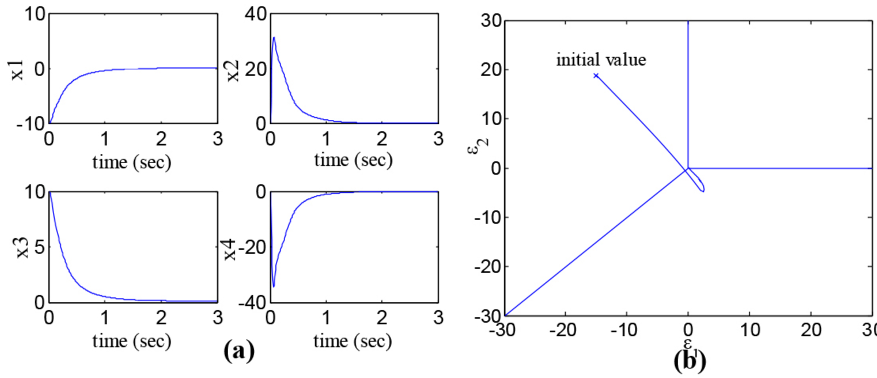

Fig. 5. The system response plots of case 4 without update law under $x_0 = [10\ 0\ 10\ 0]^T$ (a) The time response of case 4. (b) The phase response of case 4.

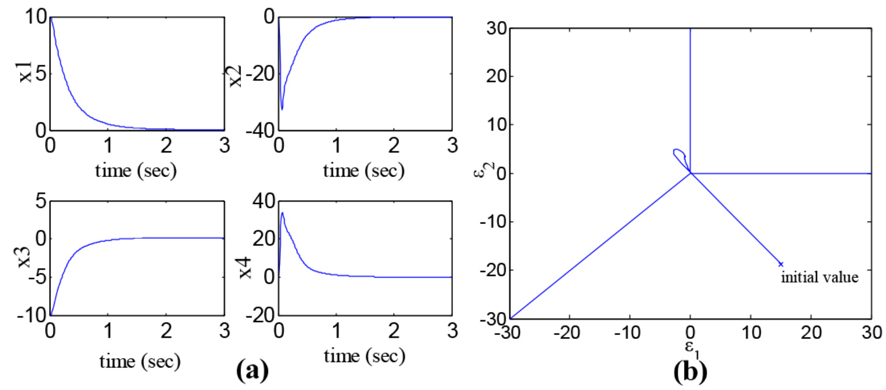

Fig. 6. The system response plots of case 5 without update law under $x_0 = [10\ 0\ 10\ 0]^T$ (a) The time response of case 5. (b) The phase response of case 5.

quantity, $du = -u_j - (Sg)^{-1}\, Sf - \eta\varepsilon$ can compensate for the magnitude of the simplex control vector u_j. It gradually increases the control vector u_j, when ε goes far away from the origin, and can gradually decrease the magnitude of the simplex control vector u_j during ε approach to the origin, which results in chattering elimination.

5. CONCLUSIONS

This paper considered the design of adaptive control based on simplex-type sliding-mode control philosophy for a nonlinear affair multivariable system. These procedures were illustrated on a nonlinear multi-input system; the results showed adaptive simplex-type sliding-mode control much improved transient responses compared with the transient responses obtained with only simplex control law. The example also indicated that a fairly large simplex control effort was required to obtain better performance in compliance with the conditions of simplex definition and Assumption 1, however, the transient responses accompany with the chattering. Adaptive control based on simplex-type sliding-mode method can reduce the control effort required and attenuate chattering circumstances. So one can conclude that both the SSMC and STASMC theoretical study and the simulation results prove that the simplex-type adaptive sliding-mode control proposed in this paper is a stable control scheme. STASMC is a good solution to the chattering problem in the simplex sliding-mode control.

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