RESEARCH ON THE FREQUENCY RELIABILITY SENSITIVITY AND RELIABILITY-BASED ROBUST DESIGN OF THE RANDOM VIBRATION SYSTEM OF CONTINUOUS ROD

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ABSTRACT

The frequency reliability sensitivity in the stochastic dynamic structure system and reliability-based robust design were studied deeply in this paper. With the criterion that the absolute value of the difference between natural frequency and forcing frequency, the frequency reliability method of avoiding resonance was proposed. Then frequency reliability sensitivity theory was presented, which provided a preliminary efficient way to analyze how each random parameter contributed to the system reliability. Moreover, the frequency reliability-based robust design method was obtained by robust and optimization technology on the basis of the frequency reliability and sensitivity research in the paper, which helped designers to establish acceptable parameter values and to determine the fluctuations of the parameters for the safe operations. Meanwhile, a numerical example of the random vibration system of continuous rod was provided and studied. The effectiveness and accuracy of the proposed methods were well demonstrated.

Keywords: resonance; frequency reliability; sensitivity; robust.

RECHERCHE SUR LA FIABILITÉ ET SENSIBILITÉ EN FONCTION DE LA FRÉQUENCE ET CONCEPTION ROBUSTE BASÉE SUR LA FIABILITÉ DE LA VIBRATION AXIALE ALÉATOIRE DES SYSTÈMES DE TIGES

RÉSUMÉ

La fiabilité et sensibilité en fonction de la fréquence dans la structure d'un système dynamique stochastique, et une conception robuste basée sur la fiabilité sont étudiées en profondeur dans cet article. Avec des critères de valeurs absolues de la différence entre la fréquence naturelle et la fréquence forcée, la méthode de fiabilité de fréquence permet d'éviter la résonnance. La théorie de la fiabilité et sensibilité en fonction de la fréquence est présentée ici. Cette théorie a apporté une façon préliminaire efficace d'analyser comment chaque paramètre aléatoire a contribué à la fiabilité d'un système. De plus, la méthode de conception robuste, basée sur la fiabilité est obtenue par une technologie d'optimisation robuste sur la base de recherche de la fiabilité et sensibilité en fonction de la fréquence. Laquelle a aidé les concepteurs dans l'établissement des valeurs des paramètres acceptables pour déterminer la fluctuation des paramètres pour la sécurité des opérations. Entretemps, un exemple numérique des vibrations aléatoires axiales des systèmes de tiges a été trouvé et analysé démontrant l'efficacité et l'exactitude des méthodes proposées.

Mots-clés : résonnance; fiabilité de la fréquence; sensibilité; robuste.

1. INTRODUCTION

Research on the stochastic dynamic structure system with random parameters is far more complex than research on the deterministic structure system. However, in recent decades, Benaroya and Rehak [1], Zhang et al. [2, 3] have made much progresses on the reliability research on the stochastic dynamic structure system based on the first-order second-moment method and stochastic finite element method (SFEM). Compared to the stochastic static structure system, stochastic dynamic structure system with random parameters are much more complex, and the vibration problem is more complex than the static problem, which bring considerable difficulties to analyze failure problem of the stochastic dynamic structure system, and study on the reliability problem is still at primary stage. In terms of the reliability of the stochastic dynamic structure system, although the reliability of the linear MDOF system with random parameters has been studied by Sara et al. [4], Kumar and Narayanan [5] proposed that reliability research on the nonlinear stochastic dynamic structure system has been limited to SDOF system by now, which is mainly because some effective methods of linear stochastic dynamic structure system are not valid for nonlinear one. So far, failure study on the stochastic dynamic structure system with random parameters have mainly two aspects, which are presented in [6–8]: one is based on the overrun structural response induced by the forced vibration, such as stress, displacement, etc., and the other is based on the structural fatigue caused by the resonance or non-resonant. Up to now, reliability study on the stochastic dynamic structure system depending on frequency is still in its infancy, so the further research is required.

The study of reliability sensitivity and robust is of great significance for the analysis of structural reliability, due to the different contribution of each random parameter to the system failure. Up to now, the study of the structural reliability sensitivity has made much progress. The basic concept of reliability sensitivity was put forward earlier in 1980s by Hohenbiche and Rackwitz [9], Bjerager and Krenk [10]; the further study of the structural reliability sensitivity has been completed, and the efficient and accurate computational reliability sensitivity methods have been presented by Sues and Cesare [11]. Mechanical system is very complex and the study of the reliability sensitivity is insufficient in comparison with simple mechanical construction, and exploratory work has been done by Zhang et al. [12] on the reliability sensitivity of nonlinear random vibration system; reliability sensitivity investigation of impact-rub for rotor-stator systems has been conducted by Zhang, et al. [13, 14], and frequency reliability sensitivity analysis for dynamic discrete structure system has been completed by Zhang et al. [15]. As far as the methods of reliability sensitivity were concerned, there are two paths in [16]: one is based on the moment method, and the other is numerical simulation on basis of Monte-Carlo. Reliability sensitivity analysis method based on the moment can directly get result of parameters, but it is strongly dependent on the state function mode; and reliability sensitivity analysis method based on the Monte-Carlo numerical simulation simulates the state function distribution, and then the sensitivity analysis can be completed.

Robust design aims at improving product quality and reducing costs by adjusting the design variables and controlling tolerance, at the same time, robust design can make the product insensitive to certain designed variables, that is to say, the product can withstand random factors interference to a certain extent which is not expected. Combining reliability method with robust design approach, the product has not only the best performance and sufficient reliability, but also the stability of reliability under the interference of various random factors. This robust design approach based on the results of reliability is a major leap of product quality, and has become the important research subject both at home and abroad. Currently, the robust design methods, which are studied by Chen [17], Lee et al. [18], Lu and Zhang [19], can be broadly divided into two categories: one is the traditional robust design method based on experimental design, with a representative of the Taguchi method, response surface methodology, the dual response surface method etc., the other is robust optimization design method based on engineering models and optimization techniques, with a representative of generalized linear model approach , tolerance polyhedron approach, sensitivity method, variation transfer method, stochastic model approach etc. The structural robust theory based on the reliability and sensitivity is so important that some scientists have focused their attention on this in recent years.

Although the theory and application of the reliability sensitivity and robust design have made remarkable achievements and have been widely used in the engineering practice, the approach of the stochastic dynamic structure reliability sensitivity and robust design based on frequency have seldom been put forward, so further research is necessary.

In this paper, the failure model and the failure probability of the random systems based on frequency are firstly defined by mechanical kinetics theory, stochastic finite element method and reliability theory, and the formulas of the frequency reliability are obtained. Based on the frequency reliability theory, the method of reliability sensitivity avoiding resonance for multi-degree-of-freedom stochastic dynamic structure system is proposed using the stochastic perturbation method, reliability theory and sensitivity technology. Furthermore, the approach on the frequency reliability-based robust is presented based on the presented frequency reliability and frequency reliability sensitivity theory. At last, a simplified example of random vibration system of continuous rod is provided and studied, which demonstrate the effectiveness and accuracy of the proposed method. The theoretical methods can provide the reasonable and necessary basis for the design, manufacturing, application and evaluation of stochastic dynamic structure systems.

2. THEORETICAL RESEARCH

2.1. Resonant Frequency Reliability

A great many of responses do not exceed the thresholds, however, the system may encounter resonance, which can cause the failure of structure system, or the state of structure system is in what may be called the quasi-failure state. In this way, the structure system is guaranteed from avoiding resonance with appropriated probability. According to the reliability theory, the state function of the stochastic dynamic structure system is defined as

$$G_{ij}(p_j, \omega_i) = |p_j - \omega_i| \quad (i = 1, 2, \ldots, n; j = 1, 2, \ldots, m), \tag{1}$$

where p_j is the jth forcing frequency. According to the relation criterion of natural frequency ω_i and forcing frequency p_j, the quasi-failure state of structure system is represented as

$$G_{ij}(p_j, \omega_i) = |p_j - \omega_i| \leq \gamma, \tag{2}$$

where γ is a specified range. Assuming $g_{ij} = p_j - \omega_i$, the mean value and the variance of g_{ij} are

$$\mu_{g_{ij}} = E(g_{ij}) = E(p_j) - E(\omega_i), \tag{3}$$

$$\sigma^2_{g_{ij}} = \mathrm{Var}(g_{ij}) = \sigma^2_{p_j} + \sigma^2_{\omega_i}. \tag{4}$$

The probability of the quasi-failure state is

$$P_f^{ij} = P^{ij}(-\gamma \leq g_{ij} \leq \gamma). \tag{5}$$

Generally, structural parameters are normal distribution, which can bring about easiness to reliability analysis and design of structure systems. Obviously, when forcing frequency and natural frequency are respectively normal distribution, the quasi-failure probability is

$$P_f^{ij} = \Phi\left(\frac{\gamma - \mu_{g_{ij}}}{\sigma_{g_{ij}}}\right) - \Phi\left(\frac{-\gamma - \mu_{g_{ij}}}{\sigma_{g_{ij}}}\right), \tag{6}$$

where $\Phi(\cdot)$ is the standardized normal distribution function. As shown previously, it is only hypothetical that forcing frequency and natural frequency are respectively normal distribution. It depends on many experimental statistics if the distribution is accurately confirmed. However, normal distribution is the most common and the first-selected distribution to the engineering probability analysis.

In structure system, resonance has occurred when any forcing frequency is in the vicinity of any natural frequency. The whole systems are considered to be in quasi-failure states. Thus, the structure system, to which forcing frequencies and natural frequencies are applied to analyze quasi-failure state, is a series system. The probability of the quasi-failure state of the entire system is represented as

$$P_f = 1 - \Pi_{i=1}^{n}\Pi_{j=1}^{m}(1 - P_f^{ij}). \tag{7}$$

The frequency reliability of the entire system is

$$R = 1 - P_f = \Pi_{i=1}^{n}\Pi_{j=1}^{m}R_{ij} = \Pi_{i=1}^{n}\Pi_{j=1}^{m}(1 - P_f^{ij}). \tag{8}$$

2.2. Resonant Frequency Reliability Sensitivity

The derivatives of the mean value and the variance of the random parameter $\bar{X}$, the frequency reliability sensitivity of the stochastic dynamic structure system can be obtained respectively as follows:

$$\begin{aligned}\frac{DR}{D\bar{X}^T} \;=\; & \sum_{u=1}^{n}\sum_{v=1}^{m}\left(\Pi_{i=1}^{n}\Pi_{j=1}^{m}\frac{R_{ij}}{R_{uv}}\right)\left\{\frac{\partial R_{uv}}{\partial \beta_{uv1}}\left[\frac{\partial \beta_{uv1}}{\partial \mu_{g_{uv}}}\left(\frac{\partial \mu_{g_{uv}}}{\partial \bar{\omega}_u}\frac{\partial \bar{\omega}_u}{\partial \bar{X}^T}\right)+\frac{\partial \beta_{uv1}}{\partial \gamma_u}\left(\frac{\partial \gamma_u}{\partial \bar{\omega}_u}\frac{\partial \bar{\omega}_u}{\partial \bar{X}^T}\right)\right]\right.\\ & \left.+\frac{\partial R_{uv}}{\partial \beta_{uv2}}\left[\frac{\partial \beta_{uv2}}{\partial \mu_{g_{uv}}}\left(\frac{\partial \mu_{g_{uv}}}{\partial \bar{\omega}_u}\frac{\partial \bar{\omega}_u}{\partial \bar{X}^T}\right)+\frac{\partial \beta_{uv2}}{\partial \gamma_u}\left(\frac{\partial \gamma_u}{\partial \bar{\omega}_u}\frac{\partial \bar{\omega}_u}{\partial \bar{X}^T}\right)\right]\right\},\end{aligned} \tag{9}$$

$$\begin{aligned}\frac{DR}{D\mathrm{Var}(X)} \;=\; & \sum_{u=1}^{n}\sum_{v=1}^{m}\left(\Pi_{i=1}^{n}\Pi_{j=1}^{m}\frac{R_{ij}}{R_{uv}}\right)\left\{\frac{\partial R_{uv}}{\partial \beta_{uv1}}\left[\frac{\partial \beta_{uv1}}{\partial \sigma_{g_{uv}}}\left(\frac{\partial \sigma_{g_{uv}}}{\partial \sigma_{\omega_u}}\frac{\partial \sigma_{\omega_u}}{\partial \mathrm{Var}(X)}\right)\right]\right.\\ & \left.+\frac{\partial R_{uv}}{\partial \beta_{uv2}}\left[\frac{\partial \beta_{uv2}}{\partial \sigma_{g_{uv}}}\left(\frac{\partial \sigma_{g_{uv}}}{\partial \sigma_{\omega_u}}\frac{\partial \sigma_{\omega_u}}{\partial \mathrm{Var}(X)}\right)\right]\right\},\end{aligned} \tag{10}$$

where

$$\beta_{uv1} = \frac{\gamma_u - u_{g_{uv}}}{\sigma_{g_{uv}}}, \tag{11}$$

$$\beta_{uv2} = \frac{-\gamma_u - u_{g_{uv}}}{\sigma_{g_{uv}}}, \tag{12}$$

$$\frac{\partial R_{uv}}{\partial \beta_{uv1}} = -\varphi(\beta_{uv1}) = -\frac{1}{\sqrt{2\pi}}e^{-\frac{\beta_{uv1}^2}{2}}, \tag{13}$$

$$\frac{\partial R_{uv}}{\partial \beta_{uv2}} = \varphi(\beta_{uv2}) = \frac{1}{\sqrt{2\pi}}e^{-\frac{\beta_{uv2}^2}{2}}, \tag{14}$$

$$\frac{\partial \beta_{uv1}}{\partial \mu_{g_{uv}}} = \frac{\partial \beta_{uv2}}{\partial \mu_{g_{uv}}} = \frac{-1}{\sigma_{g_{uv}}}, \tag{15}$$

$$\frac{\partial \beta_{uv1}}{\partial \gamma_u} = \frac{1}{\sigma_{g_{uv}}}, \tag{16}$$

$$\frac{\partial \beta_{uv2}}{\partial \gamma_u} = -\frac{1}{\sigma_{g_{uv}}}, \tag{17}$$

$$\frac{\partial \mu_{g_{uv}}}{\partial \bar{\omega}_u} = -1, \tag{18}$$

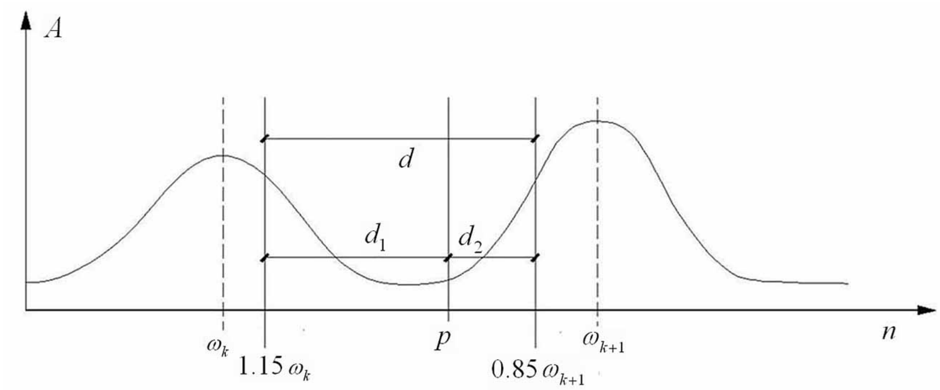

Fig. 1. Distance of reliability.

$$\frac{\partial \gamma_u}{\partial \bar{\omega}_u} = 0.1 - 0.15, \tag{19}$$

$$\frac{\partial \beta_{uv1}}{\partial \sigma_{g_{uv}}} = \frac{u_{g_{uv}} - \gamma_u}{\sigma^2_{g_{uv}}}, \tag{20}$$

$$\frac{\partial \beta_{uv2}}{\partial \sigma_{g_{uv}}} = \frac{u_{g_{uv}} + \gamma_u}{\sigma^2_{g_{uv}}}, \tag{21}$$

$$\frac{\partial \sigma_{g_{uv}}}{\partial \sigma_{w_u}} = \frac{\sigma_{w_u}}{\sqrt{\sigma^2_{p_v} + \sigma^2_{w_u}}}, \tag{22}$$

$$\frac{\partial \sigma_{\omega_u}}{\partial \mathrm{Var}(X)} = \frac{1}{2\sigma_{\omega_u}} \left(\frac{\partial \bar{\omega}_u}{\partial X} \right)^2. \tag{23}$$

When the given values and reliability analysis results are substituted into Eqs. (9–23), the reliability sensitivity $DR/D\bar{\mathbf{X}}^T$ and $DR/D\mathrm{Var}(X)$ are obtained.

2.3. Resonant Frequency Reliability-Based Robust Design

In terms of reliability-based robust design problem, the first step is to deal with reliability requirement of the structure system (i.e. reliability index), which can be either incorporated into the objective function and treated as a multi-objective optimization problem, or incorporated into the constraints of the optimization problem and treated as inequality constraints. Then the optimization method is used and the optimal solutions of parameters can be obtained. It can guarantee that the designed stochastic dynamic structure system is insensitive to the fluctuations of parameters (i.e. robustness), that is to say, when tiny fluctuation of parameters occurs, the robustness of the system performance is ensured in the processing and using course.

Based on the basic optimization model, the frequency reliability index is treated as inequality constraint in this paper, the frequency reliability sensitivity is attached to the objective function and the frequency reliability-based robust design is reduced to some multi-objective optimization problem which can meet the reliability requirement.

Suppose that an external excitation force is applied to the stochastic dynamic structure system, in which γ is 0.15 times of its corresponding natural frequency. As can be seen from Fig. 1, different rotating speeds correspond to different amplitudes. The amplitude of the forcing vibration is important in the engineering practice. If the amplitude exceeds the allowable limit, large alternating stress will be caused which can result in fatigue failure or have a bad influence on the performance of the structure system. Therefore, it is necessary to restrict the maximum amplitude when designing a stochastic dynamic structure system. The resonance range should be limited by means of that the difference between the natural frequency and the forcing frequency is large enough, where the maximum amplitude of a forcing vibration occurs. That is to say, the non-resonant interval range (reliability interval d in Fig. 1) should be adequately large to ensure safe and reliable operation of the system. Thus, the mathematical model of frequency reliability-based robust design can be expressed as

$$\left.\begin{array}{ll} \min f(\bar{\mathbf{X}}) = \sum_{k=1}^{n} w_k f_k(\bar{\mathbf{X}}) & \\ \text{S.t.}\Pi_{u=1}^{n}\left[1-\Phi\left(\frac{\gamma_u - u_{g_u}}{\sigma_{g_u}}\right)+\Phi\left(\frac{-\gamma_u - u_{g_u}}{\sigma_{g_u}}\right)\right] \geq R_0 & \\ q_i(\bar{\mathbf{X}}) \geq 0 & (i=1,\ldots,l) \\ h_j(\bar{\mathbf{X}}) = 0 & (j=1,\ldots,m) \end{array}\right\} \quad (24)$$

The objective function in Eq. (24) usually includes three sub-objective functions, namely $f_1(\bar{\mathbf{X}}), f_2(\bar{\mathbf{X}})$ and $f_3(\bar{\mathbf{X}})$. $f_1(\bar{\mathbf{X}})$ and $f_2(\bar{\mathbf{X}})$ are the differences between the forcing frequencies and the boundary points of the nearest resonance intervals, and both are negative, i.e. $f_1(\bar{\mathbf{X}}) = 1.15\omega_k - p$, $f_2(\bar{\mathbf{X}}) = p - 0.85\omega_{k+1}$, and $f_3(\bar{\mathbf{X}})$ is the square root of the squares sum which is sensitivity with respect to the mean values of the design parameters vector $\mathbf{X} = [x_1, x_2, \ldots, x_m]^T$, namely

$$f_3(\bar{\mathbf{X}}) = \sqrt{\sum_{i=1}^{m}\left(\frac{\partial R}{\partial x_i}\right)^2}.$$

When the forcing frequency is less than 0.85 times of the first natural frequency of the system, only two sub-objective functions are taken, one is $f_3(\bar{\mathbf{X}})$ which is the same as above, and the other sub-objective function is $f(\bar{\mathbf{X}}) = p - 0.85\omega_1$. w_k is the weighted factor of the objective function $f_k(\bar{\mathbf{X}})$, which depends on the order of magnitude and importance of corresponding sub-objective function. In this paper, w_k is determined by image set method of the weighted combination method, namely

$$\left.\begin{array}{l} w_1 = \dfrac{f_k(x^{*1}) - f_k(x^{*k})}{[f_1(x^{*k}) - f_1(x^{*1})] + [f_2(x^{*(k-1)}) - f_2(x^{*2})] + \cdots + [f_k(x^{*1}) - f_k(x^{*k})]} \\ w_2 = \dfrac{f_{k-1}(x^{*2}) - f_{k-1}(x^{*(k-1)})}{[f_1(x^{*k}) - f_1(x^{*1})] + [f_2(x^{*(k-1)}) - f_2(x^{*2})] + \cdots + [f_k(x^{*1}) - f_k(x^{*k})]} \\ \vdots \\ w_k = \dfrac{f_1(x^{*k}) - f_1(x^{*1})}{[f_1(x^{*k}) - f_1(x^{*1})] + [f_2(x^{*(k-1)}) - f_2(x^{*2})] + \cdots + [f_k(x^{*1}) - f_k(x^{*k})]} \end{array}\right\}, \quad (25)$$

where the subscript k is the number of sub-objective functions, x^{*k} is the optimal point obtained from the kth sub-objective function, and $f_k(\cdot)$ is the value of the kth sub-objective function.

3. NUMERICAL EXAMPLE

The simplified example of the random vibration system of continuous rod is described as below.

The top of the rod is fixed, and a motor with the gross mass (M), the rotate speed (n), the eccentric mass (m) and the eccentricity (e) is attached to the bottom of the rod. The parameters of the rod are the unit volume mass $(\rho(x))$, the length (L), the sectional area $(A(x))$ and elastic modulus (E) respectively. The random parameters ρ, A, L, M, E and P are the mutual independent random variables. The statistic quantities of

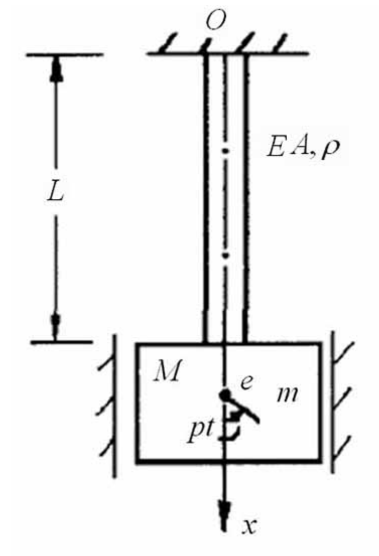

Fig. 2. System model.

the random parameters are $(\mu_\rho, \sigma_\rho) = (7.8\times10^3, 3.9\times10^2)$ kg/m^3, $(\mu_A, \sigma_A) = (5\times10^{-5}, 2.5\times10^{-7})$ m^2, $(\mu_L, \sigma_L) = (2, 0.01)$ m, $(\mu_M, \sigma_M) = (0.85, 0.0425)$ kg, $(\mu_E, \sigma_E) = (2.1\times10^{11}, 1.05\times10^{10})$ pa and $(\mu_n, \sigma_n) = (18000, 900)$ r/min respectively. From experience, the specified range of should be chosen between 10 and 15% of the mean value of the relevant natural frequency.

(1) *Analysis on frequency reliability and sensitivity*
The differential equation of the natural frequency is

$$\frac{\rho AL}{M} = \frac{\omega_i L}{a}\tan\frac{\omega_i L}{a} \quad (i = 1, 2, \ldots, n), \tag{26}$$

where $a = \sqrt{E/\rho}$.

The explicit expression formula of the natural frequency can be matched by Back Propagation network.

The quasi-failure probabilities of the subsystems are respectively

$$P_f^{11} = 0.0179, \quad P_f^{21} = 0, \quad P_f^{31} = 0, \quad \ldots. \tag{27}$$

The quasi-failure probability of the entire system is

$$P_f^{11} = 0.0179. \tag{28}$$

That is, the reliability of the entire system is

$$R = 0.9821. \tag{29}$$

From the above conclusions, it is shown that system resonance may not occur.

The reliability sensitivity of structural system is

$$\frac{DR}{D\bar{\mathbf{X}}^T} = \begin{bmatrix} DR/D\rho \\ DR/DA \\ DR/DL \\ DR/DM \\ DR/DE \end{bmatrix} = \begin{bmatrix} 0.0061218 \\ 0.0122682 \\ 0.0116828 \\ -0.01091556 \\ 0.0225144 \end{bmatrix}. \tag{30}$$

(a)

(b)

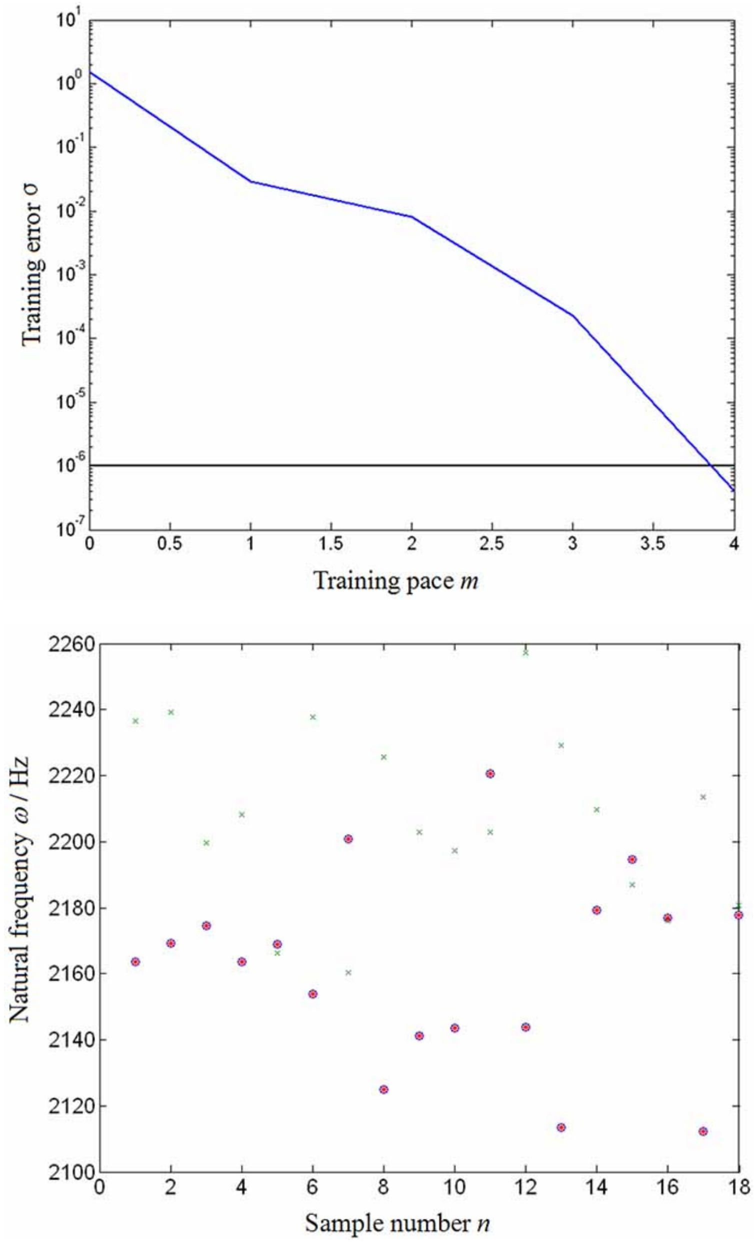

Fig. 3. Data fitting.

The differences of the sensitivity values indicate the different influence on the structure system reliability. In general, the higher absolute values show greater effect on system reliability, which is helpful to control the big parameters for us when we study the robustness of the stochastic dynamic structure system based on the reliability sensitivity. As can be seen from the sensitivity matrix $DR/D\bar{\mathbf{X}}^T$), when the mean values of the unit volume mass (ρ), the sectional area (A), the length (L) and elastic modulus (E) increase, the system will become more reliable, and vice versa. However, the influence of the gross mass (M) on system reliability is opposite, that is, the system reliability will decrease with the increase of the mean value of M. Additionally, we can see that the most significant variables are A, L, and M as compared with the other parameters, and they are the dominant variables with regard to sensitivity. In other words, the system

Table 1. Numerical results before and after robust design

Name	Before	After
R	0.9821	0.99987
$DR/D\rho$	0.0061218	0.16620e-5
DR/DA	0.0122682	0.08781e-5
DR/DL	0.0116828	0.00016e-5
DR/DM	–0.01091556	–0.00082e-5
DR/DE	0.0225144	0.09657e-5

reliability is more sensitive to these parameters.

(2) *Analysis on frequency reliability-based robust*
Considering the forcing frequency is 0.85 times less than the first-order natural frequency, only two sub-objective functions are taken in Eq. (24), which are respectively

$$f_1(\bar{\mathbf{X}}) = p - 0.85\omega_1, f_2(\bar{\mathbf{X}}) = \sqrt{\left(\frac{\partial R}{\partial \rho}\right)^2 + \left(\frac{\partial R}{\partial A}\right)^2 + \left(\frac{\partial R}{\partial L}\right)^2 + \left(\frac{\partial R}{\partial M}\right)^2 + \left(\frac{\partial R}{\partial E}\right)^2},$$

reliability constraint is $R_0 = 0.999$, and the inequality constraints of the random parameters are

$$\begin{cases} 6630 \le \rho \le 8970 \\ 4.925e-5 \le A \le 5.075e-5 \\ 1.97 \le L \le 2.03 \\ 0.7225 \le M \le 0.9775 \\ 1.785e11 \le E \le 2.415e11 \end{cases} \tag{31}$$

After the robust design, the means of the random parameters are respectively

$$\begin{cases} \rho = 7.8e3 \text{ kg/m}^3 \\ A = 0.5075e-4 \text{ m}^2 \\ L = 2.0037 \text{ m} \\ M = 0.8659 \text{ kg} \\ E = 2.1e11 \text{ pa} \end{cases} \tag{32}$$

The reliability and sensitivity can be calculated according to the values of the random parameters which are obtained by the robust modification.

$$R = 0.99987, \tag{33}$$

$$\frac{DR}{D\bar{\mathbf{X}}^T} = \begin{bmatrix} DR/D\rho \\ DR/DA \\ DR/DL \\ DR/DM \\ DR/DE \end{bmatrix} = \begin{bmatrix} 0.16620\text{e-5} \\ 0.08781\text{e-5} \\ 0.00016\text{e-5} \\ -0.00082\text{e-5} \\ 0.09657\text{e-5} \end{bmatrix}. \tag{34}$$

As can be seen from Table 1, the reliability of the system (R) becomes larger after the frequency reliability-based robust design, which indicates that system reliability increase. By comparing the frequency reliability sensitivity $(DR/D\bar{\mathbf{X}}^T)$ before with those after the frequency reliability-based robust design, we can see that the sensitivity value is significantly smaller after robust design, which indicates that the influence of the variation of design variables on system reliability is less sensitive than before, so the system becomes much more stable and robust.

4. CONCLUSIONS

(1) The failure mode of stochastic dynamic structure system was defined based on frequency, and the frequency reliability method of avoiding resonance was proposed by mechanical dynamics, stochastic finite element method and reliability theory in the paper. Furthermore, the quantitative analysis method how each random parameter contributed to the system reliability, namely the frequency reliability sensitivity method was presented based on the frequency reliability research. What's more, on the basis of the quantitative analysis results of frequency reliability sensitivity, the frequency reliability-based robust design approach was studied by reliability theory, sensitivity technology and optimization method, which can help designers to establish acceptable parameter values and to determine the fluctuations of the parameters for the safe operations.

(2) The frequency reliability and its related theories were applied to the random vibration system of continuous rod, the numerical results of reliability (R), reliability sensitivity ($DR/D\bar{\mathbf{X}}^T$) before and after reliability-based robust design were presented in the paper, which demonstrated that the proposed methods were efficient and accurate. The research can provide reasonable and necessary theoretical basis for the design, manufacture, operating and evaluation of the stochastic dynamic structure system.

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