A GENERAL CAD/CAM MODEL FOR APT-LIKE ROTARY MILLING CUTTERS

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ABSTRACT

This paper presents a general mathematical model for NC-machining an APT-LIKE rotating cutter. The design model of cutting-edge and helical groove is also developed by using the principles of differential geometry and kinematics. Based on the envelope condition, approaches for solving the inverse problems that are related to the manufacturing models are also presented. The velocities of the cutter in the radial and axial directions are then determined based on the gouging avoidance of grooves. The results of the numerical experiment indicate that the proposed systematic design method by using 2-axis NC machine setup is feasible and reliable.

Keywords: helical groove; rotating cutter; mathematical model; grinding wheel; inverse problem.

UN MODÈLE GÉNÉRAL CAD/CAM DU GENRE APT POUR UN OUTIL DE FRAISAGE ROTATIF

RÉSUMÉ

Cet article présente un modèle mathématique général du genre APT d'un outil de fraisage rotatif pour un appareil d'usinage à commande numérique (NC-machining). Un modèle de bord tranchant et de rainure hélicoïdale est aussi développé en utilisant le principe de la géométrie différentielle et la cinématique. En se basant sur la condition de l'enveloppe, les approches pour résoudre les problèmes inverses qui sont reliés aux modèles d'usine sont aussi présentées. Les vitesses de l'outil de fraisage dans les directions radiale et axiale sont alors déterminées en se basant sur l'évitement de gougeage des rainures. Les résultats des expériences numériques indiquent que la méthode de conception systématique proposée par l'utilisation du réglage de la machine 2-axes NC est faisable et fiable.

Mots-clés : rainure hélicoïdale; outil de fraisage rotatif; modèle mathématique; meule; problème inverse.

NOMENCLATURE

a	the distance between the origins of frame $\bar{\sigma}$ and σ_1
b	screw lead
$f(u)$	radius function of section of cutter
F_g	radial feeding
l	length of land
L	lead of helical cutting edge
p	pitch of helical cutting edge
z_h	parameter of conic surface
dr	tangent vector of cutting edge
δr	tangent vector of the generatrix
r^{**}	the locus of contact points
r_c	profile curve of the grinding wheel
$\hat{r}$	profile equation in the coordinate system σ_1
r^*	the surface equation of groove
r_1	equation of the sphere part of revolving cutter
r_2	equation of the conic surface of revolving cutter
$\bar{r}$	radius of inner arbor
$\bar{r}_1$	radius of hook part of groove for curling chip
$\bar{r}_2$	radius of segment of groove for chip flow
R	radius of spherical head
$\bar{R}$	the maximum sectional radius of cutter
S	equation of revolving surface
$S_{\bar{\varphi}}$	grinding wheel profile equation in frame σ_1
u	parameter of revolving surface
$u(\xi)$	unit step function
α_e	relief angle
α_E	secondary relief angle
γ	radial rake angle
ϕ_1 ,θ_1	parameters of revolution surface S_1
ϕ_2	parameter of revolution surface S_2
ψ	helix angle of the cutting edge
$\bar{\sigma}$	the stationary machine frame
σ_1	the coordinate system connects to grinding wheel
$\bar{\theta}$	the parameter of the spiral surface
$\bar{\mu}$	magnitude factor of common normal at the contact point between grinding wheel and groove
υ	parameter of groove surface in frame σ_1
ω	angular velocity of blank

1. INTRODUCTION

Rotary millingcutter is the most important tool in the manufacturing industry. The performance and quality of the cutters have a direct impact on cutting efficiency, precision and quality of products.Therefore, continuous innovation of cutting tools is the driving power of technological progress. With the increase of tasks and the demands for higher precision in machining the sculptured surfaces, researches for longer life of the tool and its higher quality have attracted more attention.

The aspects of design and NC grinding of helical groove have been recently brought to light since the helical groove is beneficial to tool's life and machining properties. Ehmann [7] proposed an analytical solution to determine the wheel profile for the grinding of helical drill grooves, and explained the undercutting phenomenon. Tang et al. [14] investigated the models for 2-axis NC machining of cutter with constant angle. Ehmann [7] also developed a generalized groove machining model for the analytical resolution of the inverse problem using CAD.

Most of the previous researches concentrated on this kind of cutter with constant helical angle. Liu and Liu [9] presented the mathematical model for the helical grooves of a cone-type ball-end cutter. Chyan and Ehmann [3] developed an engagement model to characterize both cylindrical and tapered-web helical groove machining. Wu and Chen [10, 11] investigated the similarity and the difference between the manufacturing models for rotating cutter corresponding to different definitions of the helical angle. Wu and Chen also proposed the manufacturing models of ball-end type rotating cutter with a constant helical angle [26]. In the existing literature, the thorough process for design, manufacturing and post-process of the special revolving cutters is not much discussed. Therefore, in this paper we developed a systematic approach for CAD/CAM of the rotating cutter. In addition, the integrated mathematical models are presented.

2. GEOMETRICAL MODELS

For the purpose of extending to other similar researches conveniently, the general mathematical models of helical cutting edge with constant helical angle will be introduced in this section by utilizing the principles of differential geometry.

2.1. General Mathematical Model of Helical Cutting Edge

In working condition, the outer profile of the revolving cutter is obviously a revolving surface and can be expressed as

$$S = \{f(u)\cos\phi, f(u)\sin\phi, g(u)\} \tag{1}$$

Utilizing the principles of differential geometry, the mathematical models of helical cutting edge can be deduced as

$$\phi = \tan\psi \int_{u_0}^{u} \frac{\sqrt{f'^2 + g'^2}}{f} du + \phi_0 \tag{2}$$

where ϕ_0 is the initial value of parameter ϕ.

It should be noted that the definition for helical angle ψ of the cutting edge refers to the angle between the helical curve and the generator curve of the revolving surface. Thus, the equation of cutting edge with a helical angle ψ may be obtained by substituting Eq. (2) into Eq. (1).

2.2. General Mathematical Models of Cutting Edge

Cutters, in common use, comprises three major portions, including the truncated cone shank, the spherical head and the revolving surface defined by different generatrix curves according to different requirements. For the purpose of formulating each commonly used cutter conveniently, the X–Z sectional profile of the general revolving cutter has been defined as shown in Fig. 2. The sectional profile can be defined by six parameters $(\alpha, r, R, h, H, \beta)$. Cone type torus cutter can be defined as $\alpha = 90$, $h = 0$. Cylindrical type torus cutter should be obtained while$\alpha = 90$, $h = 0$, $\beta = 0$. Flat end milling cutter can be defined as $\alpha = 90$, $h = 0$, $r = 0$ and ball-end type is defined as $\alpha = 90$, $h = 0$, $R = 0$.

The following are the outer profile equations of the revolving cutter (Fig. 1):

S1: Cone type

$$r_1 = \{z\tan\alpha\cos\phi, z\tan\alpha\sin\phi, z\}, \quad z \in [h_1 \sim h] \tag{3}$$

S2: Circular arc type

$$r_2 = \{(R + r\cos\theta)\cos\phi, (R + r\cos\theta)\sin\phi, h + r\sin\alpha + r\sin\theta\}, \quad \theta \in [-\alpha \sim -\beta] \tag{4}$$

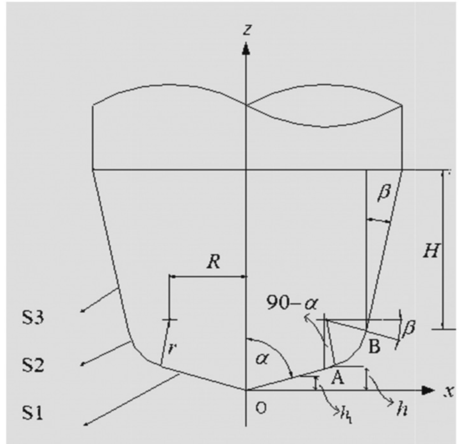

Fig. 1. Integrated revolving cutter.

S3: Cone type

$$r_3 = \begin{Bmatrix} (R + r\cos\beta + \bar{z}\tan\beta)\cos\phi, \\ (R + r\cos\beta + \bar{z}\tan\beta)\sin\phi, \\ h + r\sin\alpha - r\sin\beta + \bar{z} \end{Bmatrix}, \quad \bar{z} \in [0 \sim H] \tag{5}$$

The general revolving surface can be defined as

$$r = \{f(u)\cos\phi, f(u)\sin\phi, g(u)\} \tag{6}$$

Thus, the corresponding equations are

$$S1: \quad f(z) = z\tan\alpha, \quad g(z) = z \tag{7}$$

$$S2: \quad f(\theta) = R + r\cos\theta, \quad g(\theta) = h + r\cos\alpha + r\sin\theta \tag{8}$$

$$S3: \quad f(\bar{z}) = R + r\cos\beta + \bar{z}\tan\beta, \quad g(\theta) = h + r\sin\alpha - r\sin\beta + \bar{z} \tag{9}$$

Following now are the deduced equations of the ideal helical cutting edge. The coefficients of the first fundamental form can be obtained as

$$E = r_{\mathrm{u}}^2 = f'(u)^2 + g'(u)^2, F = r_{\mathrm{u}} \cdot r_\phi = 0, G = r_\phi^2 = f(u)^2$$

$$\begin{cases} r_u = \{f'(u)\cos\phi, f'(u)\sin\phi, g'(u)\} \\ r_\phi = \{f(u)\sin\phi, f(u)\cos\phi, 0\} \end{cases} \tag{10}$$

From the definition of the angle between two curves on the surface, let the helical angle of the cutting edge be ψ, we can obtain the following equations:

$$\phi = \tan\psi \int_{u_0}^{u} \frac{\sqrt{f'^2 + g'^2}}{f} du + \phi_0 \tag{11}$$

where ϕ_0 is the initial value of parameter ϕ.

From the S1 conical revolving surface equation, we obtain

$$\begin{cases} r_z = \{\tan\alpha\cos\phi, \tan\alpha\sin\phi, 1\} \\ r_\phi = \{-z\tan\alpha\sin\phi, z\tan\alpha\cos\phi, 0\} \end{cases} \tag{12}$$

Substitute $f(z) = z\tan\alpha$, $f'(z) = \tan\alpha$, $g'(z) = z\tan\alpha$into Eq. (11), we obtain

$$\phi = \tan\psi \int_{h_1}^{z} \sqrt{\frac{\tan^2\alpha+1}{z^2\tan^2\alpha}}dz + \phi_1 = \frac{\tan\psi}{\sin\alpha}(\ln|z| - \ln|h_1|) + \phi_1 \tag{13}$$

As $z = h_1$, we substitute $\phi_1 = 0$ into above equation:

$$\phi = \frac{\tan\psi}{\sin\alpha}(\ln|z| - \ln|h_1|) \tag{14}$$

From the S2 circular arc revolving surface equation, we obtain

$$\begin{cases} r_\theta = \{-r\sin\theta\cos\phi, -r\sin\theta\sin\phi, r\cos\theta\} \\ r_\phi = \{-(R+r\cos\theta)\sin\phi, (R+r\cos\theta)\cos\phi, 0\} \end{cases} \tag{15}$$

Thus, we substitute $f(\theta) = (R+r\cos\theta)$, $f'(\theta) = -r\sin\theta$, $g'(\theta) = r\cos\theta$ into Eq. (11) and we obtain

$$\phi = \cos\bar{\alpha}^*\tan\psi \int_{-\alpha}^{\theta} \frac{1}{1+\cos\bar{\alpha}^*\cos\theta}d\theta + \phi_2, \tag{16}$$

where $\cos\bar{\alpha}^* = r/R$.

Considering the continuity of helical cutting edge between these two connected revolving surfaces, the boundary value of ϕ_2 corresponding to surface S2 ($\theta = -\alpha$) should be equal to that of ϕ_1 when z = H. We obtain

$$\phi_2 = \frac{\tan\psi}{\sin\alpha}(\ln|H| - \ln|h_1|) \tag{17}$$

The cutting edge of revolving surface S2 can be deduced. Similarly, from S3 we deduce

$$\begin{cases} r_{\bar{z}} = \{\tan\beta\cos\phi, \tan\beta\sin\phi, 1\} \\ r_\varphi = \{-(R+r\cos\beta+\bar{z}\tan\beta)\sin\phi, (R+r\cos\beta+\bar{z}\tan\beta)\cos\phi, 0\} \end{cases} \tag{18}$$

We obtain $f'(\bar{z}) = \tan\beta$,$g'(u) = 1$

$$\phi = \sec\beta\tan\psi \int_{0}^{\bar{z}} \frac{1}{R+r\cos\beta+\bar{z}\cos\beta}d\theta + \phi_3 \tag{19}$$

Considering the continuity of helical cutting edge between these two connected revolving surfaces, the boundary value of ϕ_2 corresponding to surface S2 ($\theta = -\beta$) should be equal to that of ϕ_3 when $\bar{z} = 0$. We obtain

$$\phi_3 = \cos\bar{\alpha}^*\tan\psi \int_{-\alpha}^{-\beta} \frac{1}{1+\cos\bar{\alpha}^*\cos\theta}d\theta + \frac{\tan\psi}{\sin\alpha}(\ln|H| - \ln|h_1|) \tag{20}$$

The cutting edge of revolving surface S3 can be deduced.

3. INVERSE PROBLEM OF GROOVE MACHINING

The objective of this inverse problem is to determine the tool profile for a desired helical groove. When the sizes of transverse sections of cutter are not uniform, the solutions of the inverse problem will not the same, i.e. in order to grind the groove precisely to meet the demand groove cross-section, the grinding wheel for each different cross-section has different profile and dimension. However, it is not only impossible but also needless. Besides, the most important effects for a groove are suitable rake angle for cutting, enough strength, chip curling, chip capacity and chip ejection. A groove, which meets the basic demanded effects, is thought to be suitable. Therefore, to complete grooving, we just need to adopt the grinding wheel with radius $\bar{R}$ determined by the desired groove profile and the inverse problem solving. The desired profile will be defined in the following subsection.

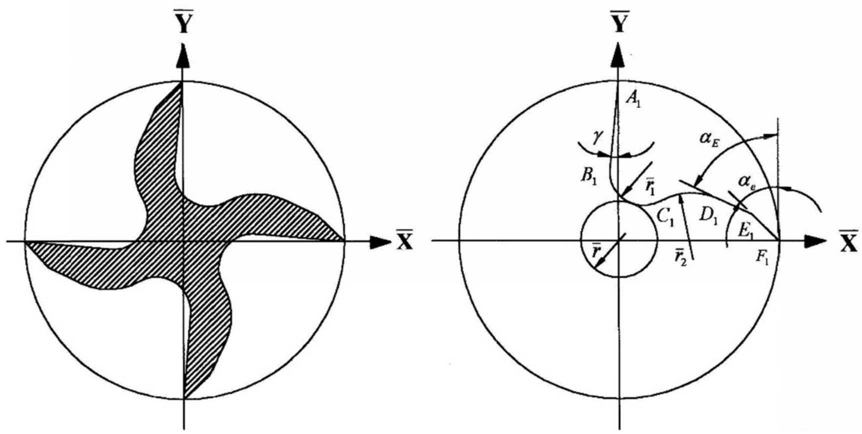

Fig. 2. Cross-section profile of groove.

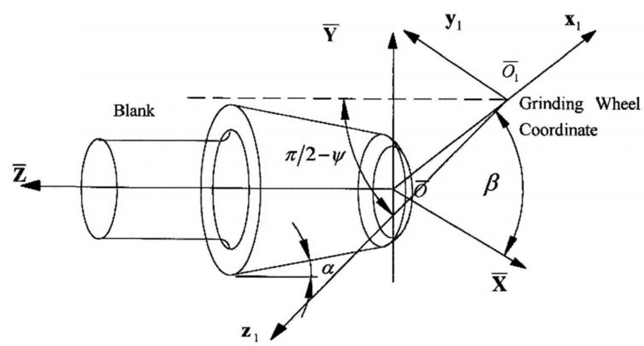

Fig. 3. Tool coordinate frame assignment.

3.1. Geometrical Model of Groove Profile

Based on the basic demands and the force analysis in [1], the desired sectional profile of the helical groove has been modeled in [12, 13]. As shown in Fig. 2, the maximal outer radius of the section is denoted as $\bar{R}$ and the corresponding thickness of web is $\bar{r}$. In this paper, a four-groove cutter is considered.

3.2. Analytical Solution for the Resolution Profile of Grinding Wheel

Two right-handed coordinate frames are used to mathematically describe the relationship between the grinding wheel and helical groove as indicated in Fig. 3. The initial tool coordinate system is defined by $Rot\ (Z\beta)$, $Trans\ (a,0,0)$ and $Rot\ (x_1,\pi/2-\psi)$ in the stationary machine frame.

Thus, in general, the equation of helical groove surface for cylindrical cutter is:

$$\begin{aligned} S^* &= \left\{x(\xi)\cos\bar{\theta} - y(\xi)\sin\bar{\theta}\ ,\ x(\xi)\sin\bar{\theta} + y(\xi)\cos\bar{\theta}, \bar{R}\bar{\theta}\cot\psi\right\} \\ &= \{x^*, y^*, z^*\} \end{aligned} \tag{21}$$

where $\{x(\xi), y(\xi)\}$ is the sectional profile as defined in Fig. 2 and ξ, $\bar{\theta}$ represent the curvilinear coordinates of the surface.

In approaching the solution to the inverse problem, we shall utilize a fundamental relationship that characterizes the groove machining equation. Referring to Fig. 3, the equation of line $\bar{O}_1z_1$ can be expressed in the coordinate system $\bar{\sigma}$ as

$$r_{Z_1} = \{a\cos\beta, a\sin\beta, 0\} + \bar{\lambda}\,\{\sin\beta\cos\psi\ ,-\ \cos\beta\cos\psi, \sin\psi\} \tag{22}$$

The normal vector of an arbitrary point on the helical groove surface in the stationary machine frame is in the form

$$\hat{r} = S^* + \bar{\mu} S^*_{\xi} \times S^*_{\bar{\theta}} = \{x^* + \bar{\mu} N_x, y^* + \bar{\mu} N_y, z^* + \bar{\mu} N_z\} \tag{23}$$

where N_x, N_y, N_z are the fractions of the normal vector, and $\bar{\mu}$ is the length parameter of the normal vector.

According to the fact that the common normal vector of any point on the instant contact curve between the profile surface of grinding wheel and the spiral surface must intersect the axis $\bar{O}_1 z_1$ of the grinding wheel, it follows that

$$\begin{cases} a\cos\beta + \bar{\lambda}\sin\beta\cos\psi = x^* + \bar{\mu} N_x \\ a\sin\beta + \bar{\lambda}\cos\beta\cos\psi = y^* + \bar{\mu} N_y \\ \bar{\lambda}\sin\psi = z^* + \bar{\mu} N_z \end{cases} \tag{24}$$

In this way, the engagement condition, which the points on the surface of Eq. (21) are also on the profile surface of the grinding wheel, can be defined.

Thus, by eliminating $\bar{\lambda}$ and $\bar{\mu}$, the engagement relationship $F = f(\xi, \bar{\theta}) = 0$ for grinding wheel and helical groove is formed as

$$(aN_z + (z^* N_x - x^* N_z)\cos\beta + (N_y z^* - N_z y^*)\sin\beta) - (x^* N_y - N_x y^* - aN_y\cos\beta + aN_x\sin\beta)\tan\psi = 0 \tag{25}$$

By solving the engagement relationship (25), a nonlinear transcendental equation, in terms of $\bar{\theta}$ and substituting it into Eq. (21), the locus of the contact points is obtained and expressed as

$$r^{**} = \left\{x(\bar{\theta}), y(\bar{\theta}), z(\bar{\theta})\right\} \tag{26}$$

The contact curve of Eq. (26) is on the profile surface of grinding wheel. By sweeping the contact curve around the z_1-axis, the profile equation in the grinding wheel coordinate system $\sigma_1 = \left[\bar{O}_1\ ;\ x_1, y_1, z_1\right]$ can be expressed as

$$\begin{aligned} S_{\bar{\varphi}} &= \left\{x_{\bar{\varphi}}, y_{\bar{\varphi}}, z_{\bar{\varphi}}\right\} \\ &= \{x_1\cos\bar{\varphi} - y_1\sin\bar{\varphi}, x_1\sin\bar{\varphi} + y_1\cos\bar{\varphi}, z_1\} \end{aligned} \tag{27}$$

The generatrix curve of grinding wheel profile can be obtained by intersecting the profile surface equation (27) to the plane $y_1 = 0$. Thus, the generatrix curve can be expressed as

$$r_c = \{x_c, 0, z_c\} = \left\{\sqrt{x_1^2 + y_1^2}, 0, z_1\right\} \tag{28}$$

3.3. Analytical Resolution for Helical Groove

From the viewpoints of geometric model, the actually obtained groove surface is the enveloping surface of profile of grinding wheel at the motion relative to cutter. The equation of grinding wheel profile need to be expressed in the coordinate system $\bar{\sigma} = \left[\bar{O}; \bar{X}, \bar{Y}, Z\right]$ and the motion of grinding wheel should also be transmitted into the coordinate system $\bar{\sigma}$. By sweeping the axial cross-section of the required grinding wheel profile r_c around the z_1-axis, the profile equation can be expressed in the coordinate system $\sigma_1 = \left[\bar{O}_1; x_1, y_1, z_1\right]$ as follows:

$$\hat{r} = \{\hat{x}, \hat{y}, \hat{z}\} = \{x_c\cos\upsilon, x_c\sin\upsilon, z_c\} \tag{29}$$

In order to express the profile equation in $\bar{\sigma} = \left[\bar{O}\ ;\ \bar{X}, \bar{Y}, Z\right]$, we utilize the relative coordinate transformation. Assume that the obtained profile equation is $r^* = \{x^*, y^*, z^*\}$. Thus, the condition for the existence of intersecting surface envelope is given by

$$\left(r^*_{x_c}, r^*_{\upsilon}, r^*_u\right) = 0 \tag{30}$$

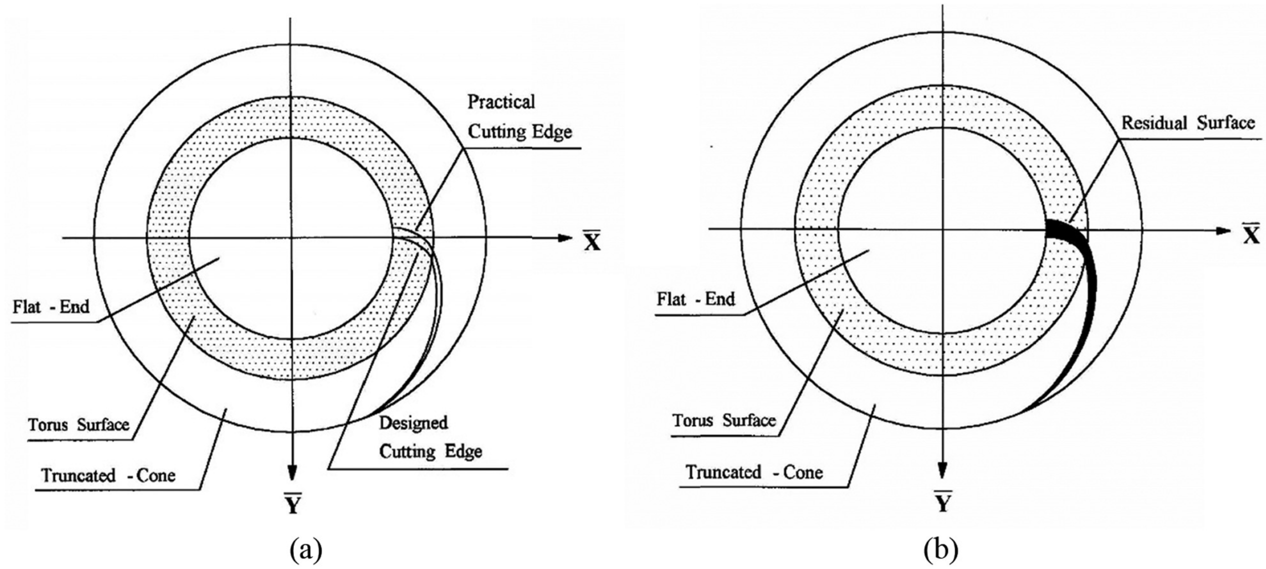

Fig. 4. Difference between proposed and practical cutting-edge and corresponding residual material.

The actual surface of the groove will be defined as

$$\begin{cases} r^* = \{x^*, y^*, z^*\} \\ \quad = \left\{ \begin{array}{l} x^* = x_c \cos \upsilon \cos\left(\frac{\pi}{4}+\phi\right) - x_c \sin \upsilon \sin \psi \sin\left(\frac{\pi}{4}+\phi\right) \\ \qquad + a \cos\left(\frac{\pi}{4}+\phi\right) + z_c \cos \psi \sin\left(\frac{\pi}{4}+\phi\right) + F_g \cos\left(\frac{\pi}{4}+\phi\right) \\ y^* = x_c \cos \upsilon \sin\left(\frac{\pi}{4}+\phi\right) + x_c \sin \upsilon \sin \psi \cos\left(\frac{\pi}{4}+\phi\right) \\ \qquad + a \sin\left(\frac{\pi}{4}+\phi\right) - z_c \cos \psi \cos\left(\frac{\pi}{4}+\phi\right) + F_g \sin\left(\frac{\pi}{4}+\phi\right) \\ z^* = x_c \sin \upsilon \cos \psi + z_c \sin \psi + g(u) \end{array} \right\} \\ \sin \psi g'(u) - \left(\cos \psi \sin \upsilon g'(u) + \cos \upsilon F_g'\right) z_c' \\ -\left(\left(a+F_g\right) z_c' \sin \upsilon \sin \psi + \cos \psi \left(a + x_c \cos \upsilon + F_g + z_c z_c' \cos \upsilon\right)\right) = 0 \end{cases} \tag{31}$$

4. SIMULATION

The following examples are presented to validate the manufacturing models presented in the previous section. The sectional profile of grinding wheel determined by solving the inverse problem is presented. A circular-arc type ball-end cutter is taken as an example for simulating the results related to the inverse problem.

Using the grinding wheel determined above, the cutting edge of an actually obtained groove is simulated in virtual manufacturing environment. In order to compare the difference between the designed cutting edge and the actually obtained cutting edge, these two cutting edge curves are simultaneously depicted in Fig. 4(a). It is clear that the actually obtained cutting edge deviates slightly from the proposed cutting edge. This leads to the result that residual material remained between the cutting edge and the flank of the neighbor groove as shown in Fig. 4(b). Besides, referring to the transverse sections of spherical head, the cutter land shortened gradually and then may vanish near the ball-nose part. It is not only because the deviation is slight but also the fundamental demands for helical groove such as suitable rake angle, chip curling and smooth chip ejection are satisfied. Thus, the authors propose that the result is acceptable.

5. CONCLUSIONS

For the conventional machining method, the major disadvantage is the improper groove profile. According to the discussion and machining simulation by Chyan and Ehmann [3] using the conventional machining method, the groove profile machined by a double conical wheel will result in over-cutting. Based on the basic demands for cutting properties and cutting force analysis in [1], the actually obtained groove profile is obviously not proper for cutting and chip curling. Such a problem can be handled by constructing the inverse problem. In this research, the objective of inverse problem is to determine the profile of a grinding wheel for a desired helical groove.

Besides, higher cost is another disadvantage. As mentioned in [5], the rake face of the cutter is machined first and then the other surfaces gradually. For the conventional methods, a three-axis or four-axis NC machine tool is adopted in most cases. In this way, one complete groove has to be machined through multiple processes, so that the manufacture cost of the cutter is correspondingly higher.

On the basis of the above discussions, this paper developed a systematic method for CAD/CAM (computer aided design and manufacture) of the rotating tool. A series of general models of a rotating cutter, including design of cutting edge and groove section, definition of the section and relative speeds of grinding wheel, are presented.The general models for the design of rotating cutter with constant helical angle are presented in this research. The cutting edge and sectional profile of grinding wheel are also developed. By means of analysis, the correlative formulae of the proposed general models for the design and NC machining of revolving cutter with constant helical angle have been proposed which are found valuable by similar researches.

The manufacturing cost is obviously reduced while the manufacturing models are established for machining the basic groove mainly in once by 2-axis NC machine. Based on the engagement relationship between groove surface and grinding wheel surface, general mathematical approaches for analytical resolutions of inverse problems have been developed by utilizing the principles of differential geometry. Detailed models and simulation results for this cutter are also provided.

The simulation results indicate that the practical cutting edge deviates from the proposed cutting edge. Because the deviation is slight, it is deemed to be harmless to the groove properties. The results of numerical experiments reveal that the proposed approaches are feasible and reliable.

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