SYNCHRONIZATION OF DUAL HOMODROMY ROTORS WITH ECCENTRIC MASSES IN A NONLINEAR VIBRATING SYSTEM

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Received July 2015, Accepted March 2016
No. 15-CSME-76, E.I.C. Accession 3851

ABSTRACT

The motion equations of a nonlinear vibrating system are given firstly. The nonlinear stiffness of springs is equivalently linearized as a function of the response of system by the asymptotic method. The synchronization criterion of dual homodromy rotors with eccentric masses is derived theoretically, as well as the stability criterion. It is shown that the phase difference is close to zero during a sub-resonant state, otherwise it approaches π. The nonlinear characteristics of system are discussed numerically. By the comparisons among theory, numeric and simulation, the validity of the theory method used is examined.

Keywords: synchronization; stability; nonlinear vibrating system; rotors with eccentric masses (REM).

SYNCHRONISATION DE BIROTORS HOLONOMIQUES AVEC MASSES EXCENTRIQUES DANS UN SYSTÈME NONLINÉAIRE VIBRANT

RÉSUMÉ

Les équations de mouvement d'un système vibrant sont établies en premier lieu. La rigidité non-linéaire des ressorts est linéarisée parallèlement comme une fonction de réponse du système par la méthode asymptotique. Les critères de synchronisation des birotors avec masses excentriques sont dérivés de manière théorique, ainsi que les critères de stabilité. Il est démontré que la différence de phase s'approche du zéro dans un état sous-résonant, sinon près de π. Les caractéristiques non-linéaire du système sont discutées du point de vue numérique. En se servant des comparaisons entre la théorie et la simulation numérique, la validité de la méthode utilisée est examinée.

Mots-clés : synchronisation; stabilité; système vibrant non-linéaire; rotors avec masses excentriques.

NOMENCLATURE

f_i	damping constant of rotor of the motor i, $i = 1,2$
f_x	damping constants in x-direction
J_{0i}	moment of inertia of REMs i, $i = 1,2$
k_x	constants of springs in x-direction
m	mass of the rigid frame
m_0	mass of the standard REM
m_i	mass of the REM i, $i = 1,2$
M	mass of the main vibrating body including the mass of the rigid frame m and that of two REMs, m_i, $i = 1,2$, $M = m + m_1 + m_2$
r_i	eccentric radius of two REMs, $r_i = r$, $i = 1,2$
r_m	ratio of mass between the standard REM (m_0) and the total system (M), $r_m = m_0/M$
T_{ej}	electromagnetic torque of the motor j, $j = 1,2$
T_{e0j}	electromagnetic torque of the induction motor operating steadily at the angular velocity ω_{m0}, $T_{e0j} = n_p \frac{L_{mj}^2/U_{s0}^2}{L_{sj}^2 R_{rj}} \frac{(\omega_s - n_p\omega_{m0})}{1+\sigma_j^2\tau_{rj}^2(\omega_s - n_p\omega_{m0})^2}$, $j = 1,2$
z_x	ratio of frequency between the operation frequency and the natural one, $z_x = \omega_{m0}/\omega_{nx}$
ω_{m0}	synchronous angular velocity of two REMs when the vibrating system operates in the steady state
ω_{nx}	natural frequency of a vibrating system in x-direction, $\omega_{nx} = \sqrt{k_x'/M}$
ξ_{nx}	critical damping ratio of a vibrating system in x-direction
$(\dot{\bullet})$	$d\bullet/dt$
$(\ddot{\bullet})$	$d^2\bullet/dt^2$

1. INTRODUCTION

Synchronization phenomena existing in nature have been giving rise to great interests among many scholars, such as synchronization and resonances in neuronal electrical activities [1], chaotic burst synchronization in neuronal network [2], synchronization in self-sustained electromechanical devices [3], and that in the other fields including neuronal electrical activities and communication engineering [4, 5], etc.

Vibrations in engineering generally are clarified by vibration suppression and utilization. As for the former, it is mainly related to the vibration isolation. For example, elimination of the vibration of shield tunnel boring machine cutterhead driving system [6] and dynamic vibration absorber was suspended to reduce vibration [7]. While in light of the vibration utilization engineering, synchronization theory of REMs, caused by vibrations, is widely used in various industries, including vibrating conveyors, feeders, dryers, coolers, screenings.

In connection with the synchronization of REMs, the most representative researchers are Blekhman [8, 9] and Wen [10–12]. The former gave firstly the theoretical explanation of two identical REMs by using the method of direct separation of motions and many engineering solutions were carried out. Wen further developed and completed such theory of synchronization in a vibrating system with small damping, and applied it to engineering successfully, for example, also the improvement of the processing effect of the largest self-synchronous vibrating screen in the world, and the optimization design of vibrating centrifugal dehydrators with hard nonlinear characteristics of springs, and so on.

According to the ratio of the operating frequency and natural frequency (this ratio is set as temporarily, z = operating frequency/natural frequency), vibrating mechanical systems, generally speaking, are classified by three types: sub-resonant ($z < 1$), near-resonant (z is in the vicinity of 1), and super-resonant ($z > 1$) vibrating system. Vibrating machines used in engineering generally operate by one of the above three resonant types to meet certain requirements. One of the main advantages of vibrating machines operating in a super-resonant state lies in the fact of its reliability of the amplitudes of responses. For that operating in a sub-resonant state, it can achieve the aim at saving energy. As referred to in [11], under the condition of the same amplitudes of responses, the exciting force in a sub-resonant state is only one-fifth to one-third of that in a super-resonant one. Thus, the power of motors is relatively decreased, and the energy is saved as well. It is, therefore, significant to study the synchronization theory of REMs in various resonant states, especially with nonlinear characteristics of springs. The nonlinear restoring forces of springs are the results of their behaviors associated with moderate deformations, which can generally be manifested by two types, hardening and softening characteristics. These characteristics can adjust and influence the stability and reliability of the amplitude of response. For the hardening, it can stabilize the amplitude of response in a sub-resonant or near sub-resonant state. Utilizing this principle, the ideal working point areas can be selected to improve the performances of a certain vibrating centrifugal dehydrators in engineering. While in light of the latter (softening), it can adjust the amplitude of response in a super-resonant or near super-resonant state, such that designing vibrating shakers complying with this principle.

Using the method of direct separation of motions, the synchronization theory of two identical REMs has been given by Blekhman [8, 9]. Based on the averaging method of small parameters, synchronization theory of two non-identical REMs was also further studied in the super-resonant vibrating system with linear springs [13]. Synchronization of two eccentric rotors driven by hydraulic motors in a vibrating system is studied by Zhang [14]. Synchronization theory of REMs in a sub-resonant or near-resonant vibrating system, especially with nonlinear springs, however, is less considered.

To make up the above drawbacks, in this paper, we focus on investigating the synchronization of two homodromy REMs in a nonlinear vibrating system (NVS), in which the NVS characteristics are referred to as nonlinear restoring forces of springs characterized by piecewise linear. Firstly, taking a dynamical model with two homodromy REMs, for example, the differential equations of motion are described. In the following, the nonlinear stiffness of springs is equivalently linearized as a function of the response of the system, which is followed by giving a criterion of implementing synchronization and stability. Numeric and simulation analyses are presented in Section 4. Finally, conclusions are provided.

2. DIFFERENTIAL EQUATIONS OF MOTION OF THE SYSTEM

Figure 1 shows a dynamical model of the considered vibrating system, in which the main vibrating body is connected to the foundation by springs and guide plate. Springs consist of two types ones:the coil spring k_x and the gap-activated spring Δk_x with the average gap e. Two REMs are mounted on m and driven by two co-rotating motors separately. Due to the limiting role of the guide plate, we can assume that the total system exhibits one degree of freedom along with x-direction, denoted by x; REMs 1 and 2 rotate their own spin axes, denoted by φ_1 and φ_2, respectively.

Substituting the kinetic energy of the system (T), the potential energy (V), and the viscous dissipation function (D), into the following Lagrange equation:

$$\frac{d}{dt}\frac{\partial(T-V)}{\partial \dot{q}_i} - \frac{\partial(T-V)}{\partial q_i} + \frac{\partial D}{\partial \dot{q}_i} = Q_i, \quad i = x, \varphi_1, \varphi_2 \tag{1}$$

If $\mathbf{q} = \{x, \varphi_1, \varphi_2\}^T$ is chosen as the generalized coordinates, the generalized forces are: $Q_x = 0$, $Q_{\varphi_1} = T_{e1}$, $Q_{\varphi_2} = T_{e2}$. According to Wen et al. [11], $m_i \ll m$. Hence, the inertia coupling stemming from asymmetry of two REMs can be neglected. Applying T, V, D to Eq. (1) yields the following differential equations of motion of the system:

$$\begin{aligned}
&M\ddot{x} + F_1(\dot{x}) + F_2(x) = m_1 r_1(\dot{\varphi}_1^2 \sin\varphi_1 - \ddot{\varphi}_1 \cos\varphi_1) + m_2 r_2(\dot{\varphi}_2^2 \sin\varphi_2 - \ddot{\varphi}_2 \cos\varphi_2) \\
&J_{01}\ddot{\varphi}_1 + f_1\dot{\varphi}_1 = T_{e1} - m_1 r_1 \ddot{x} \cos\varphi_1 \\
&J_{02}\ddot{\varphi}_2 + f_2\dot{\varphi}_2 = T_{e2} - m_2 r_2 \ddot{x} \cos\varphi_2
\end{aligned} \tag{2}$$

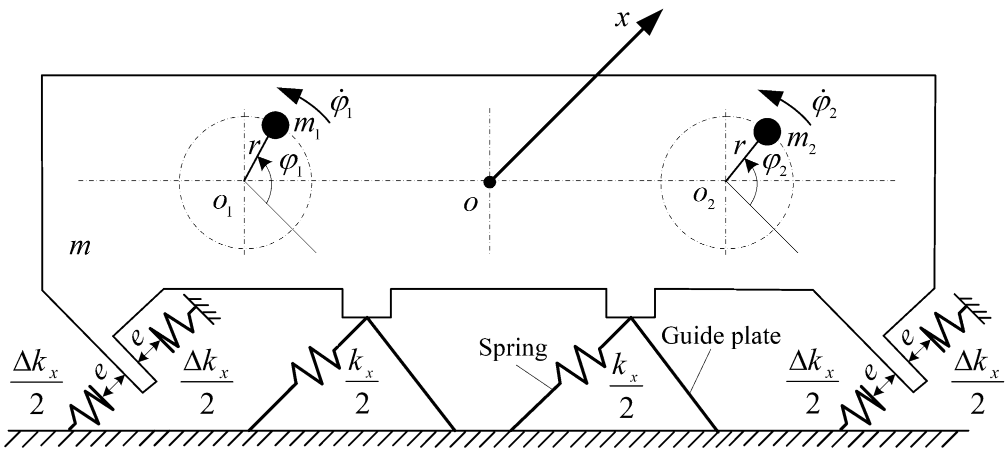

Fig. 1. Dynamical model of a nonlinear vibrating system.

where

$$M = m + m_1 + m_2, \quad J_{0j} = m_i r^2, \quad r_1 = r_2 = r$$

In Eq. (2), the nonlinear restoring force $F_2(x)$, is expressed as

$$F_2(x) = \begin{cases} k_x x, & -e \leq x \leq e \\ k_x x + \Delta k_x (x - e), & x \geq e \\ k_x x + \Delta k_x (x + e), & x \leq -e \end{cases} \tag{3}$$

It should be noted that the nonlinear damping force $F_1(\dot{x})$ in Eq. (2) has also weakly nonlinear characteristics in essence. In general, vibrating machines with very small damping, $F_1(\dot{x})$ can be replaced by the linear damping force $f_x \dot{x}$, i.e., $F_1(\dot{x}) = f_x \dot{x}$. Here, f_x is the equivalent damping coefficient, which can only be obtained by way of experiment [10, 11]. While in light of the nonlinear restoring force, $F_2(x)$, it must be equivalently linearized as follows.

The synchronous angular velocity of two exciters in the steady state is set as ω_{m0}. The average phase between two exciters and their phase difference are assumed to be φ and 2α, respectively, i.e.,

$$\varphi_1 = \varphi + \alpha, \quad \varphi_2 = \varphi - \alpha \tag{4}$$

Setting the equivalent stiffness of springs as k'_x, and considering $F_1(\dot{x}) = f_x(\dot{x})$, the first formula in Eq. (2) can be rewritten by

$$M\ddot{x} + f_x \dot{x} + k'_x(x) = m_1 r_1 (\dot{\varphi}_1^2 \sin\varphi_1 - \ddot{\varphi}_1 \cos\varphi_1) + m_2 r_2 (\dot{\varphi}_2^2 \sin\varphi_2 - \ddot{\varphi}_2 \cos\varphi_2) \tag{5}$$

The natural frequency of the system in x-direction is

$$\omega_{nx} = \sqrt{\frac{k'_x}{M}} \tag{6}$$

Based on the asymptotic method [10], the first approximate solution of Eq. (5) can be expressed as

$$x = \lambda_x \cos(\omega t + \theta) = \lambda_x \cos \nu \tag{7}$$

where

$$\omega = \omega_{m0}, \quad \theta = \operatorname{arccot}\left(\frac{f_x/2M}{\omega_{nx}^2 - \omega_{m0}^2}\right)$$

with θ being the phase angle of the response.

Denoting by ν_0 the root of the following equation:

$$e = \lambda_x \cos\nu \tag{8}$$

where $\nu_0 = \arccos(e/\lambda_x)$, $\lambda_x > 0$ and $\lambda_x > e$ in Eq. (8), which means that ν_0 corresponds to the value of ν for the initial condition of $x = e$ in Eq. (7).

Assuming that

$$F_2(x) = \varepsilon f_2(x) = \varepsilon f_2(\lambda_x \cos\nu) \tag{9}$$

where ε is the small parameter introduced by the asymptotic method.

Hence, the nonlinear restoring force $F_2(x)$ can be replaced by

$$\varepsilon f_2(\lambda_x \cos\nu) = \begin{cases} k_x\lambda_x\cos\nu, & \nu_0 \le \nu \le \pi - \nu_0 \text{ and } \pi + \nu_0 \le \nu \le 2\pi - \nu_0 \\ k_x\lambda_x\cos\nu + \Delta k_x(\lambda_x\cos\nu - e), & 2\pi - \nu_0 \le \nu \le 2\pi \text{ and } 0 \le \nu \le \nu_0 \\ k_x\lambda_x\cos\nu + \Delta k_x(\lambda_x\cos\nu + e), & \pi - \nu_0 \le \nu \le \pi + \nu_0 \end{cases} \tag{10}$$

The equivalent stiffness, therefore, can be derived as

$$k_x' = \frac{1}{\pi\lambda_x}\int_0^{2\pi} \varepsilon f_2(\lambda_x\cos\nu)\cos\nu d\nu \tag{11}$$

Considering Eq. (10), the equivalent stiffness for $\lambda_x > e$, k_x' can be presented in the form

$$k_x' = k_x + \frac{2\Delta k_x}{\pi}\left[\arccos\left(\frac{e}{\lambda_x}\right) - \left(\frac{e}{\lambda_x}\right)\sqrt{1 - \left(\frac{e}{\lambda_x}\right)^2}\right] \tag{12}$$

3. SYNCHRONIZATION OF TWO REMS AND STABILITY OF THE SYNCHRONOUS STATES

If two REMs can operate with the synchronous state, their synchronous angular velocity is $\dot\varphi = \omega_{m0}$. Furthermore, $\ddot\varphi_1$ and $\ddot\varphi_2$ can be neglected in the first formulae of Eq. (2) in the steady state. Considering Eq. (4) and assuming $m_1 = m_0$ and $m_2 = \eta m_0$ $(0 \le \eta \le 1)$, the first formula of Eq. (2) can be replaced by

$$M\ddot x + f_x\dot x + k_x' x = m_0 r\omega_{m0}^2[\sin(\varphi + \alpha) + \eta\sin(\varphi - \alpha)] \tag{13}$$

The response of Eq. (13) can be readily presented by

$$x = \frac{z_x^2 r_m r}{\sqrt{(1 - z_x^2)^2 + (2\xi_{ns}z_x)^2}}[\sin(\varphi + \alpha - \gamma_x) + \eta\sin(\varphi - \alpha - \gamma_x)] \tag{14}$$

where

$$\gamma_x = \begin{cases} \arctan\frac{2\xi_{nx}z_x}{1 - z_x^2}, & 1 - z_x^2 > 0 \\ \pi + \arctan\frac{2\xi_{nx}z_x}{1 - z_x^2}, & 1 - z_x^2 < 0 \end{cases} \qquad r_m = \frac{m_0}{M}, z_x = \frac{\omega_{m0}}{\omega_{nx}}, \omega_{nx} = \sqrt{\frac{k_x'}{M}}$$

3.1. Synchronization Criterion of Two REMs

Inserting the second-order derivative of Eq. (14), $\ddot{x}$, into the last two formulae of Eq. (2), considering Eq. (4) and then integrating them over $\varphi = 0 \sim 2\pi$, the average balanced equations of two motors can, after rearrangement, be given as

$$
\begin{aligned}
T_{e01} - f_1\omega_{m0} &= T_u \frac{r_m z_x^2}{z_x^2 - 1}[\sin\gamma_x + \eta\sin\gamma_x\cos(2\alpha) - \eta\cos\gamma_x\sin(2\alpha)] \\
T_{e02} - f_2\omega_{m0} &= T_u \frac{r_m z_x^2}{z_x^2 - 1}[\eta^2\sin\gamma_x + \eta\sin\gamma_x\cos(2\alpha) + \eta\cos\gamma_x\sin(2\alpha)]
\end{aligned} \tag{15}
$$

Here $T_u = m_0 r^2 \omega_{m0}^2/2$ denotes the energy of standard REM. The expression of T_{e0j} $(j = 1,2)$ can be found in the Nomenclature or in [13].

Addition and subtraction of the two formulae of Eq. (15), and rearranging them, yield Eqs. (16) and (17) as follows, respectively

$$T_{e01} + T_{e02} - (f_1 + f_2)\omega_{m0} = T_L \tag{16}$$

$$\sin(2\alpha) = \frac{T_D}{T_C} \tag{17}$$

where

$$T_L = \frac{T_u r_m z_x^2}{z_x^2 - 1}[\sin\gamma_x(1+\eta^2) + 2\eta\sin\gamma_x\cos(2\alpha)], \quad T_{R1} = T_{e01} - f_1\omega_{m0} - \frac{T_u r_m z_x^2}{z_x^2 - 1}\sin\gamma_x$$

$$T_{R2} = T_{e02} - f_2\omega_{m0} - \frac{T_u r_m z_x^2 \eta^2}{z_x^2 - 1}\sin\gamma_x, \quad T_C = -\frac{2T_u r_m \eta\cos\gamma_x z_x^2}{z_x^2 - 1}, \quad T_D = T_{R1} - T_{R2}$$

Equation (16) represents the torque balance equation of the total vibrating system operating in the steady state. T_L denotes the load torque acted on two motors; T_{R1} and T_{R2} are the residual torques of motors 1 and 2, respectively; T_D denotes their difference; and T_C denotes the capture frequency torque.

For Eq. (17), since $|\sin(2\alpha)| \le 1$, we have that the synchronization criterion of two REMs is

$$|T_C| \ge |T_D| \tag{18}$$

As such, the solution of the phase difference between two REMs reads

$$2\alpha = \arcsin\left(\frac{T_D}{T_C}\right) \tag{19}$$

There are two solutions with respect to 2α in Eq. (19): one is stable and the other is not stable, which will be discussed in the following sections. Here the stable 2α is denoted by $2\alpha^*$.

3.2. Stability Criterion of Two REMs

The kinetic energy (T) of the total system

$$T = M\dot{x}^2/2 \tag{20}$$

and the potential energy (V)

$$V = k_x' x^2/2 \tag{21}$$

Hamilton's average action amplitude over one periodic can be expressed by

$$I = \frac{1}{2\pi}\int_0^{2\pi}(T - V)d\varphi = \frac{1}{2\pi}\int_0^{2\pi}\left(\frac{1}{2}M\dot{x}^2 - \frac{1}{2}k_x' x^2\right)d\varphi \tag{22}$$

Table 1. Stability and motion types of the vibrating system.

Parameters of the system			Stability (2α)	States of the system	Motion of the system
$\eta \neq 1$	$z_x > 1$	$H < 0$	$2\alpha \in (\pi/2, 3\pi/2)$	Super-resonant state	Vibration stemming from partly negative superposition
	$z_x < 1$	$H > 0$	$2\alpha \in (-\pi/2, \pi/2)$	Sub-resonant state	Vibration resulting from partly positive superposition
$\eta = 1$	$z_x > 1$	$H < 0$	π	Super-resonant state	No vibration
	$z_x < 1$	$H > 0$	0	Sub-resonant state	Vibration stemming from completely positive superposition

According to Wen et al. [11], the stable synchronous solution of 2α should correspond to a minimum of Hamilton's average action amplitude. In other words, the second-order derivative of **I** should be greater than zero, i.e.,

$$\frac{d^2 I}{d\varphi^2} > 0 \tag{23}$$

Inserting Eqs. (20)–(22) into Eq. (23), the stability criterion of the synchronous states can be expressed as

$$H\cos(2\alpha) > 0 \tag{24}$$

where

$$H = \frac{T_u r_m z_x^2 \eta}{1 - z_x^2} \tag{25}$$

and H is defined as the coefficient of ability of stability, z_x is the ratio of frequency, $z_x = \omega_{m0}/\omega_{nx}$.

From Eq. (24), one can see that $z_x < 1$, which means that the phase difference between two REMs is stabilized at $2\alpha \in (-\pi/2, \pi/2)$, which belongs in a sub-resonant state. $z_x > 1$ represents that the stable phase difference $2\alpha \in (\pi/2, 3\pi/2)$, and the vibrating system is in a super-resonant state.

Particularly, when the parameters of two motors are completely identical (i.e., $\eta = 1$), 2α is stabilized in the vicinity of zero for $z_x < 1$, otherwise in the neighborhood of π, i.e.,

$$2\alpha^* = \begin{cases} 0, & H = \frac{T_u r_m z_x^2}{1 - z_x^2} > 0 \\ \pi, & H = \frac{T_u r_m z_x^2}{1 - z_x^2} < 0 \end{cases} \tag{26}$$

The above discussions are presented in a simplified way in Table 1.

4. NUMERIC DISCUSSIONS

In this section, some numerical discussions are given to verify the validity of the above theoretical results.

Here, two motors are assumed to be the same; the model is three-phase squirrel-cage (50 Hz, 380 V, 6-pole, 0.75 kW, rated speed 980 r/min). The parameters of two motors are: the rotor resistance $R_r = 3.40\ \Omega$, the stator resistance $R_x = 3.35\ \Omega$, the rotor inductance $L_r = 170$ mH, the stator inductance $L_x = 170$ mH, the mutual inductance $L_m = 164$ mH, and $f_1 = f_2 = 0.05$. Two REMs are identical, i.e., $\eta = 1$, $m_1 = m_2 = m_0 = 10$ kg; the other parameters of the vibrating system are: $r = 0.15$ m, $m = 1200$ kg, $f_x = 7.6$ kN·s/m, $\xi_{nx} = 0.07$, $e = 0.001$ m.

4.1. Numerical Discussions on Dynamical Characteristics

When two REMs operate in the steady state, according to Eq. (14), the amplitude of the response in x-direction can be simplified in the form

$$\lambda_x = \begin{cases} \frac{2 z_x^2 r_m r}{\sqrt{(1 - z_x^2)^2 + (2\xi_{nx} z_x)^2}}, & H > 0 \\ 0, & H < 0 \end{cases} \tag{27}$$

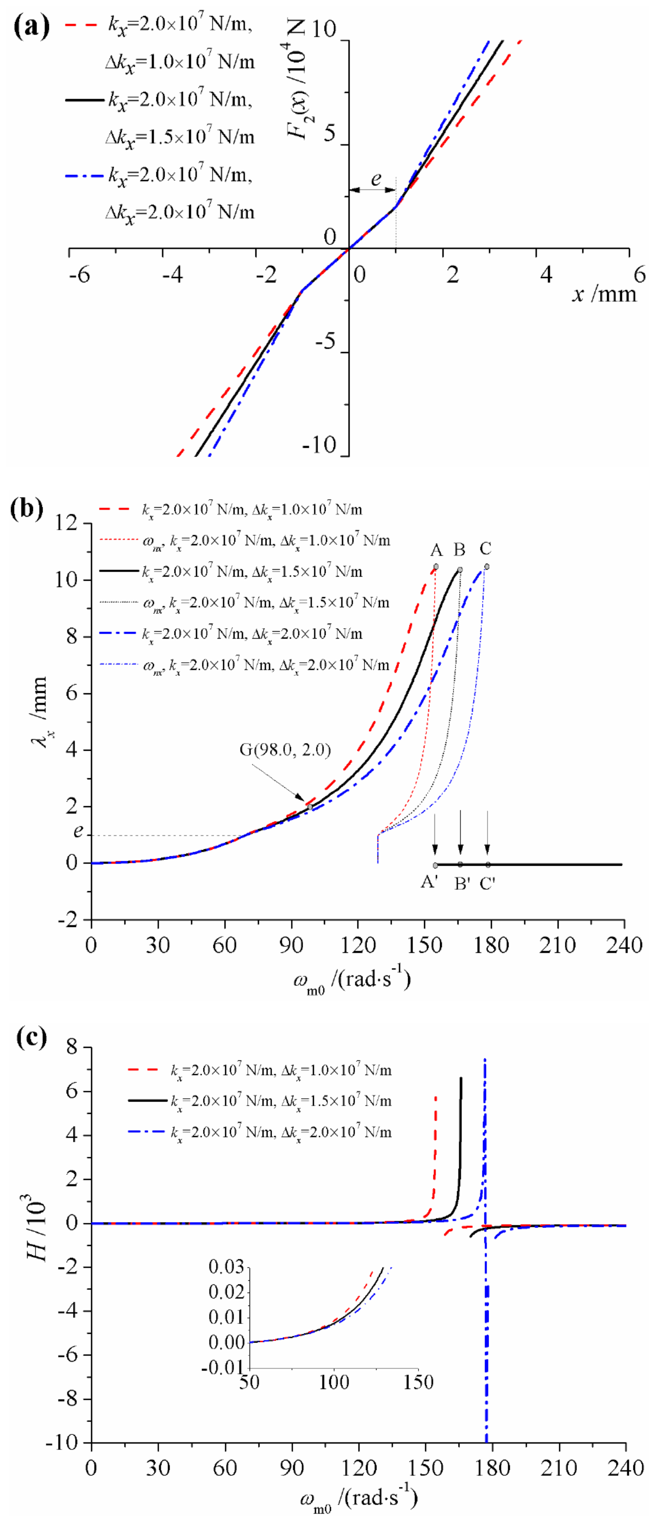

Fig. 2. Dynamical characteristic of the vibrating system versus Δk_x under the condition of $k_x = 2.0 \times 10^7$ N/m. (a) Restoring forces of springs versus displacements; (b) frequency-amplitude relationship in the steady state; (c) coefficient of ability of stability.

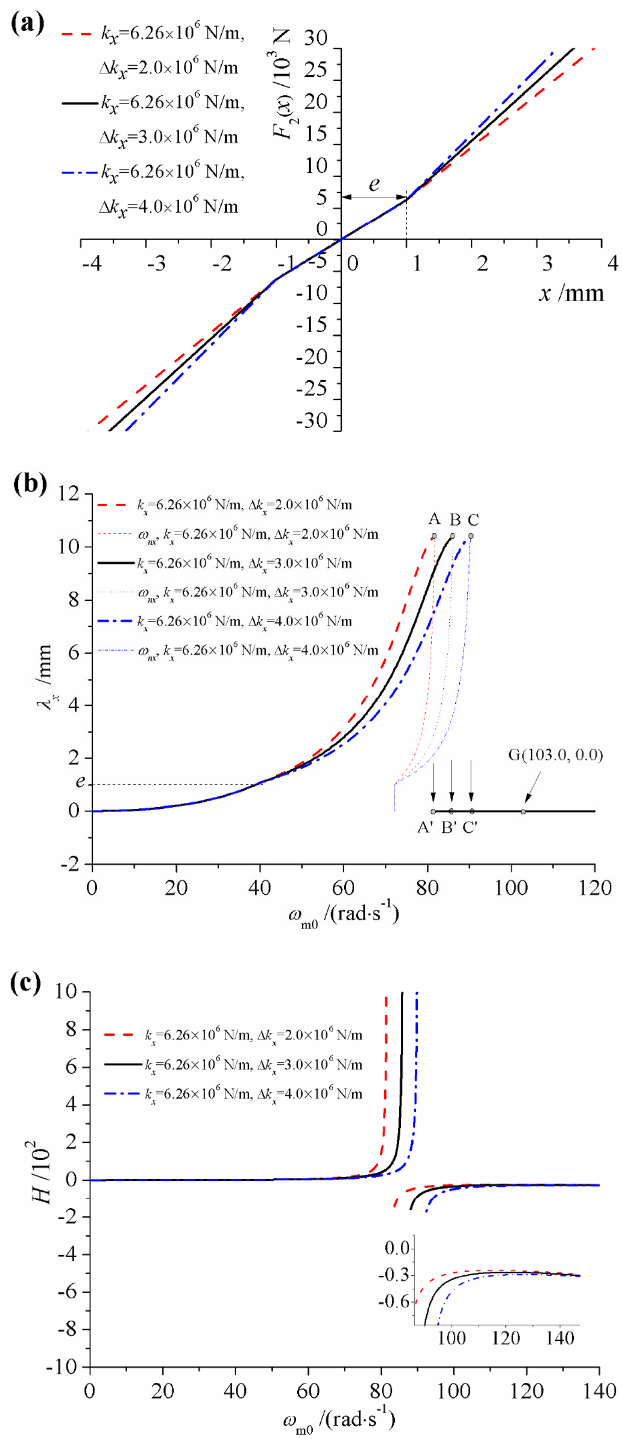

Fig. 3. Dynamical characteristic of the vibrating system versus Δk_x under the condition of $k_x = 6.26 \times 10^6$ N/m. (a) Restoring forces of springs versus displacements; (b) frequency-amplitude relationship in the steady state; (c) coefficient of ability of stability.

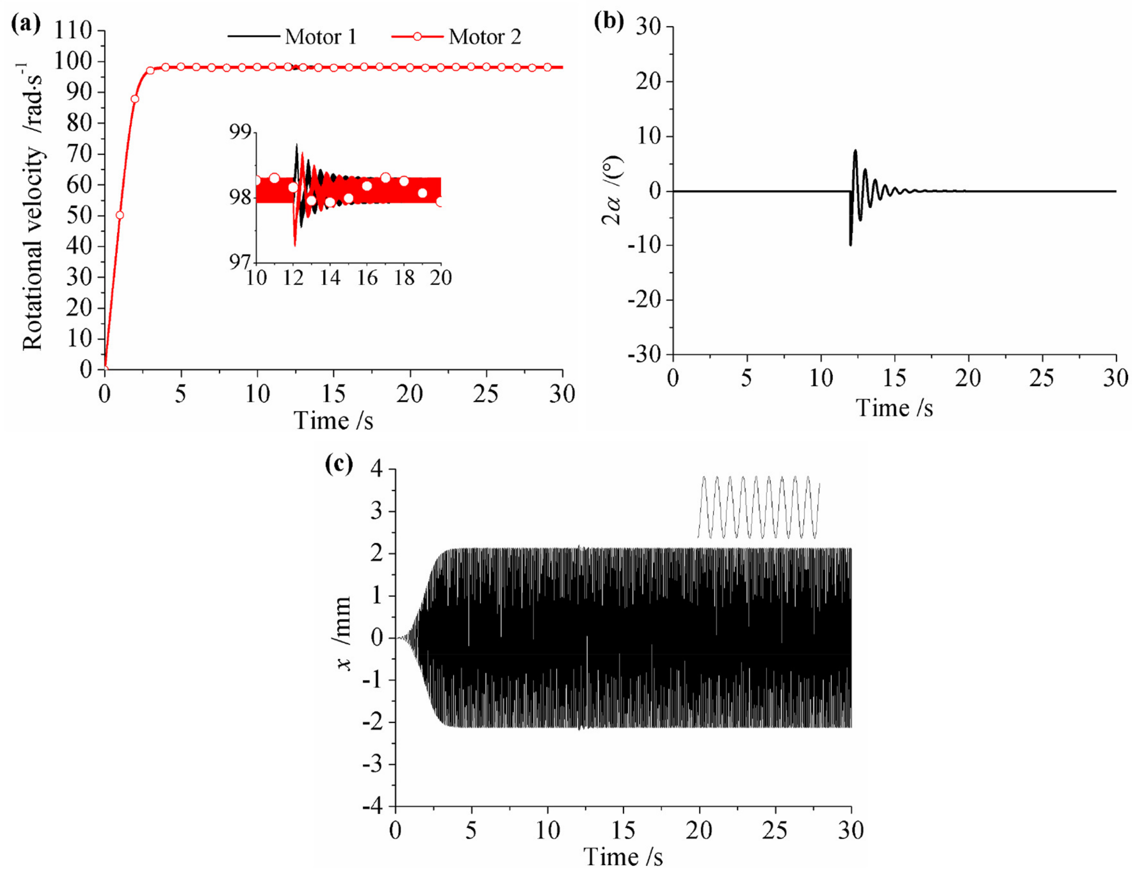

Fig. 4. Simulation results of two identical REMs in a sub-resonant state ($k_x = 2.0 \times 10^7$, $\Delta k_x = 1.5 \times 10^7$). (a) Rotational velocities of two REMs; (b) phase difference between two REMs ($2\alpha = \varphi_1 - \varphi_2$); (c) displacement in x-direction.

We can respectively fix $k_x = 2.0 \times 10^7$ N/m and $k_x = 6.26 \times 10^6$ N/m to discuss the dynamical characteristic of the vibrating system versus Δk_x, as shown in Figs. 2 and 3.

Based on Eq. (3), one can find that the nonlinear restoring forces of springs reflect obviously the hardening nonlinear characteristic, not softening one, as shown in Figs. 2(a) and 3(a).

From Eq. (27) we know that ω_{m0} is the function of λ_x. Solving Eq. (27), we obtain the frequency-amplitude relationship curve corresponding to the response of the system, as shown in Figs. 2(b) and 3(b), in which the hardening nonlinear characteristics of the system are obviously reflected. Taking the curve with $\Delta k_x = 1.5 \times 10^7$ N/m, for example, in Figs. 2(b) and 2(c), it is easy to locate the zones of useful and useless amplitudes in engineering. Thus, the interval GB response curve corresponds to the useful one, due to its greater and stable amplitude of the response. In this case, the coefficient of ability of stability, $H > 0$, as shown in Fig. 2(c), 2α is stabilized in the vicinity of zero, and two REMs operate in a sub-resonant state. At point B, there occurs a break in amplitude, and jump down from B to B', at this moment, the vibrating system comes into being in a super-resonant state, $H < 0$ in Fig. 2(c), 2α is stabilized in the neighborhood of π, and the response of the system is zero duo to the mutually offsetting of two exciting forces.

In Fig. 3, $k_x = 6.26 \times 10^6$, some corresponding results are similar with those in Fig. 2, but this will not be further discussed in this article.

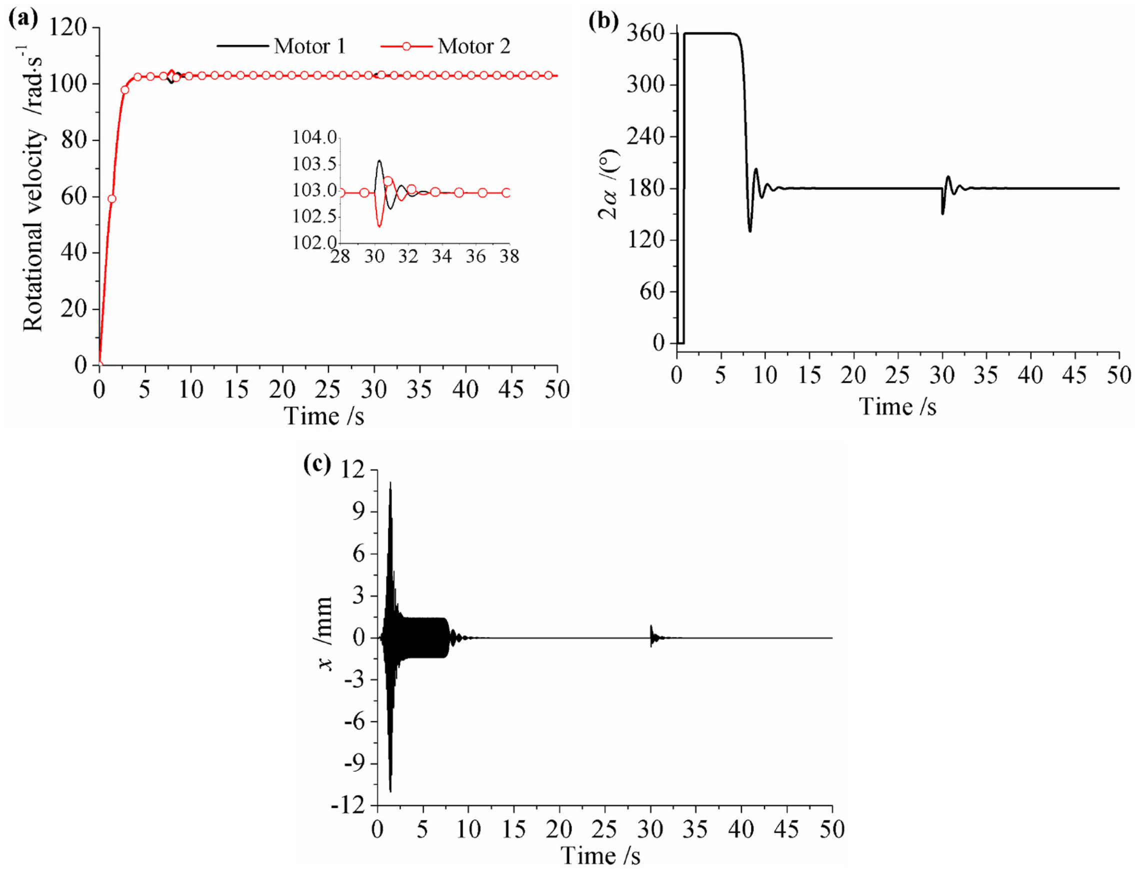

Fig. 5. Simulation results of two identical REMs in a super-resonant state ($k_x = 6.26 \times 10^6$, $\Delta k_x = 2.0 \times 10^6$). (a) Rotational velocities of two REMs; (b) phase difference between two REMs ($2\alpha = \varphi_1 - \varphi_2$); (c) displacement in x-direction.

4.2. Computer Simulations

In Section 4.1, some numerical discussions corresponding to theoretical analytical results in the simplified form, including restoring forces of nonlinear springs, frequency-amplitude relationship and ability of stability, are given.

In this section, two simulation results (in the sub-resonant and super-resonant states) are carried out by applying a forth order Runge–Kutta routine to Eq. (2), in order to compare quantitatively numerical results in Section 4.1.

In Fig. 4, $k_x = 2.0 \times 10^7$, $\Delta k_x = 1.5 \times 10^7$, two REMs operate in a sub-resonant state. During the starting process, the angular acceleration of motor 2 is equal to the other one since their inertial moments are identical. For the time being, under the precondition of the resonant phenomenon not occurring, two REMs begin to reach synchronization and stabilize rapidly. In this case, the stable states: $2\alpha = 0°$, the synchronously rotational velocity approaches 98.0 rad/s, as shown in Figs. 4(a) and 4(b), and the response in x-direction is the stronger sub-resonant harmonic vibration. These facts are the desired ones in engineering, as illustrated in Fig. 4(c). At 12 s, a disturbance of $\pi/18$ is added to motor 2 and, after about 4 s, the states of the system gradually return to the previous one, which indicate that the vibrating system has a very strong stability. The above facts agree approximately with what is shown in point G of Figs. 2(b) and 2(c).

As shown in Fig. 5, $k_x = 6.26 \times 10^6$, $\Delta_x = 2.0 \times 10^6$, two REMs operate in a super-resonant state. It should be noted that, at 30 s, a disturbance of $\pi/6$ is also added to motor 2. In stable and synchronous states, $2\alpha = 180°$, the synchronously rotational velocity approaches 103.0 rad/s, as shown in Figs. 5(a) and 5(b), which coincide with the numerical results in Fig. 3 (shown in Point G of Fig. 3b). In contrast with the results in Fig. 4, the two REMs in Fig. 5 are stabilized in the operation with the opposite phases, and the obvious transient phenomenon of passing through the resonant region of the system occurs obviously. Also, the corresponding amplitude of the response in *x*-direction of Fig. 5(c) is zero due to the mutual cancellation of two exciting forces, which is not desired in engineering.

5. CONCLUSIONS

Criteria of synchronization and stability should be firstly satisfied to assure synchronization of two REMs.

As for the vibrating system driven by two homodromy REMs, the phase difference is stabilized at $(-\pi/2, \pi/2)$ for being in a sub-resonant state, otherwise at $(\pi/2, 3\pi/2)$ which belongs to being in a super-resonant state. Particularly, when two REMs are identical, the stable phase difference in a sub-resonant state approaches zero, while that in a super-resonant state is close to π. In light of the former, the energy is saved, which is exactly the desire in engineering. Based on our experiences, the power supplies of motors of vibrating machines working in a sub-resonant state can be saved by some 40%.

In engineering, when two homodromy REMs are installed in short distance, the two REMs should be as identical as possible, and the working points of the vibrating machines should be selected in a sub-resonant state in order to obtain a stronger and more stable amplitude of response.

We can adjust the stiffness of nonlinear springs k_x and Δk_x to stabilize the amplitude of response of the system by utilizing the nonlinear characteristics of springs.

ACKNOWLEDGEMENTS

This project is supported by the Fundamental Research Funds for the Central Universities of China (N150304010) and National Natural Science Foundations of China (51375080).

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