# A CONTACT RATIO AND INTERFERENCE-PROOF CONDITIONS FOR A SKEW LINE GEAR MECHANISM

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ICETI-2014 J1076_SCI
No. 15-CSME-48, E.I.C. Accession 3823

## ABSTRACT

Line Gear (LG) is an innovative gear which is mainly applicable to micro mechanical systems proposed by Yangzhi Chen. A Skew Line Gear Mechanism (SLGM) is one pair of LGs transmitting force and motion between two skewed axes. In this study, a design formula of a contact ratio for a SLGM is deduced, and eight influencing parameters are found. The influences of six parameters on a contact ratio for a SLGM with non-vertical skewed axes are studied by using of two coordinate parameters given definitely. The principal influencing parameters on a contact ratio for a SLGM are obtained. Moreover, two types of interferences between the driving and the driven line teeth are discussed, then these geometric parameter formulas for the interference-proof conditions are deduced, and design formulas of a maximum line tooth number for the driving line gear are derived for different interference-proof conditions.

**Keywords:** contact ratio; influencing parameters; interference-proof conditions; skew line gear mechanism (SLGM).

# RAPPORT DE CONTACT ET CONDITIONS DE PROTECTION D'INTERFÉRENCE POUR UN MÉCANISME D'ENGRENAGE EN LIGNE ASYMÉTRIQUE

## RÉSUMÉ

Les engrenages en ligne sont des engrenages innovateurs qui sont surtout applicables aux systèmes micros mécaniques proposés par Yangzhi Chen. Un mécanisme d'engrenage en ligne asymétrique est une paire d'engrenages en ligne transmettant l'énergie et le mouvement entre deux axes asymétriques. Au cours de cette étude, une formule de conception pour un rapport de contact est déduite, et on a trouvé huit paramètres d'influence. Les influences de six paramètres sur un rapport de contact pour un mécanisme d'engrenage en ligne asymétrique sont étudiées en utilisant deux paramètres des coordonnés clairement donnés. Les principaux paramètres d'influence d'un rapport de contact sont obtenus. De plus, deux types d'interférence entre les lignes d'entraînement et les lignes entraînées sont discutés, alors ces formules de paramètres géométriques pour les conditions de protection d'interférence sont déduites, et les formules de conception pour un nombre maximum de dents pour l'engrenage d'entraînement sont dérivées pour les différentes conditions de protection d'interférence.

**Mots-clés :** rapport de contact; paramètre d'influence; conditions de protection d'interférence; mécanisme d'engrenages en ligne asymétrique.

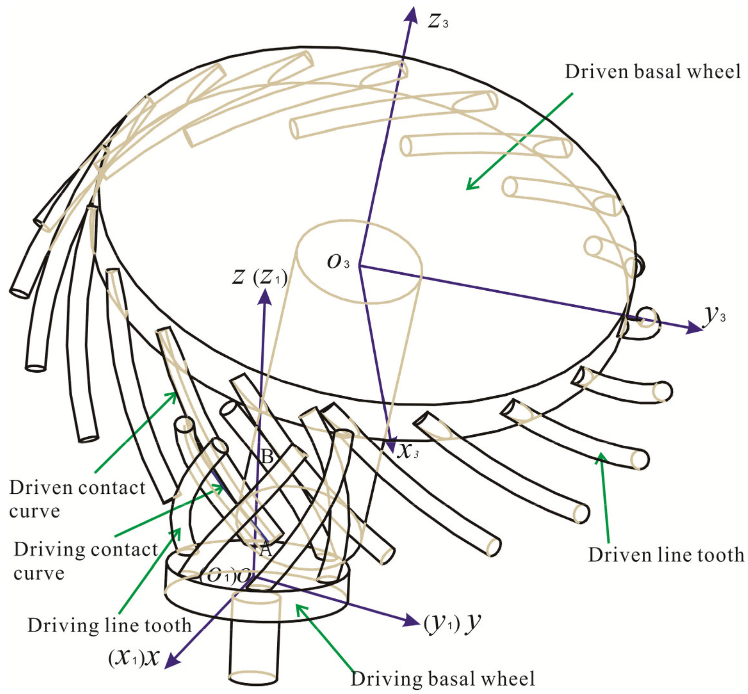


Fig. 1. The Skew Line Gear Mechanism (SLGM).

## 1. INTRODUCTION

Line Gear (LG) is an innovative gear which is based on the space curve meshing theory proposed by Yangzhi Chen [1]. With a large transmission ratio, small size and light weight, it is mainly applicable to small and micro mechanical systems. A pair of LGs consists of a driving and a driven line gears. The driving and the driven line teeth are uniformly distributed around the basal wheels of the driving and the driven line gears, respectively, as shown in Fig. 1. The shapes of the driving and the driven line teeth both are the simple entities. A pair of LGs transmits force and motion by means of point meshing between a couple of space conjugate curves, namely, a driving contact curve and a driven contact curve. Its fundamental meshing principal is totally different from the surface meshing theory for the conventional gears with complex tooth profiles. The LG has also been named as Space Curve Meshing Wheel (SCMW) or Micro Elastic Meshing Wheel (MEMW) in previous studies, including the Micro-elastic Meshing Wheel (MEMW) [1], Arbitrary Intersecting Gear (AIG) [1] and Space Curve Meshing Skew Gear (SCMSG) [2]. The Space Curve Meshing Skew Gear Mechanism (SCMSGM) is named as Skew Line Gear Mechanism (SLGM) in this study. The previous researches mainly focused on MEMW and AIG covering their meshing theory, geometric design formula, manufacture technology, transmission error analysis and strength analysis [1].

A contact ratio is an important criterion for evaluating the smoothness and continuity of gearing transmission [3, 4], so it is indispensable to gear research. The researches of a contact ratio for the conventional gearing mechanism mainly focused on the impact of a contact ratio (especially high contact ratio) on the load distribution between meshing teeth [5], the noise of the gear transmission [6], the contact strength and the bonding strength of the tooth surface [7] and the bending strength of the tooth root. To guarantee transmission continuity of the gear mechanism, a contact ratio generally must be greater than 1. Moreover, the selection of a value of a contact ratio is on demand for different applications. A contact ratio for a conventional gear mechanism is mainly depended on its addendum, number of teeth, pressure angle and center distance.

A contact ratio of a LG mechanism with two cross shafts or arbitrary intersecting shafts has been deduced, and its influencing parameters have been discussed in previous studies [8, 9]. This study mainly focuses on a contact ratio for a SLGM, and covers deducing the design formula of a contact ratio for a SLGM, analyzing its influencing parameters and influence laws and those interference-proof conditions of a SLGM. This study provides a theoretical basis to calculate and select a contact ratio and design parameters for a SLGM.

## 2. A CONTACT RATIO FOR A SLGM

The coordinate systems for a SLGM are shown in Fig. 1 [2]. $o_1 - x_1y_1z_1$ and $o_3 - x_3y_3z_3$ are fixed with the driving and driven line gears, respectively. The driving line gear rotates around axis $z_1$ with a rotation speed $\varpi_1$, whose direction is the negative direction of the $z$-axis. $\varphi_1$ is a rotation angle of the driving line gear. The driven line gear rotates around axis $z_3$ with a rotation speed $\varpi_2$, and $\varphi_3$ is a rotation angle of the driven line gear. $o$–$xyz$ is a fixed coordinate system. The distance from point $o_3$ to plane$yoz$ is $|a|$, to plane$xoy$ is $|b|$, and to plane $xoz$ is $|c|$. A supplementary angle between axis $z$ and axis $z_3$ is equal to $\theta$ in a range of $0° < \theta < 180°$.

According to Chen et al. [8], a contact ratio for a SLGM can be defined as

$$\varepsilon = \Delta\varphi_3/(2\pi/N_2) \tag{1}$$

where $\Delta\varphi_3$ is an angle through which one line tooth of the driven line gear rotates from starting meshing to ending meshing, and $N_2$ is the line tooth number of the driven line gear. Meanwhile, $\Delta\varphi_1$ is an angle covering a range of that one line tooth of the driving line gear rotates from starting meshing to ending meshing, and $N_1$ is the line tooth number of the driving line gear. A transmission ratio of a driving line gear to a driven line gear is denoted as $i_{12}$, namely, $i_{12} = \varpi_1/\varpi_2 = \varphi_1/\varphi_3$. A contact ratio of a SLGM can be deduced as follows: $\Delta\varphi_3 = \Delta\varphi_1/i_{12}$ and $N_2 = N_1/i_{12}$,

$$\varepsilon = \Delta\varphi_1 N_1/(2\pi) \tag{2}$$

According to the space curve meshing theory [1], a space curve meshing equation can be obtained as

$$c\cos\theta\cos(\varphi_1 - t) - a\cos\theta\sin(\varphi_1 - t) - (n\pi + nt - b)\sin\theta\sin(\varphi_1 - t) = 0 \tag{3}$$

where $n$ is a pitch coefficient of the driving line teeth, $n > 0$; $t$ is a parameter indicating the driving line gear, $t_s \le t \le t_e$, while $t_s$ and $t_e$ denote the parameters of the initial and the terminal meshing points, respectively. That $\Delta t = t_e - t_s$ denotes a variation range of parameter $t$ for the meshing points, $\Delta t > 0$. According to Eq. (3), Eq. (2) can be rewritten and simplified as

$$\varepsilon = \frac{\Delta t N_1}{2\pi} - \frac{N_1}{2\pi}\arctan\left(\frac{c\cos\theta\sin\theta\Delta t n}{(c\cos\theta)^2 + (a\cos\theta + (n\pi + nt_e - b)\sin\theta)(a\cos\theta + (n\pi + nt_s - b)\sin\theta)}\right) \tag{4}$$

When $\theta = 90°$, Eq. (4) is simplified as $\varepsilon = \Delta t N_1/(2\pi)$, whose design formula is the same as that of a contact ratio for arbitrary intersecting gear mechanism proposed by Chen and Chen [9]. So those influencing parameters of a contact ratio for a SLGM with two non-vertical skewed axes are mainly discussed in Section 3.

## 3. INFLUENCING PARAMETERS ON A CONTACT RATIO OF A SLGM

According to Eq. (4), a contact ratio $\varepsilon$ for a SLGM depends on parameters $N_1$, $\Delta t$, $t_s$, $c$, $a$, $b$, $n$ and $\theta$. From Eq. (4), because a value range of the arctan function is $[-\pi/2, \pi/2]$, a value range of a contact ratio $\varepsilon$ for a SLGM is decided as below: $[\Delta t N_1/(2\pi) - N_1/4, \Delta t N_1/(2\pi) + N_1/4]$, which means parameters $N_1$ and $\Delta t$ have a significant impact on a contact ratio $\varepsilon$ for a SLGM.

### 3.1. The Relationship between Parameters *a* and *b*

According to Chen et al. [2], an equation of the driven contact curve in $o_3 - x_3y_3z_3$ is expressed as follows:

When $0^\circ < \theta < 90^\circ$,

$$\begin{cases} x_M^{(3)} = -[\cos(\varphi_1 - t)m + a]\cos\varphi_3\cos\theta - [n\pi + nt - b]\cos\varphi_3\sin\theta + [\sin(\varphi_1 - t)m + c]\sin\varphi_3 \\ y_M^{(3)} = -[\cos(\varphi_1 - t)m + a]\sin\varphi_3\cos\theta - [n\pi + nt - b]\sin\varphi_3\sin\theta - [\sin(\varphi_1 - t)m + c]\cos\varphi_3 \\ z_M^{(3)} = [\cos(\varphi_1 - t)m + a]\sin\theta - [n\pi + nt - b]\cos\theta \end{cases} \tag{5}$$

When $90^\circ < \theta < 180^\circ$,

$$\begin{cases} x_M^{(3)} = -[\cos(\varphi_1 - t)m + a]\cos\varphi_3\cos\theta - [n\pi + nt - b]\cos\varphi_3\sin\theta - [\sin(\varphi_1 - t)m + c]\sin\varphi_3 \\ y_M^{(3)} = [\cos(\varphi_1 - t)m + a]\sin\varphi_3\cos\theta + [n\pi + nt - b]\sin\varphi_3\sin\theta - [\sin(\varphi_1 - t)m + c]\cos\varphi_3 \\ z_M^{(3)} = [\cos(\varphi_1 - t)m + a]\sin\theta - [n\pi + nt - b]\cos\theta \end{cases} \tag{6}$$

where $m$ is a meshing radius of a driving contact curve in a range of $m > 0$. Eqs. (5) and (6) can be simplified as follows:

$$\begin{cases} x_M^{(3)} = -P_r\cos(\varphi_3 + \zeta) \\ y_M^{(3)} = -P_r\sin(\varphi_3 + +\zeta) \\ z_M^{(3)} = [\cos(\varphi_1 - t)m + a]\sin\theta - [n\pi + nt - b]\cos\theta \end{cases} \tag{7}$$

$$\begin{cases} x_M^{(3)} = -P_r\sin(\varphi_3 - \zeta) \\ y_M^{(3)} = P_r\cos(\varphi_3 - \zeta) \\ z_M^{(3)} = [\cos(\varphi_1 - t)m + a]\sin\theta - [n\pi + nt - b]\cos\theta \end{cases} \tag{8}$$

where

$$P_r = \sqrt{([\cos(\varphi_1 - t)m + a]\cos\theta + (n\pi + nt - b)\sin\theta) \wedge 2 + [\sin(\varphi_1 - t)m + c] \wedge 2}$$

is a meshing radius of the driven line gear, and

$$\zeta = \frac{\sin(\varphi_1 - t)m + c}{[\cos(\varphi_1 - t)m + a]\cos\theta + (n\pi + nt - b)\sin\theta}$$

According to Eq. (3), Eq. (9) can be obtained as follows:

$$P_r = \sqrt{[(a\cos\theta + (n\pi + nt - b)\sin\theta) \wedge 2 + c \wedge 2](\sin(\varphi_1 - t)m + c) \wedge 2 / c \wedge 2} \tag{9}$$

According to Eq. (9), if $c \neq \pm m$, when $a\cos\theta + (n\pi + nt - b)\sin\theta = 0$, a value of $P_r$ is minimum, and not equal to 0; then $t_{(0)} = -a\cos\theta/(n\sin\theta) - \pi + b/n$ is the breaking point of the driven contact curve. Therefore, to ensure the continuity of a driven contact curve, a value range of parameter $t$ must be set as $(-\infty, t_{(0)})$ or $(t_{(0)}, \infty)$. Suppose that $t_{(0)} = 0$, then the following can be obtained: the relationship between $a$ and $b$ is presented as follows: $(a\cos\theta + (n\pi - b)\sin\theta) = 0$; $t_s \geq 0$ or $t_e \leq 0$, i.e. $t_st_e \geq 0$.

Therefore, the design formula of a contact ratio can be simplified as Eq. (10). The influencing parameters of a contact ratio for a SLGM could be reduced to be six. They are $N_1$, $\Delta t$, $t_s$, $c$, $n$ and $\theta$.

$$\varepsilon = \frac{\Delta t N_1}{2\pi} - \frac{N_1}{2\pi}\arctan\left(\frac{c\cos\theta\sin\theta\Delta t n}{c^2\cos^2\theta + n^2 t_s t_e \sin^2\theta}\right) \tag{10}$$

Denote

$$A = \Delta t N_1/(2\pi), \quad B = \frac{\Delta t N_1}{2\pi} - \frac{N_1}{2\pi}\arctan\left(\frac{\Delta t}{2\sqrt{t_s t_e}}\right) \quad \text{and} \quad C = \frac{\Delta t N_1}{2\pi} + \frac{N_1}{2\pi}\arctan\left(\frac{\Delta t}{2\sqrt{t_s t_e}}\right)$$

Table 1. The monotonicity of $\varepsilon(c)$.

| | $0<\theta<90^o$ | | | $90^o \le \theta < 180^o$ | | |
|---|---|---|---|---|---|---|
| $c$ | $(-\infty, c_1^{(0)})$ | $[c_1^{(0)}, c_2^{(0)}]$ | $(c_2^{(0)}, +\infty)$ | $(-\infty, c_1^{(0)})$ | $[c_1^{(0)}, c_2^{(0)}]$ | $(c_2^{(0)}, +\infty)$ |
| Monotonicity of $\varepsilon(c)$ | $\uparrow$ | $\downarrow$ | $\uparrow$ | $\downarrow$ | $\uparrow$ | $\downarrow$ |
| Range value of $\varepsilon(c)$ | $(A,C)$ | $[B,C]$ | $(B,A)$ | $(B,A)$ | $[B,C]$ | $(A,C)$ |
| Minimum value of $\varepsilon(c)$ | | $B$ | | | $B$ | |

Table 2. The monotonicity of $\varepsilon(n)$.

| | $c\cos\theta<0$ | | $c\cos\theta>0$ | |
|---|---|---|---|---|
| $n$ | $(0,n^{(0)})$ | $[n^{(0)},+\infty)$ | $(0,n^{(0)})$ | $[n^{(0)},+\infty)$ |
| Monotonicity of $\varepsilon(n)$ | $\uparrow$ | $\downarrow$ | $\downarrow$ | $\uparrow$ |
| Range value of $\varepsilon(n)$ | $(A,C)$ | $[A,C]$ | $(B,A)$ | $(B,A)$ |
| Minimum value of $\varepsilon(n)$ | $A$ | | $B$ | |

### 3.2. The Influence of Parameter *c* on a Contact Ratio for a SLGM

According to Eq. (10), $\varepsilon$ can be denoted as $\varepsilon(c)$ by using parameter $c$ as a variable. When $b \to -\infty$ or $b \to +\infty$, $\varepsilon(c)$ converges to $A$. $c_1^{(0)} = -\sqrt{n^2 t_s t_e \sin^2\theta/\cos^2\theta}$ and $c_2^{(0)} = \sqrt{n^2 t_s t_e \sin^2\theta/\cos^2\theta}$ are two extreme value points of $\varepsilon(c)$, while the extreme values of $\varepsilon(c)$ are $B$ and $C$. Table 1 shows the monotonicity of $\varepsilon(c)$. If $(\Delta t N_1/(2\pi) - N_1 \arctan(\Delta t/(2\sqrt{t_s t_e}))/(2\pi)) \ge 1$, then a contact ratio for a SLGM is greater than 1, no matter what values other parameters are taken as.

### 3.3. The Influence of Parameter *n* on a Contact Ratio for a SLGM

According to Eq. (10), $\varepsilon$ can be denoted as $\varepsilon(n)$ by using parameter $n$ as a variable. When $n \to 0$ or $n \to \infty$, $\varepsilon(n)$ converges to $A$. $n^{(0)} = \sqrt{c^2\cos^2\theta/(t_s t_e \sin^2\theta)}$ is an extreme value point of $\varepsilon(n)$, while the extreme values of $\varepsilon(n)$ are $B$ and $C$. Table 2 shows the monotonicity of $\varepsilon(n)$.

If $(\Delta t N_1/(2\pi) - N_1 \arctan(\Delta t/(2\sqrt{t_s t_e}))/(2\pi)) \ge 1$, then a contact ratio for a SLGM is greater than 1, no matter what values other parameters are taken as.

### 3.4. The Influence of Parameter $t_s$ on a Contact Ratio of a SLGM

According to Eq. (10), $\varepsilon$ can be denoted as $\varepsilon(t_s)$ by using parameter $t_s$ as a variable. When $t_s \to -\infty$ or $t_s \to +\infty$, $\varepsilon(t_s)$ converges to $A$. $t_s^{(0)} = -n^2\Delta t \sin^2\theta/(2n^2\sin^2\theta) = -\Delta t/2$ is an extreme value point of $\varepsilon(t_s)$, while its extreme value is that

$$\varepsilon(t_s^{(0)}) = \frac{\Delta t N_1}{2\pi} - \frac{N_1}{2\pi}\arctan\left(\frac{c\cos\theta\sin\theta\Delta t n}{c^2\cos^2\theta - n^2\Delta t\sin\theta/4}\right)$$

The extreme value of $\varepsilon(t_s)$ is $E = \Delta t N_1 - N_1\arctan(\Delta t n \sin\theta/(c\cos\theta))/(2\pi)$ at point $t_s = 0$ or $t_s = -\Delta t$ because $t_s t_e \ge 0$, i.e. $t_s > 0$ or $t_s < -\Delta t$. Table 3 shows the monotonicity of $\varepsilon(t_s)$.

### 3.5. The Influence of Parameter $\theta$ on a Contact Ratio of a SLGM

According to Eq. (10), $\varepsilon$ can be denoted as $\varepsilon(\theta)$ by using parameter $\theta$ as a variable. When$\theta \to 0^\circ$ or $\theta \to 180^\circ$, $\varepsilon(\theta) \to A$. $\theta_1^{(0)} = \arctan(\sqrt{c^2/(n^2 t_e t_s)})$ and $\theta_2^{(0)} = \pi - \arctan(\sqrt{c^2/(n^2 t_e t_s)})$ are the extreme value points of $\varepsilon(\theta)$, while the extreme values of $\varepsilon(\theta)$ are $B$ and $C$. Table 4 shows the monotonicity of $\varepsilon(\theta)$.

If $(\Delta t N_1/(2\pi) - N_1 \arctan(\Delta t/(2\sqrt{t_s t_e}))/(2\pi)) \ge 1$, then a contact ratio for a SLGM is more than 1, no matter what values other parameters are taken as.

Table 3. The monotonicity of $\varepsilon(t_s)$.

| | $c\cos\theta < 0$ | | $c\cos\theta > 0$ | |
|---|---|---|---|---|
| $t_s$ | $(-\infty, -\Delta t]$ | $(0, +\infty)$ | $(-\infty, -\Delta t]$ | $(0, +\infty)$ |
| Monotonicity of $\varepsilon(t_s)$ | $\uparrow$ | $\downarrow$ | $\downarrow$ | $\uparrow$ |
| Range value of $\varepsilon(t_s)$ | $(A, E)$ | $[A, E]$ | $(E, A)$ | $(E, A)$ |
| Minimum value of $\varepsilon(t_s)$ | $A$ | | $E$ | |

Table 4. The monotonicity of $\varepsilon(\theta)$.

| | $c \le 0$ | | | $c > 0$ | | |
|---|---|---|---|---|---|---|
| $\theta$ | $(0, \theta_1^{(0)})$ | $[\theta_1^{(0)}, \theta_2^{(0)}]$ | $(\theta_2^{(0)}, \pi)$ | $(0, \theta_1^{(0)})$ | $[\theta_1^{(0)}, \theta_2^{(0)}]$ | $(\theta_2^{(0)}, \pi)$ |
| Monotonicity of $\varepsilon(\theta)$ | $\uparrow$ | $\downarrow$ | $\uparrow$ | $\downarrow$ | $\uparrow$ | $\downarrow$ |
| Range value of $\varepsilon(\theta)$ | $(A, C)$ | $[B, C]$ | $(B, A)$ | $(B, A)$ | $[B, C]$ | $(A, C)$ |
| Minimum value of $\varepsilon(\theta)$ | | $B$ | | | $B$ | |

### 3.6. The Influence of Parameter $\Delta t$ on a Contact Ratio for a SLGM

According to Eq. (10), $\varepsilon$ for a SLGM can be denoted as $\varepsilon(\Delta t)$ by using parameter $\Delta t$ as a variable. Because $\Delta t > 0$, then the point at $\Delta t' = -(c^2\cos^2\theta + n^2 t_s^2 \sin^2\theta)/(n^2 t_s \sin^2\theta)$ is a break point of $\varepsilon(\Delta t)$. $\varepsilon(\Delta t)$ is monotone increasing , if $\Delta t \neq \Delta t'$.

### 3.7. The Influence of Parameter $N_1$ on a Contact Ratio for a SLGM

According to Eq. (10), $\varepsilon$ can be denoted as $\varepsilon(N_1)$ by using parameter $N_1$ as a variable. $\varepsilon(N_1)$ is proportional to the parameter $N_1$, which means the greater the line tooth number of the driving line gear is, the greater a contact ratio for a SLGM is.

## 4. CALCULATION EXAMPLES

According to Section 3, if $(a\cos\theta + (n\pi - b)\sin\theta) = 0$ and $t_s t_e \ge 0$, then the following two points can be concluded. 1) If $c\cos\theta < 0$, no matter what values of parameters $c$, $\theta$, $t_s$, $n$ are taken as, when $(\Delta t N_1/(2\pi)) \ge 1$, then a contact ratio for a SLGM is greater than 1. 2) If $c\cos\theta > 0$, because $c\cos\theta\sin\theta\Delta t n \ge 0$, $c^2\cos^2\theta + n^2 t_s t_e \sin^2\theta \ge 0$and$((c^2\cos^2\theta + n^2 t_s t_e \sin^2\theta) - (c\cos\theta\sin\theta\Delta t n)) \ge 0$, then

$$\frac{c\cos\theta\sin\theta\Delta t n}{c^2\cos^2\theta + n^2 t_s t_e \sin^2\theta} \le 1$$

namely,

$$\left(\frac{\Delta t N_1}{2\pi} - \frac{N_1}{8}\right) \le \varepsilon \le \frac{\Delta t N_1}{2\pi}$$

That means, if

$$\left(\frac{\Delta t N_1}{2\pi} - \frac{N_1}{8}\right) \ge 1$$

then a contact ratio for a SLGM is greater than 1 no matter what the values of parameters $c$, $\theta$, $t_s$, $n$ are.

In this section, eight pairs of SLGMs are designed by using the parameters in Table 5, and the models of these SLGMs are shown in Fig. 2 by using Pro/E software.

In Table 5, comparing (a) with (b), whose parameters are the same except $N_1$, it can be concluded that a contact ratio for a SLGM is proportional to parameter $N_1$. Comparing (a) with (d) or (c) with (d), which have different parameters of $c$, $\theta$ and $t_s$, it can be conclude that if $(\Delta t N_1/(2\pi)) \ge 1$, then $\varepsilon \ge 1$.

Table 5. Parameters of SLGMs.

| | $\theta$ (°) | $c$ (mm) | $m$ (mm) | $i_{12}$ | $n$ (mm) | $a$ (mm) | $b$ (mm) | $t_s$ | $\Delta t$ | $N_1$ | $\varepsilon$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (a) | 45 | $-17$ | 5 | 3 | 6 | 16.9647 | 35.8142 | $1.57\pi$ | $0.5\pi$ | 4 | 1.0704 |
| (b) | | $-17$ | 5 | 3 | 6 | 16.9647 | 35.8142 | $1.57\pi$ | $0.5\pi$ | 6 | 1.6055 |
| (c) | 155 | 12 | 5 | 3 | 6 | 21.0181 | $-26.224$ | $-2.8\pi$ | $0.5\pi$ | 4 | 1.0522 |
| (d) | | 12 | 5 | 5 | 6 | 36.2373 | $-58.8615$ | $-5\pi$ | $2\pi$ | 1 | 1.0255 |
| (e) | 20 | 8 | 5 | 5 | 6 | 23.4283 | 83.2182 | $3.775\pi$ | $4.525\pi$ | 4 | 1.4698 |
| (f) | | 15 | 5 | 3 | 4 | 4.2093 | 24.1314 | $\pi$ | $1.75\pi$ | 5 | 1.3764 |
| (g) | 155 | $-9$ | 5 | 3 | 6 | 16.3288 | $-16.1677$ | $-2.15\pi$ | $0.5\pi$ | 8 | 1.8587 |
| (h) | | $-9$ | 5 | 3 | 6 | 17.2394 | $-18.1204$ | $-2.275\pi$ | $0.75\pi$ | 4 | 1.3928 |

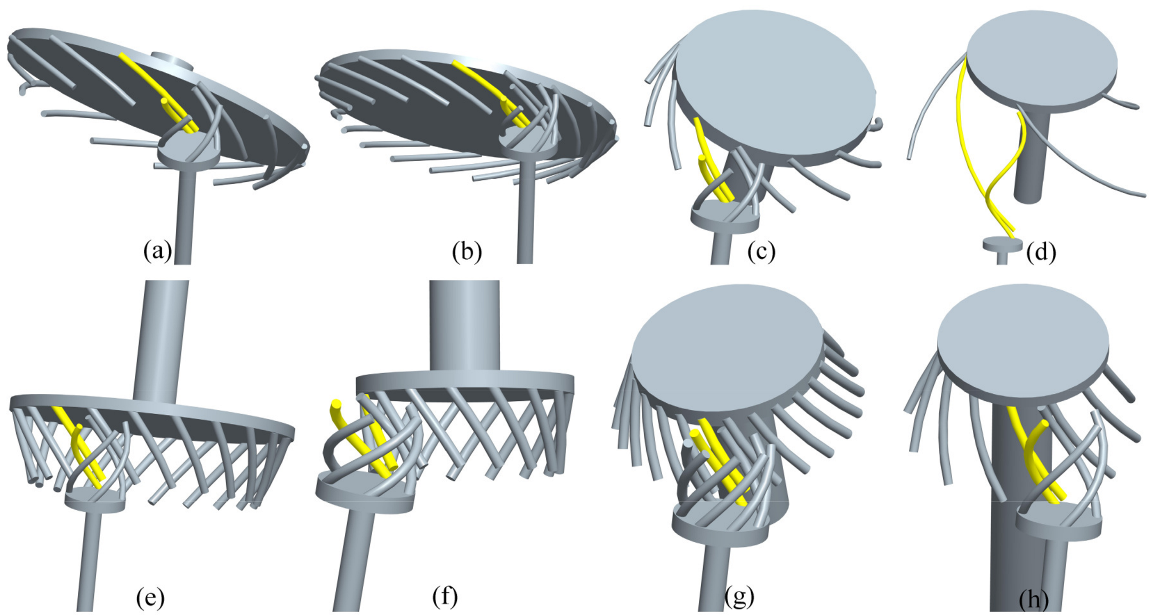


Fig. 2. Models of SLGMs.

Comparing (e), (f), (g) and (h), a conclusion can be obtained that if $c\cos\theta > 0$ and $(\Delta t N_1/(2\pi) - N_1/8) \geq 1$ then $\varepsilon \geq 1$.

When a contact ratio for a SLGM is greater than 1, the least line tooth number of the driving line gear is 1, which is shown in Fig. 2(d).

## 5. INTERFERENCE-PROOF CONDITIONS FOR A SLGM

According to Sections 3 and 4, a SLGM with a high contact ratio can be designed by increasing a line tooth number of the driving line gear. However, to interference-proof between the meshing line tooth and other non-meshing line tooth, a line tooth number of the driving line gear $N_1$ cannot be too big, due to an extreme small space between two adjacent line teeth, as shown in Fig. 3.

Two types of interferences for a SLGM are classified. One is an interference between a driven meshing line tooth and a driving non-meshing line tooth, while the other is an interference between a driving meshing line tooth and a driven non-meshing line tooth. These two types of interferences will be discussed as below and their interference-proof conditions are given, respectively.

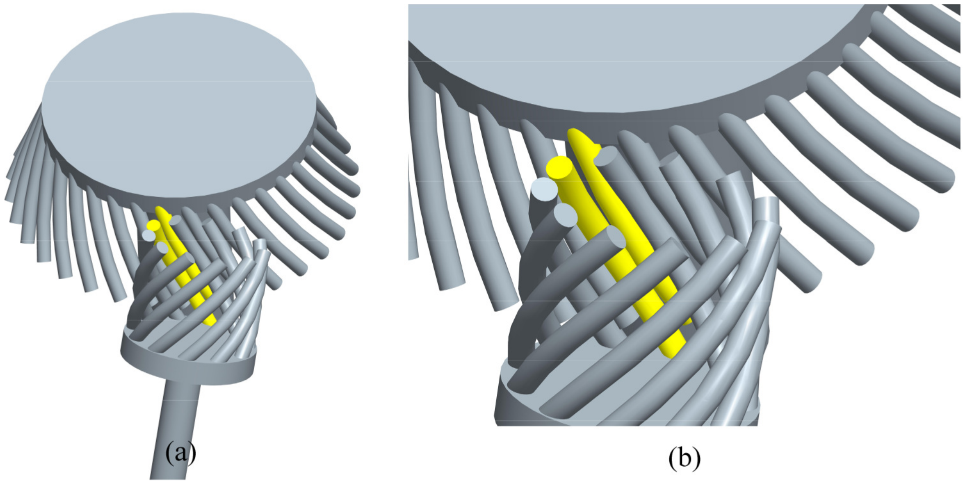


Fig. 3. An interference of a SLGM due to an extreme small space between two adjacent line teeth.

### 5.1. An Interference between a Driven Meshing Line Tooth and a Driving Non-Meshing Line Tooth

An interference between the driven meshing line tooth and the driving non-meshing line tooth is mainly caused by an extreme small space between two adjacent driving line teeth. This type of interference for the arbitrary intersecting LG mechanism has been discussed in reference [10]. Similarly, Eq. (11) can be used as an interference-proof condition for a SLGM, with consideration of the different shapes of the driving contact curve and the positions of the meshing point for a pair of arbitrary intersecting LG and a SLGM.

$$m\psi_1 \sin\lambda_1 > 2k_1(r_1 + r_2) \tag{11}$$

where $\psi_1$ is a driving line tooth included angle, namely, $\psi_1 = 2\pi/N_1$; $\lambda_1$ is a helix angle of the driving line tooth, namely, $\lambda_1 = \tan(n/m)$; $r_1$ and $r_2$ are the radii of the driving and the driven line teeth, respectively; $k_1$ is a coefficient which is decided by a curving degree of the driven line tooth, generally in a value range of $1 \le k_1 \le 1.5$. According to Eq. (11), a maximum line tooth number of the driving line gear can be decided by Eq. (12) as follows:

$$N_1 < (\pi mn/(k_1(r_1 + r_2)\sqrt{m^2 + n^2})) \tag{12}$$

### 5.2. An Interference between a Driving Meshing Line Tooth and a Driven Non-Meshing Line Tooth

An interference between a driving meshing line tooth and a driven non-meshing line tooth is mainly caused by an extreme space between two adjacent driven line teeth. This type of interference for an arbitrary intersecting LG mechanism has been discussed in [10]. Similarly, Eq. (13) can be used as an interference-proof condition for a SLGM, with consideration of the different shapes of the driving contact curve and the positions of the meshing point for a pair of arbitrary intersecting LG and a SLGM.

$$P_r\psi_2 \sin\lambda_{2t} > 2k_2\,(r_1 + r_2) \tag{13}$$

where $\psi_2$ is a driven line tooth included angle, $\psi_2 = 2\pi/N_2 = 2\pi/(i_{21}N_1)$; $\lambda_{2t}$ is a helix angle of driven line tooth; $k_2$ is a coefficient which is decided by a curving degree of a driving line tooth, generally, $1 \le k_1 \le 1.5$.

According to Eq. (9), the meshing radius of the driven line gear $P_r$ can be obtained as

$$P_r = \sqrt{[(nt\sin\theta)^2 + c^2](\sin(\varphi_1 - t)m + c)^2/c^2} \tag{14}$$

Moreover, a helix angle of the driven line tooth $\lambda_{2t}$ is defined as

$$\lambda_{2t} = \arcsin\left(\frac{\left|z_M^{(2)'}\right|}{\sqrt{(x_M^{(2)'})\wedge 2 + (y_M^{(2)'})\wedge 2 + (z_M^{(2)'})\wedge 2}}\right) \tag{15}$$

When $0 < \theta < 90^\circ$, according to Eqs. (5) and (15), it can be obtained as

$$\sin\lambda_{2t} = \frac{|(-\sin(\varphi_1 - t)m(\varphi_1' - 1)\sin\theta - n\cos\theta)|}{sqrt((\frac{[\sin(\varphi_1 - t)m + c]}{\sin(\varphi_1 - t)}\varphi_3'\cos\theta + m(\varphi_1' - 1))^2 + ([\sin(\varphi_1 - t)m + c]\,\varphi_3'\sin\theta - n)^2)} \tag{16}$$

When $90^\circ < \theta < 180^\circ$, according to Eqs. (6) and (15), it can be obtained as

$$\sin\lambda_{2t} = \frac{|(-\sin(\varphi_1 - t)m(\varphi_1' - 1)\sin\theta - n\cos\theta)|}{sqrt((\frac{[\sin(\varphi_1 - t)m + c]}{\sin(\varphi_1 - t)}\varphi_3'\cos\theta - m(\varphi_1' - 1))^2 + ([\sin(\varphi_1 - t)m + c]\,\varphi_3'\sin\theta + n)^2)} \tag{17}$$

According to Eq. (13), a maximum line tooth number of the driving line gear can be decided by Eq. (18) as follows:

$$N_1 < (\pi P_r \sin(\lambda_{2t})/(k_2 i_{21}(r_1 + r_2))) \tag{18}$$

It is worth mentioning that, if $0 < \theta < 90^\circ$, at point $t = t_s$, a value of $P_{rs}\psi_2\sin\lambda_{2t_s}$ is minimum, namely, a maximum line tooth number of the driving line gear must yield $N_1 < (\pi P_{rs}\sin(\lambda_{2t_s})/(k_2 i_{21}(r_1 + r_2)))$; and if $90 < \theta < 180^\circ$, a minimum value of $P_{rs}\psi_2\sin\lambda_{2t_s}$ is set at the point $t = t_e$, namely, a maximum line tooth number of the driving line gear must yield $N_1 < (\pi P_{re}\sin(\lambda_{2t_e})/(k_2 i_{21}(r_1 + r_2)))$.

Those parameters in Table 5(g) are taken as an example. If $k_1 = 1$ and $k_2 = 1$, according to Eq. (12), it can be obtained as $N_1 < (\pi mn/(k_1(r_1 + r_2)\sqrt{m^2 + n^2}) = 10.06)$; meanwhile, according to Eq. (18), it can be obtained as $N_1 < (\pi P_{re}\sin(\lambda_{2t_e})/(k_2 i_{21}(r_1 + r_2)) = 8.4951)$. Therefore, a maximum line tooth number of the driving line gear is recommended not more than 8. A line tooth number of the driving line gear in Fig. 2(g) is 8, and the SLGM works well; while a line tooth number of the driving line gear in Fig. 3(a) is 11, the SLGM does not work well because of that an interference occurs between line teeth.

## 6. CONCLUSIONS

A formula of a contact ratio for a SLGM is deduced in this study. By analyzing six influencing parameters on a contact ratio for a SLGM, the following conclusions can be drawn:

1. A contact ratio for a SLGM with two vertical skewed axes could be designed flexibly by changing a line tooth number of the driving line gear or a variation range of parameter $t$ for meshing points.

2. A value range of a contact ratio for a SLGM with two non-vertical skewed axes is obtained as $[\Delta t N_1/(2\pi) - N_1/4, \Delta t N_1/(2\pi) + N_1/4]$, and parameters $N_1$ and $\Delta t$ have significant impact on it. A contact ratio increases with parameter $\Delta t$ and is proportional to parameter $N_1$.

3. When $(a\cos\theta + (n\pi - b)\sin\theta) = 0$ and $t_s t_e \geq 0$, if $c\cos\theta < 0$, no matter what values of parameters $c$, $\theta$, $t_s$ and $n$ are taken as, it ensures that $(\Delta t N_1/(2\pi)) \geq 1$, then a contact ratio of the designed SLGM with two non-vertical skewed axes is greater than 1; if $c\cos\theta > 0$, no matter what values of parameters $c$, $\theta$, $t_s$ and $n$ are taken as, it ensures that $(\Delta t N_1/(2\pi) - N_1/8) \geq 1$, then a contact ratio of the designed SLGM with two non-vertical skewed axes is greater than 1.

4. If a contact ratio is greater than 1, then a least line tooth number of the driving line gear is 1, therefore, a theoretical transmission ratio of a SLGM will be greatly enlarged.

5. By analyzing two types of interferences between the line teeth, the interference-proof conditions for a SLGM have been proposed definitely and the formulas of a maximum line tooth number of the driving line gear have been deduced for ease of use in practical.


## ACKNOWLEDGEMENTS

The authors gratefully acknowledge the support from National Natural Science Foundation of China (No. 51175180). It is our honor to thank the reviewers and editors for their valuable comments.


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