OPTIMIZATION OF MODIFIED WATT'S LINKAGE FOR AN APPROXIMATELY LONG STRAIGHT PATH WITHIN LIMITED DIMENSION

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ABSTRACT

This paper presents an optimal method of aiming for an approximately straight path of the Modified Watt's Linkage (MWL) within limited dimension. A modification to the Watt's linkage and the corresponding condition are introduced, followed by the kinematic synthesis. The path generation based on the modification considering constraints from practical application is provided. Genetic algorithm is utilized to perform the constrained optimization. The centrosymmetric property of the MWL is considered in the synthesis process. Ideal parameters of the mechanism are achieved to demonstrate the effectiveness of the proposed method.

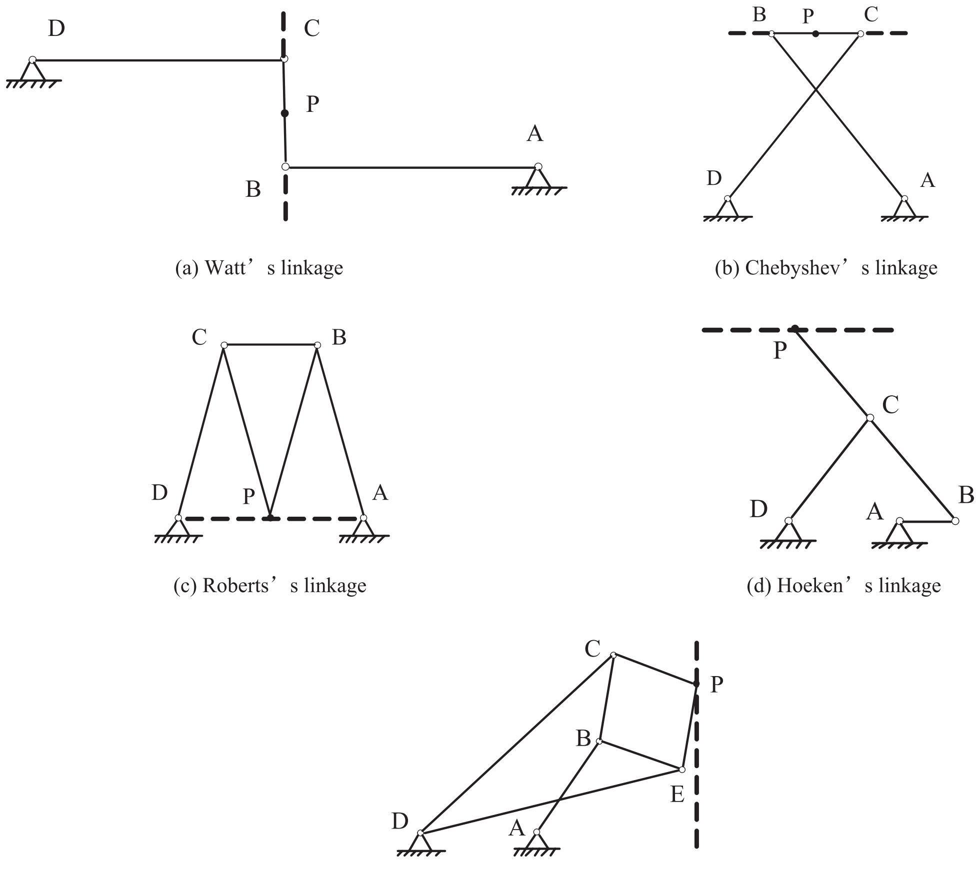
Keywords: straight coupler curve; modified Watt's linkage; optimization; genetic algorithm.

OPTIMISATION DU MÉCANISME DE WATT MODIFIÉ POUR UNE TRAJECTOIRE RECTILIGNE APPROXIMATIVE DANS DES DIMENSIONS LIMITÉES

RÉSUMÉ

Cet article présente une méthode optimale visant obtenir une trajectoire rectiligne approximative du mécanisme de Watt modifié dans des dimensions limitées. Une modification du mécanisme de Watt et les conditions correspondantes sont introduites, suivies de la synthèse cinématique. La génération de trajectoire basée sur la modification est donnée en considérant les contraintes de l'application pratique. Un algorithme génétique est utilisé pour produire l'optimisation sous contraintes. La propriété centrosymétrique du mécanisme de Watt modifié est considérée dans le procédé de synthèse. Les paramètres optimaux du mécanisme sont obtenus pour démontrer l'efficacité de la méthode proposée.

Mots-clés : courbe de coupleur droit; mécanisme de Watt modifié; optimisation; algorithme génétique.



(a) Watt's linkage (b) Chebyshev's linkage

(c) Roberts's linkage (d) Hoeken's linkage

(e) Peaucellier's linkage

Fig. 1. Straight-line linkages.

1. INTRODUCTION

Straight path is of great significance in such fields as robot design and industry. There are several mechanisms that can generate straight path, taking ball screw assembly, straight path linkage for example. In some cases, the easier designing linkage with approximately straight path can also satisfy the design demand, making the choice much wider. Compared with linkages, the ball screw assembly is of greater mass and dimension.

Linkage is a mechanism consisting of simple links and joints. It makes sense to get a straight path, or approximately straight path, with the simple four-bar linkage. Although other equally simple mechanisms like slider-cranks can also generate the same path, its main problem is that the prismatic joints are expensive to fabricate and maintain, which is similar to the ball screw assembly.

It was James Watt who invented the first straight-line linkage [1] shown in Fig. 1(a), the Watt's linkage, which led to a more convenient space saving design used in his rotary beam engine. There are additional linkages that can generate an approximately or exactly straight path. Chebyshev [2] and Richard Roberts [3] devised another two straight-line linkages shown in Figs. 1(b) and 1(c) respectively. The lengths of the links in these linkages must lie in strict proportions. One of the cognate linkages of Chebyshev's linkage is Hoeken's linkage [4] shown in Fig. 1(d). Unlike the previous three linkages, it is a Grashof crank-rocker [2]. The linkages mentioned above just generate an approximately straight path. Peaucellier [5] invented a seven-bar linkage shown in Fig. 1(e) which can generate exactly straight path.

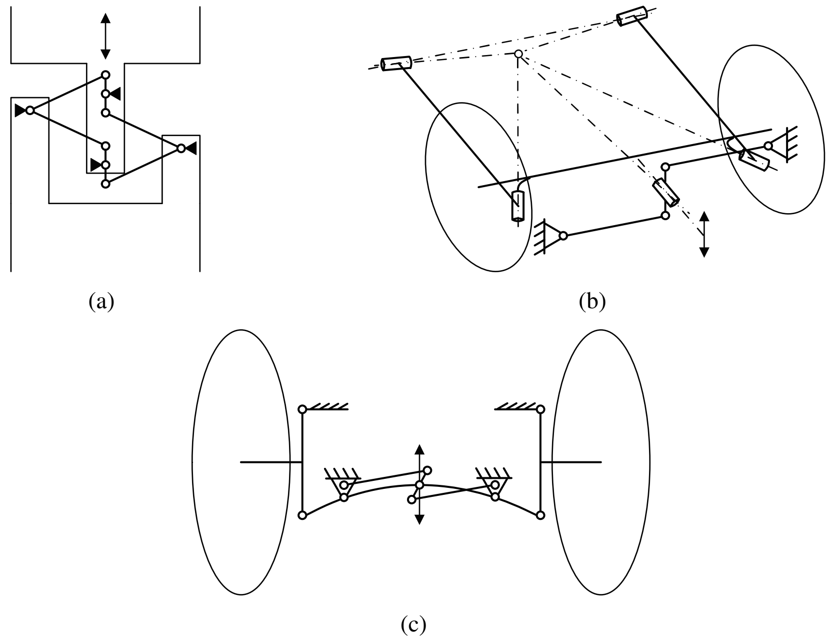


Fig. 2. Suspension systems.

Since the Watt's linkage was invented, it has been widely used in the engineering field. The most popular application is the suspension system for non-steered wheels on a motor vehicle body [6–10], in which the Watt's linkage is used to prevent axle movement in the longitudinal direction of the vehicle, but allow vertical movement, as shown in Fig. 2. The Watt's linkage is also adopted for insectlike flapping wings in hover [11], where the wing tip traces a figure eight. In some cases, the Watt's linkage is modified for better performance. Phineas Crowther [3] made a little change to the condition for the arrangement of the mechanism. Kusaka et al. [9] and Haeusler et al. [10] didn't use the basic Watt's linkage either when designing the suspension systems shown in Figs. 2(a) and 2(c), but modified it without compromising the function of the Watt's linkage in any way. Unfortunately, they didn't give the detailed description about their modification.

Path generation is a key issue for linkage mechanism. It could be extremely challenging as the coupler curve equation is highly nonlinear. There are two main approaches: synthesis of precision points and optimal synthesis [12]. Precision point synthesis implies that the coupler point traces through a certain number of desired points. The approach is limited to the case of five specified points [13]. Optimal synthesis is repeated analysis for a random mechanism, determined by a vector of design variables, and finding of the best possible one so that it can meet designer's requirement, which is mostly used for path generation.

Some efforts have been made to optimize linkages with different optimization algorithms. For example, when the objective function is smooth and of unimodal distribution and the number of optimization variables is reduced to three, the optimization can be conducted with the quasi-Newton gradient method [14]. Genetic algorithm is commonly used for linkage path generation. Cabrera et al. [15] conducted optimal synthesis for linkage mechanism with genetic algorithm. Zhou et al. [16] utilized genetic algorithm for dimensional synthesis of adjustable path generation linkages. Nariman-Zadeh et al. [17] and Khorshidi et al. [18] introduced hybrid multi-objective genetic algorithm for optimum synthesis of four-bar linkages considering the minimization of several objective functions simultaneously. Genetic algorithm is just a kind of biological motivated algorithm, which is named evolutionary algorithm. Acharyya et al. [19] analyzed the equivalences and differences of various popular evolutionary algorithms. Lin [13] combined differential evolution with genetic algorithm for path synthesis of a four-bar linkage. He also studied the special trajectories generating for the geared five-bar mechanism with the proposed GA-DE evolutionary algorithm [20]. Raste et al. [21] conducted optimum synthesis of a path generating adjustable four-bar mechanism with Differential Evolution (DE). Many researches about optimal synthesis are motivated by practical applications. Al-Smadi et al. [22] introduced the planar four-bar path generation with a coupler point load, crank static torque, crank transverse deflection and follower buckling in a modified search algorithm. Xu et al. [23] conducted optimal process by taking the penalty function method. With the optimized parameters, the authors designed a biped robot that could walk on water. All mentioned algorithms need kinematic synthesis of linkages to obtain objective functions. The method presented by Freudenstein [24] is widely adopted for its simplicity. He utilized the closed vector equation instead of the general Assur group method [25]. With the method different from what is mentioned above, Wunderlich [26] carried out the optimization in geometric method, he gave the analytical description of the coupler curve. Mundo et al. [27] presented a graphical-analytical technique for the synthesis of non-circular gears in path generating geared five-bar mechanisms.

In path generation of linkage mechanism, the desired path can be in arbitrary shape based on the practical application. Among the paths, straight path is one of the most widespread one. Various methods have been introduced for path generation with straight line target path. Mehdigholi et al. [12] used genetic algorithm for optimization of Watt's six-bar linkage to generate straight and parallel motion. In studies by Nariman-Zadeh [17], Khorshidi [18] and Lin [13], one of the synthesis cases was the straight line coupler curve generation. Shiakolas et al. [28] utilized differential evolution and the geometric centroid of precision positions for the four-bar linkage optimum synthesis, where one of the target coupler curve was a straight line. With geometric synthesis of the Watt's linkage, Wunderlich [26] succeeded in making the coupler point approximate a segment of a straight line.

Although there are other linkages that can generate a straight path or approximately straight path, they confront several drawbacks, such as the number of links is large, the transmission angle deviates much from 90°, etc. Herein, the Watt's linkage is the adopted linkage for path generation with enough reasons. What's more, the Watt's linkage is employed by people with different backgrounds, especially in the engineering field. Unfortunately, most of them consider more about adopting the mechanism instead of the relationship between performance and its parameters. So little description about the parameter definition method has been given in the literature, except Wunderlich [26] with the analytical method. Some modifications have been introduced to the basic Watt's linkage. But little theoretical description has been provided for the modifications. So there is great need for a modification considering both theoretical analysis and better practical performance. In addition, in most studies on path generation of linkages, the authors have no idea of the configuration of the mechanism before the optimization is accomplished. So the authors don't know whether the optimization result can be used in practical application or not beforehand. For the above three reasons, we have the motivation to write this paper to introduce our research in modifying the basic Watt's linkage and carrying out the path generation considering practical application.

For the Watt's linkage, the larger the mechanism dimension is, the straighter the coupler curve will be. However, it is meaningless to get an ideal approximately straight path with an immense mechanism that is improper for practical application. It is of great value to find an optimal solution with a smaller deviation stroke ratio of the approximately straight path within the limited dimension. The dimension is determined by the practical application. The detailed method will be explained in the constraints for optimization.

As the coupler curve equation is highly nonlinear, it is relatively hard to conduct the task with classical linkage synthesis approach. With the help of high speed computers, we are able to search for feasible solution more quickly with numeric method. Using the numeric optimization method, there is no need for a deep knowledge of searching space, such as whether or not it is continuous, presents local minimums or shows other mathematical characteristics demanded by traditional searching algorithms [15]. Ma et al. [29] discussed the equivalences and differences of evolutionary algorithms and found out that biogeography-based optimization, differential evolution, evolution strategy and particle swarm optimization are equal to genetic algorithm under certain conditions. Each evolution algorithm has its own advantages and disadvantages. It may perform better in a synthesis, but worse in another synthesis, which is influenced by several complex factors. What we care most is the synthesis process for modified Watt's linkage instead of the optimal algorithm itself. So the widely used genetic algorithm in optimization is utilized in our path generation.

Finally, ideal parameters for the mechanism are achieved. The symmetry character of the mechanism promotes the synthesis greatly. We have succeeded to generate longer and straighter coupler curve with the configuration and dimension of linkage mechanism restricted beforehand. With the optimal parameters obtained from the proposed method, two typical applications of the mechanism with a brief description are given at last.

The paper is organized as follows. The basic Watt's linkage and modification to it are introduced in Section 2, where the most important part is the condition for the modification. Then, the kinematic analysis of the linkage is given to obtain the objective function for optimization in Section 3. Section 4 describes the process of optimization, including objective function, constraints, optimization results and applications. Two practical applications of the mechanism are introduced. The presented method based on optimization to generate longer and straighter coupler curve of linkage mechanism is summarized in Section 5 and future studies are also presented in this section.

2. MODIFIED WATT'S LINKAGE (MWL)

2.1. The Watt's Linkage

James Watt is remembered for his pioneering work on steam engines. It was James Watt who invented the first straight-line linkage in 1784 [1]. The simplest form of the Watt's linkage is illustrated in Fig. 1(a), which consists of only three links, *AB*, *BC*, *CD*. Here, *A* and *D* are fixed in space, but free to rotate, forcing *B* and *C* to move in arcs of circles. Links *AB* and *CD* are of the same length, and the center of the coupler link *BC*, labeled *P*, is the coupler point that moves vertically in the approximate-straight-line manner.

It is worthy of noting that the length of *AB* can be unequal with the length of *CD*. This is the general form of the Watt's linkage, in which *P* is positioned with the ratio:

$$\frac{AB}{CD} = \frac{BP}{CP}. \tag{1}$$

For simplicity, *AB* and *CD* are of the same length in our research.

The useful segment is only for a small displacement of *B*, or equivalently *C*. The complete path of *P* is an eight shaped curve. What we can use is the part near the center of the curve, which is an approximately straight path.

2.2. The Approach to Modify the Watt's Linkage

When using the Watt's linkage, Phineas Crowther [3] made a slight change to the basic Watt's linkage. At the center of the stroke, when the links *AB* and *CD* are horizontal, the link *BC* is not vertical but approximately $\theta = 5^\circ$ from the vertical. The dimensions are as follows:

$AB = CD = 132$,

$BC = 27$, with *P* at the center.

The path taken by the point *P* can be calculated using kinematics of the four-bar linkage, which is illustrated in Fig. 3(b) with a dashed line.

In Crowther's engine, for *BC* it is not necessary to be vertical at the center of the stroke. However, *AB* and *CD* are also restricted to be horizontal. Based on the idea of Phineas Crowther, we can modify the Watt's

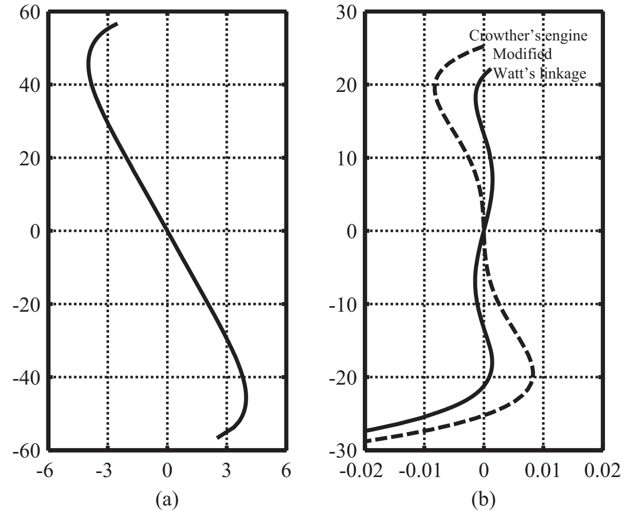


Fig. 3. The path of *P* in modified Watt's linkage.

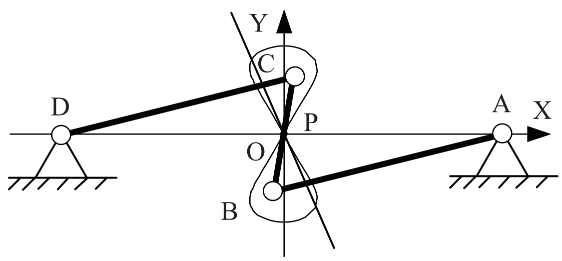


Fig. 4. Diagram of modified Watt's linkage.

linkage even further. Therefore, *AB* and *CD* are not necessarily horizontal while *BC* does not need to be vertical at the center of the stroke in our design. What is more important is the way to achieve straighter path of *P* instead of the slight difference of the linkage mechanism.

For simplicity of the kinematic synthesis, modified Watt's linkage is illustrated in Fig. 4, in which the vertical distance between the fixed pivots is zero. As a result, the location of *P* can be calculated with just five parameters, namely, lengths of three links, distance between *A*, *D* and angle of link *AB* which is measured from the axis *X* and is positive counter clockwise. Given the parameters the same as those in the Crowther's engine, the useful segment of the coupler curve is shown in Fig. 3(a), which ideally looks to be an approximately straight path.

Then, the linkage is rotated clockwise by an appropriate angle for getting a vertical approximately straight path, making the coupler curve rotated at the same time. The value of the rotating angle is determined by how the approximately straight path is close to the target vertical line, which is another parameter determining the performance of the approximately straight path besides the five parameters mentioned above. The detailed method will be clarified below. After rotating, *AB* and *CD* are not necessary to be horizontal and *BC* isn't necessary to be vertical at the center of the stroke. The coupler curve of the MWL is shown in Fig. 3(b) with a solid line while that of the Crowther's engine is also plotted with a dashed line for comparison. Both are actually quite good approximations to the target straight line, despite the appearance of Fig. 3(b) because of axes with different scales.

As the coupler curve is not an absolutely straight line, the deviation can't be avoided. We define deviation ratio as the evaluating index of the coupler curve, which is calculated with the maximal deviation at two sides divided by the stroke. In Crowther's engine, the complete stoke is 52 when $\theta = 5^\circ$, the maximal deviation at two sides is 0.016. Then the deviation ratio is $0.016/52 = 3.08 \times 10^{-4}$. By contrast, the deviation at one side of the straight line is the same as that at the other side in MWL as shown in Fig. 3(b), which is the formula to determine the rotating angle. So the maximal deviation is less than that in Crowther's engine, making the deviation ratio smaller. The deviation ratio in MWL with the coupler curve shown in Fig. 3(b) is $0.003/45 = 6.7 \times 10^{-5}$, which is a great improvement compared with Crowther's modification.

From Fig. 3(b), it is clear that the coupler curve of MWL is different from Crowther's engine in that the path traverses the objective line five times, while the number is three in Crowther's engine. Moreover, the number is one in the basic Watt's linkage. With the increasing number of intersecting points, we can find that the deviation ratio is smaller and smaller. Here comes the condition for modified Watt's linkage.

2.3. The Condition for Modified Watt's Linkage

The target of designing MWL is to get an approximately straight path that can traverse the target line five times. However, only mechanisms with specific combinations of parameters can satisfy such condition, which is defined as the condition for MWL by authors. In Fig. 4, the coordinates of A and D are $(a,0)$ and $(-a,0)$ respectively, the lengths of AB and CD are b, the length of BC is $2c$. The path of the coupler point P is called Watt's curve with implicit function as [30, 31]:

$$P_X{}^2(P_X{}^2 + P_Y{}^2 + P)^2 + P_Y{}^2(P_X{}^2 + P_Y{}^2 + Q)^2 - RP_Y{}^2 = 0, \tag{2}$$

where

$$P = c^2 - b^2 - a^2,$$
$$Q = c^2 - b^2 + a^2,$$
$$R = 4c^2a^2,$$

and the coordinate of P is (P_X, P_Y).

The intersecting points between the coupler curve and the target line in Fig. 4 satisfy

$$P_Y = kP_X, \tag{3}$$

where k is the slope of the line.

Substituting this into Eq. (2) gives

$$P_X{}^2(P_X{}^2 + k^2P_X{}^2 + P)^2 + k^2P_x{}^2(P_X{}^2 + k^2P_X{}^2 + Q)^2 - Rk^2P_X{}^2 = 0. \tag{4}$$

According to Fig. 4 and the condition for MWL, Eq. (4) should have five different real roots, among which zero is one of the roots and the others are two pairs of opposite numbers. We define $t = P_X{}^2$, $S = k^2$, and rearrange it. Eq. (4) can be written as

$$t[(1+S)^3t^2 + 2(1+S)(P+SQ)t + P^2 + SQ^2 - SR] = 0. \tag{5}$$

Obviously, Eq. (5) should have three different real roots, among which zero is one of the roots and the others are two different positive numbers. We can readily get the conclusion that there are two different positive roots for the quadratic equation

$$(1+S)^3t^2 + 2(1+S)(P+SQ)t + P^2 + SQ^2 - SR = 0. \tag{6}$$

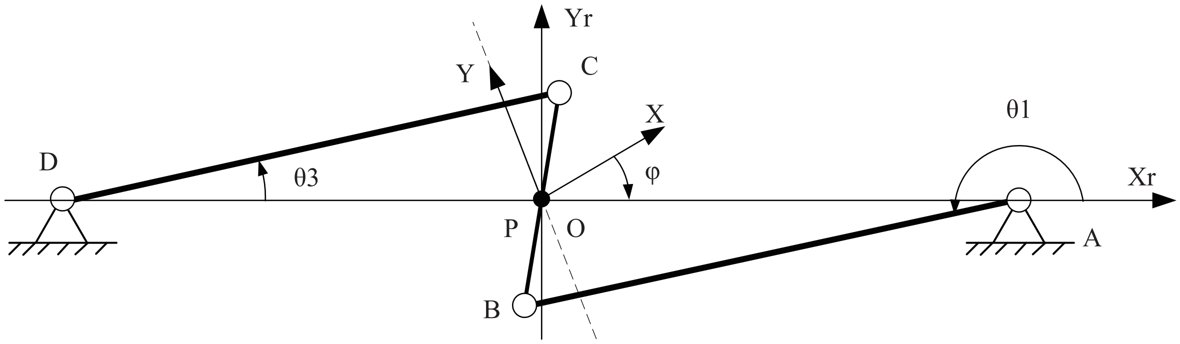


Fig. 5. Modified Watt's linkage with its design variables.

According to the characteristics of the quadratic equation, we get the inequations

$$\begin{cases} [2(1+S)(P+SQ)]^2-4(1+S)^3(P^2+SQ^2-SR)>0 \\ -\frac{2(1+S)(P+SQ)}{(1+S)^3}>0 \\ \frac{P^2+SQ^2-SR}{(1+S)^3}>0 \end{cases}, \tag{7}$$

where S is the variable, squaring the slope k. So the solution for the inequations should be an interval of positive numbers. Rearranging the inequations gives

$$\begin{cases} S[RS+R-(R-Q)^2]>0 \\ P+QS<0 \\ P^2+(Q^2-R)S>0 \end{cases}, \tag{8}$$

where Q and R are positive. Solutions for each inequation can be computed respectively as

$$S<0, \text{ or } S>\frac{-R+(P-Q)^2}{R}, \tag{9}$$

$$S<-\frac{P}{Q}, \tag{10}$$

$$\begin{cases} S<-\frac{P^2}{Q^2-R}, Q^2-R<0 \\ S>-\frac{P^2}{Q^2-R}, Q^2-R>0 \end{cases}. \tag{11}$$

To ensure that a positive interval solution exists for Eqs. (8), the condition for the parameters can be obtained as

$$\begin{cases} \frac{-R+(P-Q)^2}{R}<-\frac{P}{Q} \\ -\frac{P^2}{Q^2-R}>\frac{-R+(P-Q)^2}{R}, Q^2-R<0 \\ -\frac{P^2}{Q^2-R}<-\frac{P}{Q}, Q^2-R>0 \end{cases}, \tag{12}$$

which is also the condition for the proposed modified Watt's linkage.

3. KINEMATIC SYNTHESIS OF THE LINKAGE

As mentioned before, we modify the Watt's linkage in the state that the vertical distance between A and D is zero, shown in Fig. 5. We can conduct the kinematic synthesis easily with a few parameters [19].

At the beginning, we define the dimensions as $r_{AB} = r_1$, $r_{BC} = r_2$, $r_{CD} = r_3$, $r_{AD} = r_4$. There are two coordinate systems, global coordinate system O_{XY} and reference coordinate system O_{XrYr}. The global coordinate system O_{XY} is the actual coordinate system of the mechanism, where the axis Y is the target straight line. The global coordinate system O_{XY} rotates α clockwise about point O to the reference coordinate system O_{XrYr}, where axis Xr traces point A and D, making the axis horizontal.

If not stated specifically, the coordinate of a point refers to that in reference coordinate system O_{XrYr}. The coordinates of A and D are:

$$\begin{bmatrix} A_{Xr} \\ A_{Yr} \end{bmatrix} = \begin{bmatrix} \frac{r_4}{2} \\ 0 \end{bmatrix}, \tag{13}$$

$$\begin{bmatrix} D_{Xr} \\ D_{Yr} \end{bmatrix} = \begin{bmatrix} -\frac{r_4}{2} \\ 0 \end{bmatrix}. \tag{14}$$

From the vector $\overrightarrow{r_B} = \overrightarrow{r_A} + r_1 e^{j\theta_1}$, the coordinate of B is stated as

$$\begin{bmatrix} B_{Xr} \\ B_{Yr} \end{bmatrix} = \begin{bmatrix} \frac{r_4}{2} + r_1 \cos\theta_1 \\ r_1 \sin\theta_1 \end{bmatrix}. \tag{15}$$

Similarly, the coordinate of C is:

$$\begin{bmatrix} C_{Xr} \\ C_{Yr} \end{bmatrix} = \begin{bmatrix} -\frac{r_4}{2} + r_3 \cos\theta_3 \\ r_3 \sin\theta_3 \end{bmatrix}. \tag{16}$$

According to Freudenstein's equation [24], angle θ_3 can be solved in respect to input angle θ_1. From the vector equation $\overrightarrow{BC}^2 = (\overrightarrow{CD} + \overrightarrow{DA} + \overrightarrow{AB}) \cdot (\overrightarrow{CD} + \overrightarrow{DA} + \overrightarrow{AB})$, we can get:

$$K_1 \cos\theta_1 - K_2 \cos\theta_3 + K_3 = \cos(\theta_1 - \theta_3), \tag{17}$$

where K_1, K_2 and K_3 are:

$$K_1 = \frac{r_4}{r_3}, \quad K_2 = \frac{r_4}{r_1}, \quad K_3 = \frac{{r_1}^2 - {r_2}^2 + {r_3}^2 + {r_4}^2}{2 r_1 r_3}.$$

From Eq. (17), the solution of the equation can be computed as:

$$\theta_3 = 2\tan^{-1}\left(\frac{-R_2 \pm \sqrt{{R_1}^2 + {R_2}^2 - {R_3}^2}}{R_3 - R_1}\right), \tag{18}$$

where

$$R_1 = \frac{r_4}{r_1} + \cos\theta_1, \quad R_2 = \sin\theta_1, \quad R_3 = -\frac{r_4}{r_3}\cos\theta_1 - \frac{{r_1}^2 - {r_2}^2 + {r_3}^2 + {r_4}^2}{2 r_1 r_3}.$$

According to the geometric relationship of the links, we can get the application capable solution as:

$$\theta_3 = 2\tan^{-1}\left(\frac{-R_2 - \sqrt{{R_1}^2 + {R_2}^2 - {R_3}^2}}{R_3 - R_1}\right). \tag{19}$$

As P locates at the middle of the coupler link BC with the relationship

$$\overrightarrow{r_P} = \frac{\overrightarrow{r_B} + \overrightarrow{r_C}}{2} = \frac{r_1 e^{j\theta_1} + r_3 e^{j\theta_3}}{2}, \tag{20}$$

the coordinate of P is:

$$\begin{bmatrix} P_{Xr} \\ P_{Yr} \end{bmatrix} = \begin{bmatrix} \frac{r_1 \cos\theta_1 + r_3 \cos\theta_3}{2} \\ \frac{r_1 \sin\theta_1 + r_3 \sin\theta_3}{2} \end{bmatrix}. \tag{21}$$

With angle θ_1 varying from minimal to maximal valve, the point P travels along its slantwise approximately straight path, which is similar to the curve in Fig. 3(a). So we rotate the linkage clockwise to get a vertical straight path. By contraries, we rotate the reference coordinate system O_{XrYr} with α counter clockwise about point O to the global coordinate system O_{XY}, making the axis Y to be the objective line. So the coordinate of P in global coordinate system O_{XY} is [32]:

$$\begin{bmatrix} P_X \\ P_Y \end{bmatrix} = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} P_{Xr} \\ P_{Yr} \end{bmatrix}. \tag{22}$$

This formulation of the coordinate of the coupler point P will be used below to express the objective function for optimization.

4. OPTIMIZATION OF THE LINKAGE

Optimization of mechanisms is considered one of the notable subjects in mechanism design. Because of nonlinearity of objective functions and optimization constraints and lack of explicit relations for their gradients, these optimization problems are very complex using conventional analytical method [33]. Genetic algorithm use the distributing of objective function values instead of its gradients to scheme search direction, making it highly capable for optimization of nonlinear, incontinuous, non-differentiable problems. The reasons why it is adopted among several optimization algorithms have been stated before.

4.1. Objective Function Formulation

In path generation researches, the most widely used objective function is the error between the points tracked by the coupler and its desired path, named tracking error (TE) [17]. The synthesis process involves in finding the vector of design variables $X = [l_1, l_2, l_3, \alpha, \theta_1^i]$ so that tracking error is minimized for an ideal coupler curve. Among the design variables, l_1, l_2, l_3 are the lengths of the links AB and CD, the link BC and the link AD respectively, namely, $l_1 = r_1 = r_3$, $l_2 = r_2$, $l_3 = r_4$. α is the angle that the global coordinate system O_{XY} is rotated to the reference coordinate system O_{XrYr}. θ_1^i is the ordered sequence $(i = 1, 2, ..., N)$ of input angles, where N is the number of the points to be synthesized.

Tracking error is computed by the sum of the squares of the Euclidean distances between each P_d^i and the corresponding P^i, where $\{P_d^i\}$ is a set of target points that are supposed to be traced by the coupler point of the mechanism. They can be stated in the global coordinate system O_{XY} in Fig. 5 as

$$P_d^i = \begin{bmatrix} P_{Xd}^i \\ P_{Yd}^i \end{bmatrix}. \tag{23}$$

$\{P^i\}$ is a set of positions of the coupler point P in the designed mechanism for a set of input angles $\{\theta_1^i\}$

$$P^i = \begin{bmatrix} P_X^i \\ P_Y^i \end{bmatrix} = \begin{bmatrix} P_X(\theta_1^i) \\ P_Y(\theta_1^i) \end{bmatrix}. \tag{24}$$

Therefore, the objective function, TE, can be readily computed as

$$TE = \sum_{i=1}^{N} [(P_{Xd} - P_X)^2 + (P_{Yd} - P_Y)^2]. \tag{25}$$

4.2. Constraint Functions

The constraints of the optimization locate in these areas:

1. The condition to constitute a four-bar linkage. To generate a four-bar linkage, the lengths of the links should match the condition that the length of the longest link should be less than the sum of the remaining lengths. Observing Fig. 5, we know the fixed link *AD* is the longest one. Therefore, the condition can be written as
$$2l_1 + l_2 > l_3. \tag{26}$$

2. The sequence of input angles [18]. To insure that the target points are traced in the desired order, it is necessary for the corresponding ${\theta_1}^i$ to follow this order as well, which is equivalent to the following constrains:
$${\theta_1}^i < {\theta_1}^{i+1}. \tag{27}$$

3. The ranges for design variables. The constraint can be readily stated as
$$x_i \in [\mathrm{Min}(x_i), \mathrm{Max}(x_i)], \tag{28}$$
where $x_i \in X = [l_1, l_2, l_3, \alpha, \theta_1^i]$. Here, the maximal dimension of the linkage mechanism is given as
$$\mathrm{Dim} = \mathrm{Max}(l_3) \cdot \cos[(\mathrm{Min}(\alpha))]. \tag{29}$$
This is how we arrange the mechanism within limited dimension, making the synthesis result more capable for practical application.

4. The range of the input angle. For a MWL with l_1, l_2, l_3, α settled, we need to get the minimal and maximal value of the input rocker angle θ_1 as the input rocker *AB* can't rotate 360°. When joints *A*, *B* and *C* are collinear, we get the minimum value of θ_1 as
$$\min(\theta_1) = \pi - \arccos\frac{(l_1+l_2)^2 - {l_1}^2 + {r_3}^2}{2(l_1+l_2)l_3}. \tag{30}$$
Similarly, the maximum value of θ_1 is calculated when joints *B*, *C* and *D* are collinear.
$$\max(\theta_1) = \pi + \arccos\frac{{l_1}^2 - (l_1+l_2)^2 + {l_3}^2}{2l_1 l_3}. \tag{31}$$

5. The condition to generate an eight shaped path [26]. The center point *P* of the link *BC* traces an eight shaped path on condition that
$$l_2 < l_3, \tag{32}$$
and
$$2\left|l_1 - \frac{l_2}{2}\right| < l_3 < 2\left(l_1 + \frac{l_2}{2}\right). \tag{33}$$
The four-bar linkage with parameters satisfying Eqs. (32) and (33) has similar coupler curve with the Watt's linkage. This is the way in which the configuration of the mechanism is restricted beforehand.

6. The most important constraint is the condition for the proposed MWL, which will improve the optimization performance dramatically. According to the description of the modification, the condition is

$$\begin{cases} \frac{-R+(P-Q)^2}{R} < -\frac{P}{Q} \\ -\frac{P^2}{Q^2-R} > \frac{-R+(P-Q)^2}{R}, Q^2-R<0 \\ -\frac{P^2}{Q^2-R} < -\frac{P}{Q}, Q^2-R>0 \end{cases}, \tag{34}$$

which is the same as Eq. (12) with different meanings of the variables as

$$P = \left(\frac{l_2}{2}\right)^2 - l_1{}^2 - \left(\frac{l_3}{2}\right)^2,$$
$$Q = \left(\frac{l_2}{2}\right)^2 - l_1{}^2 + \left(\frac{l_3}{2}\right)^2,$$
$$R = 4\left(\frac{l_2}{2}\right)^2\left(\frac{l_3}{2}\right)^2.$$

4.3. Optimization Results

With the objective function and constraints specified, we conducted two synthesis cases. One case was the conventional straight path generation for the proposed MWL. The other case took advantage of the centrosymmetric character of the MWL. Afterwards, comparison between the two synthesis results was given to get a better optimal result.

(1) Six target points
In linkage path generation, the more target points are given to the target path, the better the coupler curve will be obtained. But more target points means more design variables, making the optimization more complex. Based on the researches in the literature, six target points were utilized here. They lay on the vertical line Y in the global coordinate system O_{XY} with coordinates as

$$\{(P_{Xd}, P_{Yd})\} = \{(0,15),(0,10),(0,5),(0,-5),(0,-10),(0,-15)\}. \tag{35}$$

The point $(0,0)$ was not given as a target point as the coupler point P would surely pass the point. The lower and upper bounds for the vector of design variables:

$$X = [l_1, l_2, l_3, \alpha, \theta_1^1, \theta_1^2, \theta_1^3, \theta_1^4, \theta_1^5, \theta_1^6], \tag{36}$$

were chosen, respectively, to be:

$$[\text{Min}, \text{Max}] = \{[40,50],[28,35],[85,105],[18,21],[170,230],[170,230], [170,230],[170,230],[170,230],[170,230]\}. \tag{37}$$

Table 1. Optimization results with six target points.

Parameter	Value	Parameter	Value
l_1	43.5684	$\theta_1{}^2$	187.030°
l_2	30.8691	$\theta_1{}^3$	193.621°
l_3	88.8841	$\theta_1{}^4$	206.801°
α	20.2907°	$\theta_1{}^5$	213.568°
$\theta_1{}^1$	180.157°	$\theta_1{}^6$	220.433°

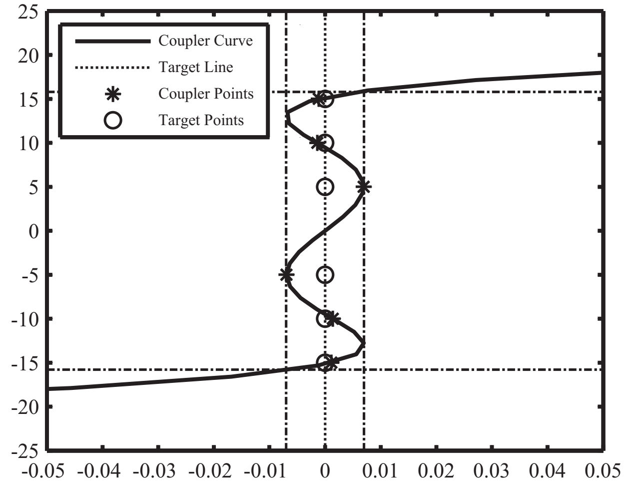


Fig. 6. Optimized coupler curve with six target points.

Table 2. Optimization results with three target points.

Parameter	Value	Parameter	Value
l_1	42.6911	$\theta_1{}^1$	178.9349°
l_2	29.2268	$\theta_1{}^2$	186.0151°
l_3	87.2567	$\theta_1{}^3$	192.7668°
α	19.545°		

The proposed conditions for the dimension, configuration and MWL make the constraints highly complex and nonlinear. So the optimization results are highly sensitive to the ranges of the design variables. Improper ranges may render the mechanism impractical. Therefore, the bounds were chosen based on preliminary experiments and trial parameters assignments and expanded to a proper extent.

Table 1 shows the optimization result. Figure 6 shows the coupler curve of the resulted mechanism which traces quite close to the target points, despite the appearance because of axes with different scales. The dimension of the optimized mechanism is

$$88.8841 \times \cos(20.2907^\circ) = 83.3684. \tag{38}$$

(2) Three target points

The centrosymmetric character of the MWL makes the coupler curve centrosymmetric too, which is confirmed by Fig. 6. In this way, the target points could be reduced to three with coordinates given as

$$\{(P_{Xd}, P_{Yd})\} = \{(0,15),(0,10),(0,5)\}. \tag{39}$$

The design variables could be reduced to seven with only three input angles. After synthesis, the optimized parameters are listed in Table 2 with the corresponding coupler curve shown in Fig. 7. The dimension of the optimized mechanism is

$$87.2567 \times \cos(19.545^\circ) = 82.2289. \tag{40}$$

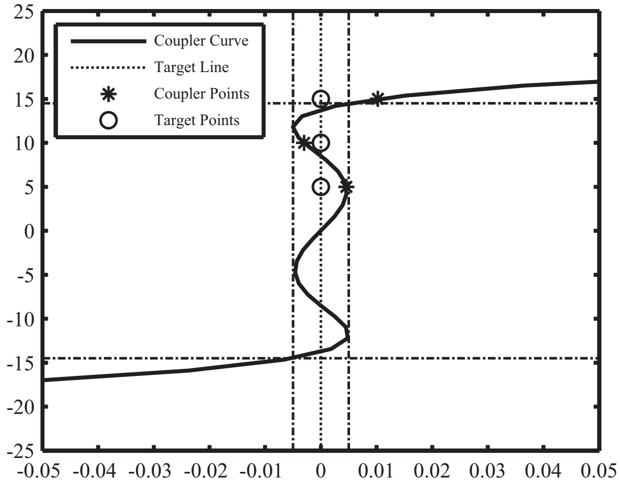


Fig. 7. Optimized coupler curve with three target points.

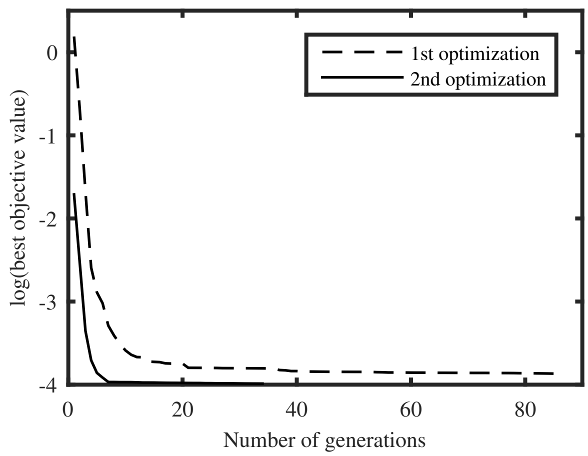


Fig. 8. History of objective function.

Figure 8 shows convergence plots of the optimizations, in which the best (least) objective function value obtained in a generation is plotted in logarithmic scale against the number of generation. It is obvious that both optimizations are effective in approximating the target path. But the second optimization offers faster and better convergence. The value of the objective function of the best individual in the second optimization almost converges to a minimum value within only nine generations.

The performance comparison of the modification and optimizations is given in Table 3. We define stroke ratio as the evaluating index of the potential application, which is calculated with the stroke divided by the dimension. We can see the stroke ratios of the optimizations are much greater than those of the Crowther's engine and the MWL. So the optimized results are of greater potential application, which enable the significance of the optimization. Between two optimization results, the second one offers better performance with the following reasons:

1. The second optimization is much faster than the former one with fewer design variables.

Table 3. Performance comparison.

Items	Crowther's engine [3]	MWL	1st optimization	2nd optimization
TE	–	–	1.35×10^{-4}	1.0297×10^{-4}
Terminal generation	–	–	87th	34th
Stoke	52	45	31.6	29
Deviation	0.016	0.003	0.014	0.01
Deviation ratio	3.08×10^{-4}	6.7×10^{-5}	4.43×10^{-4}	3.45×10^{-4}
Dimension	261.622	261.612	83.3684	82.2289
Stoke ratio	0.199	0.172	0.379	0.353

2. The dimension of the second optimization is smaller.
3. The deviation ratio in the second optimization is a little smaller.

The goal of the synthesis is to find a solution with a smaller deviation ratio of the approximately straight path within limited dimension, which guarantees the potential practical application. What's more, the coupler curve meets the condition for the proposed MWL. Above all, an ideal result is achieved. Analyzing other studies on straight path generation for linkages, it can be seen that most studies concern tracking error only. Our study is the first one that takes the deviation in unit stroke into consideration, namely the deviation ratio. What's more, we have the configuration of the optimal mechanism in mind beforehand, and the dimension of the mechanism is constrained from practical application, which haven't been seen in other studies on path generation as far as we are concerned.

4.4. Applications

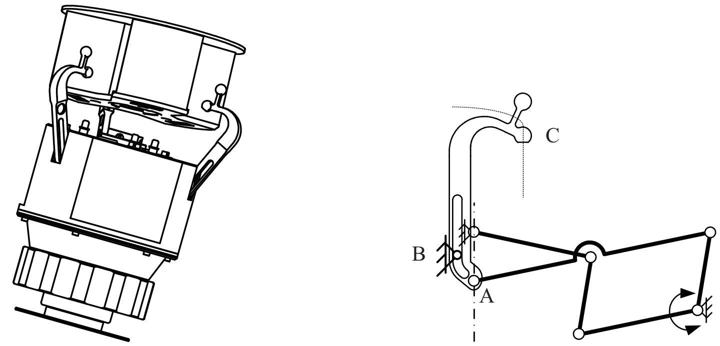
The synthesis's goal is not only the length of the links in a specific mechanism, but also the proportions of the links that can be used in other mechanisms. With this optimization, we can easily arrange another similar mechanism with different strokes. To generate a mechanism with the stroke equal to $29k$, the parameters can be readily given as

$$\begin{bmatrix} l_1 \\ l_2 \\ l_3 \\ \alpha \end{bmatrix} = \begin{bmatrix} 42.6911k \\ 29.2268k \\ 87.2567k \\ 19.545^\circ \end{bmatrix}, \tag{41}$$

where k is the stroke factor.

We have developed a space on-orbit end-effector with the optimized linkage mechanism. The end-effector and its grapple assembly are shown in Fig. 9. The gripper was specially designed with the shape shown in Fig. 9(b), making the actuation relatively simple. As the groove on the gripper was constrained by the fixed joint B, making the gripper conduct plane motion and linear translation sequentially with the simple linear actuation. The gripper tip C traced the curve shown in Fig. 9(b). We combined the synthesized straight line linkage with a pantograph mechanism to generate enlarged stroke within limited dimension. The axis dimension of the end-effector was thereby reduced. The straight path with stoke as 34 could be generated with the axial dimension as 15, resulting in smaller volume compared to those designs with ball screw or other components. Moreover, the linkage mechanism was of greater stability than the conventional ball screw mechanism [34]. The unique actuation also left the central space empty, making it easier to arrange other subsystems along the central axis.

Another potential application is a mechanism that may be employed in walking mechanism. According to the Robert–Chebyshev theorem, three different planar four-bar linkages will trace identical coupler



(a) (b)

Fig. 9. End-effector and grapple assembly.

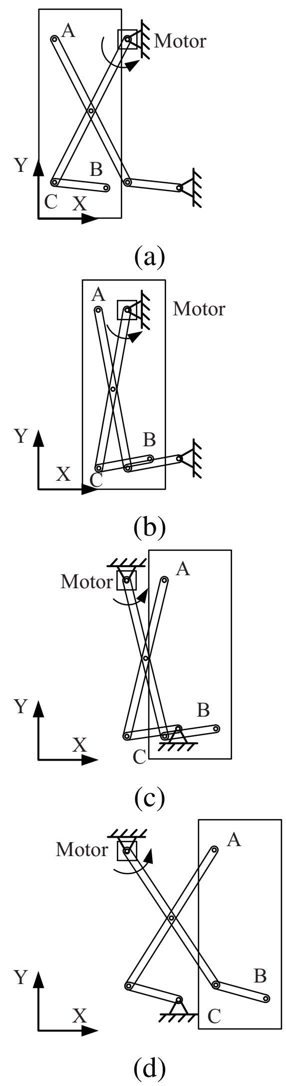
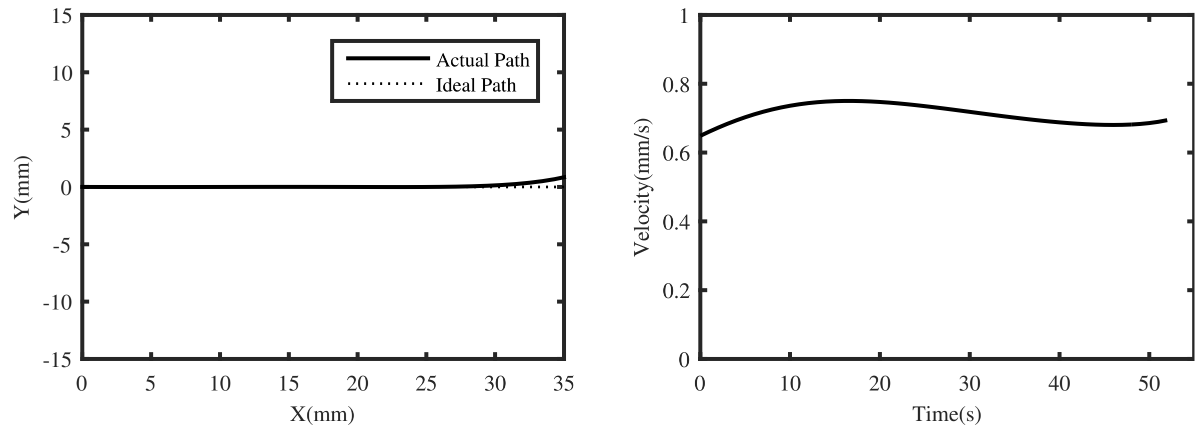


Fig. 10. Walking procedure.



(a) (b)

Fig. 11. Simulation results of the walking mechanism.

curve [35]. After combining with a four-bar linkage's cognate linkages and rearranging them, we can get a translational link [12]. In this way, we get a six-bar linkage with the translational link AB shown in Fig. 10(a). Here, curves of points A and B are parallel and synchronous, resulting the translational motion of link AB.

The walking mechanism was simulated in ADAMS, utilizing the parameters in Eq. (41) with $k = 1$. The motor rotated with the velocity of $1^\circ/s$. The walking procedure was illustrated in Fig. 10. The combined linkage mechanism transferred the rotatory motion of the motor into the translational motion of link AB. The path and velocity of point C was obtained as illustrated in Figs. 11(a) and 11(b), respectively. The coordinate system O_{XY} was located at the original place of point C, with axis X horizontal. From the figures, we can conclude that the simulated path traces quite well to the ideal path within the useful stroke of 29 and point C can move at an approximately constant velocity. If mounted on a body with another identical mechanism, the body can move with approximately constant velocity, which is an ideal walking mechanism.

5. CONCLUSION

The paper has presented an optimal method to generate a longer and straighter coupler curve of linkages. Modification and optimization are the key aspects of our study. After comparing the advantages and disadvantages of existing widely used straight-line linkage, the Watt's linkage was adopted for our approximately straight path generation. According to the envelope of Watt's curve, a modification to the Watt's linkage and the corresponding condition were introduced to improve the effect of the straight path. Regarding the complexity of the coupler curve equation, genetic algorithm was adopted to perform the highly nonlinear optimization. The path generation considering configuration and dimension of the mechanism was proposed. Several constraints were taken into consideration, where the most significant one was the proposed condition for the MWL. At last, ideal parameters for the mechanism were achieved. Considering the centrosymmetric property of the MWL, target points number was reduced, resulting faster and better optimization. The effectiveness of generating longer and straighter coupler curve of linkage mechanism within limited dimension with the proposed method was demonstrated. Two potential applications were given with detailed simulation results at last. In addition, the deviation in unit stroke is the first time introduced to evaluate straight path generation. In the future, we can readily specify the parameter of the mechanism with different strokes based on this optimization result.

The intention of generating an approximately straight path is reached with the modification and the optimization. There are many other things we can do besides utilizing the approximately straight path. Comparison of different optimal algorithms for better optimization may be one of our future directions. Combining the mechanism with a pantograph mechanism [3], we will get arbitrary amplificatory or contractible stoke with parameters of the mechanism unchanged. In addition, more applications from the translational motion besides the walking mechanism can be achieved with the obtained mechanism.

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