VIBRATION OF ROAD BRIDGES UNDER MOVING VEHICLES: A COMPARATIVE STUDY BETWEEN SINGLE CONTACT POINT AND TWO CONTACT POINT MODELS

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ABSTRACT
The study of dynamic response of bridges under the action of moving vehicles provides a tool for structural designing as well as damage identification. Bridge-vehicle interaction is generally modelled through single contact point for mathematical simplicity, although the interaction actually takes place through front and rear wheels. In the present work, dynamic deflection of the bridge modelled through two contact points is investigated for a range of vehicle velocity and wheel to wheel distance. It is observed that for wheelbase to bridge span ratio greater than a certain limit, two contact point model gives significantly different results particularly at higher velocity range. The results, presented in terms of non-dimensional parameters, can be used as a design monograph for any bridge-vehicle-velocity combination.

Keywords: bridge-vehicle interaction; moving mass on a beam; dynamics of road bridges.

VIBRATION DES PONTS ROUTIERS SOUS LES VÉHICULES EN MOUVEMENT : UNE ÉTUDE COMPARATIVE ENTRE UN POINT DE CONTACT UNIQUE ET DEUX MODÈLES DE POINT DE CONTACT

RÉSUMÉ
L'étude de la réponse dynamique des ponts sous l'action des véhicules en mouvement fournit un outil pour la conception structurelle ainsi que l'identification des dommages. L'interaction véhicule-pont est généralement modélisée au moyen d'un point de contact unique pour la simplicité mathématique, bien que l'interaction passe par le contact des roues avant et arrière. Dans le présent travail, la déviation dynamique du pont modélisée par deux points de contact est étudiée pour une de vitesse de véhicule variée ainsi que l'empattement du véhicule. On observe que pour une distance plus grande entre les essieux que le ratio de la dimension de la travée du pont il y a une certaine limite. Deux points de contact modèle donnent des résultats assez différents, en particulier quand la vitesse est plus élevée. Les résultats, présentés en termes de paramètres sans dimension, peuvent être utilisés comme monographie de conception pour n'importe quelle combinaison véhicule-pont-vitesse.

Mots-clés : interaction véhicule-pont; masse en mouvement sur une poutre; dynamique des ponts routiers.

1. INTRODUCTION

The study of dynamic response of highway bridges under the action of moving vehicles has become an important research area as it not only helps in structural designing of the bridges, it also provides a tool for damage identification and structural health monitoring. Initially the bridge-vehicle interaction was modelled as a moving load problem [1], where only the weight of the vehicle was considered as a moving force. Later on the model was improved [2, 3] by including the moving mass inertia effects, i.e., inertial, centripetal and coriolis effects. Vibration response was considered in the form of an asymptotic series using modal analysis and the resulting set of coupled differential equations in terms of modal co-ordinates were numerically solved and synthesised to obtain the dynamic response of the bridge vibration. Atkin and Mofid [4] studied a beam under moving mass for different boundary conditions of the beam and showed that the response under moving mass effect can be significantly different from simple moving load predictions. Lee [5] showed that the inertial effects of the moving mass are not negligible even when the velocity of the moving mass is small.

Michaltsos et al. [6] used an iterative approach for solving the coupled differential equations of beam vibration. Excitation pertaining to moving mass acceleration was constructed from moving load response. Rao [7] employed a perturbation technique using method of multiple scales for studying the dynamic response considering the inertia effects of the moving mass. Majumder and Manohar [8] considered a finite element model and proposed a new model reduction scheme which resulted in reduced set of nonlinear equations with time varying coefficients. Vehicle was considered to be a moving oscillator and a time domain approach was employed to detect damages in bridge structures using the vibration data. Rahimzadeh and Ali [9] observed that the effect of higher vibrational modes may not be negligible in certain velocity ranges. Variation is shown in the critical velocity (the velocity at which maximum deflection occurs) depending on the mass ratio and number of modes considered. Dehestani et al. [10] investigated vibration response in terms of two parameters; coefficient of inertia effect (CIE) and coefficient of influential speed (CIS). Esen and Mevkii [11] considered an accelerating mass moving on a simply supported beam. They combined classical finite element method with a moving finite element representing the accelerating mass with both inertial as well as damping effect. Response was obtained numerically and it was observed that with increasing acceleration, deflection increases and deflection curve shifts to the right. Pala and Reis [12] extended the iterative method for beams with general boundary conditions, where all the three excitation terms (inertial, centripetal and coriolis) have been considered.

Billello et al. [13] and Billelo and Bergman [14] presented experimental results for a scaled model of a highway bridge (in the form of a simple beam), in which the dynamic response was measured at 7/16L location of the beam for different velocities. It was observed that experimental values are quite close to moving mass deflection but measured responses exhibit oscillation of greater magnitude than theoretically predicted. Similar results have been also reported in the experimental work of [15].

2. MATHEMATICAL MODELLING OF BRIDGE-VEHICLE INTERACTION DYNAMICS

A vehicle travelling on a bridge-like structure is generally modelled as a single contact point moving mass as shown in Fig. 1. The bridge is modelled as a beam with ideal boundary conditions and its dynamic response is evaluated under the weight and inertia force effects of the vehicle mass M. The vehicle is considered to be moving with constant velocity u and its contact always remains maintained with the bridge during the traversal. The governing equation of motion of the beam,considering both the weight and inertia effects of the moving mass, is given by [17]

$$EI\frac{\delta^4 y}{\delta x^4}+\rho A\frac{\delta^2 y}{\delta t^2}+M\left\{\frac{\delta^2 y}{\delta t^2}+2u\frac{\delta y}{\delta x}\frac{\delta y}{\delta t}+u^2\frac{\delta^2 y}{\delta x^2}\right\}\delta(x-ut)=Mg\delta(x-ut) \tag{1}$$

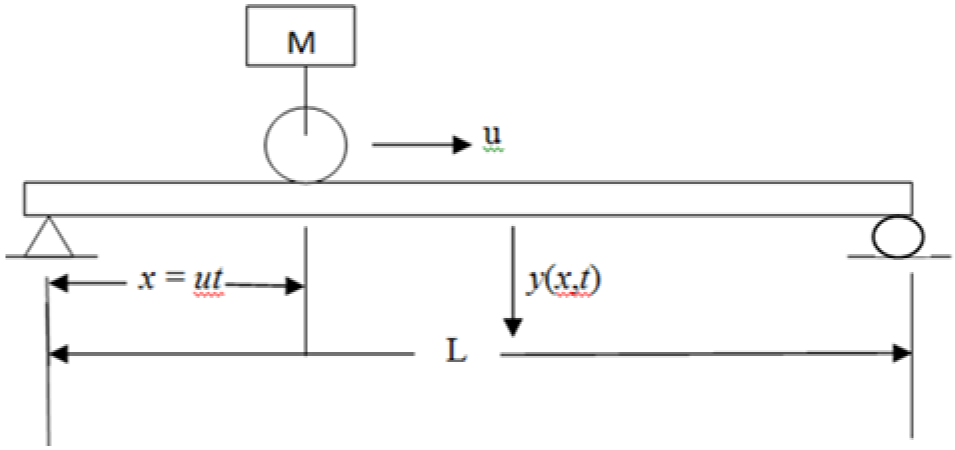


Fig. 1. A travelling mass M moving on a beam with constant velocity.

Here, $y(x,t)$ is the dynamic deflection, ρ is the mass density of the beam, A and I are the area and second moment of area of the beam cross section, respectively. E is the elastic modulus of the beam material. M is the vehicle mass and its velocity is u. δ represents the well known Dirac delta function. Whereas the right hand side expression represents the moving load effect, the bracketed terms on the left hand side represent the inertia effect of the moving mass.

Figure 2 shows a two point contact model of the vehicle during the above mentioned three phases of bridge traversal. In the dynamic response modelling for this case, the following assumptions are considered:

(i) The beam is simply supported at both the ends.

(ii) The vehicle acts as a rigid mass and moves with a constant velocity, u.

(iii) The vehicle vibration is translational only. The rotational vibration is taken to be negligible and not considered.

(iv) At any time, only one vehicle traverses the bridge with zero initial conditions.

The governing differential equation of motion for the two contact point model can be obtained by extending Eq. (1) for multiple points loading as

$$\begin{aligned} & EI\frac{\delta^4 y}{\delta x^4}+\rho A\frac{\delta^2 y}{\delta t^2}+M_1\left\{\frac{\delta^2 y}{\delta t^2}+2u\frac{\delta y}{\delta x}\frac{\delta y}{\delta t}+u^2\frac{\delta^2 y}{\delta x^2}\right\}\delta(x-ut) \\ & +M_2\left\{\frac{\delta^2 y}{\delta t^2}+2u\frac{\delta y}{\delta x}\frac{\delta y}{\delta t}+u^2\frac{\delta^2 y}{\delta x^2}\right\}\delta(x+d-ut) \\ & =M_1 g\delta(x-ut)+M_2 g\delta(x+d-ut) \end{aligned} \tag{2}$$

where d is the wheel base distance and M_1 and M_2 are the mass loading on front wheels and rear wheels. We seek a solution to this equation assuming the response in a series form as

$$y(x,t)=\sum_{j=1}^{\infty} Y_j(x)*q_j(t) \tag{3}$$

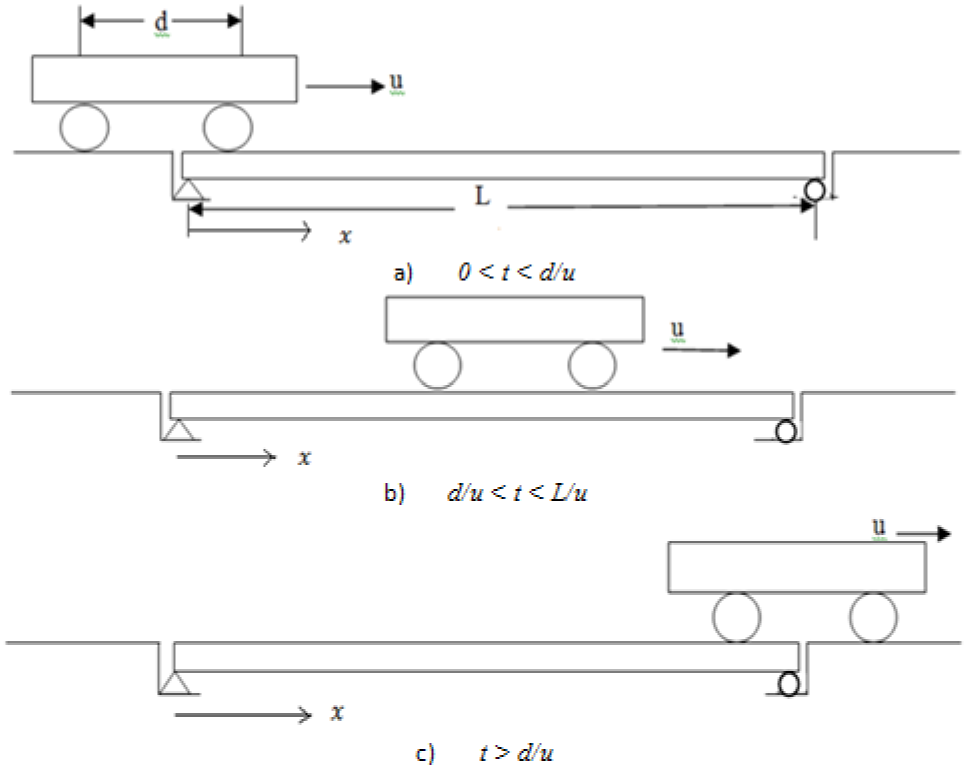


Fig. 2. Three phases of two contact point model.

where $q_j(t)$ are the modal coordinates and $Y_j(x)$ are mass normalised mode shapes for a simply supported beam given by

$$Y_j(x) = \sqrt{\frac{2}{\rho AL}} \sin\left\{\frac{j\pi x}{L}\right\} \tag{4}$$

Figure 3 shows the trend of response convergence with increasing number of modes (i.e., terms in the series given in Eq. (3)). It can be seen that the response almost converges with first three modes and hence, in all the computational work, the series given in Eq. (3) is limited to $j = 3$ only.

Now, substitution of Eq. (3), with the series truncated after third mode, into Eq. (2) gives

$$\begin{aligned} &EI\sum_{j=1}^{3}\frac{d^4Y_j(x)}{dx^4}q_j(t) + \rho A\sum_{j=1}^{3}Y_j(x)\ddot{q}_j(t) \\ &+\sum_{k=1}^{2}M_k\alpha_k\left\{\sum_{j=1}^{3}Y_j(x)\ddot{q}_j(t) + 2v\sum_{j=1}^{3}\frac{dY_j(x)}{dx}\dot{q}_j(t) + v^2\sum_{j=1}^{3}\frac{d^2Y_j(x)}{dx^2}q_j(t)\right\}\delta(x+(k-1)d-vt) \\ &=\sum_{k=1}^{2}M_k\alpha_k g\delta(x+(k-1)d-vt) \end{aligned} \tag{5}$$

such that $\alpha_k = 1$, when $0 < ut-(k-1)d < L$ and $\alpha_k = 0$ otherwise. Now multiplying each term of Eq. (5)

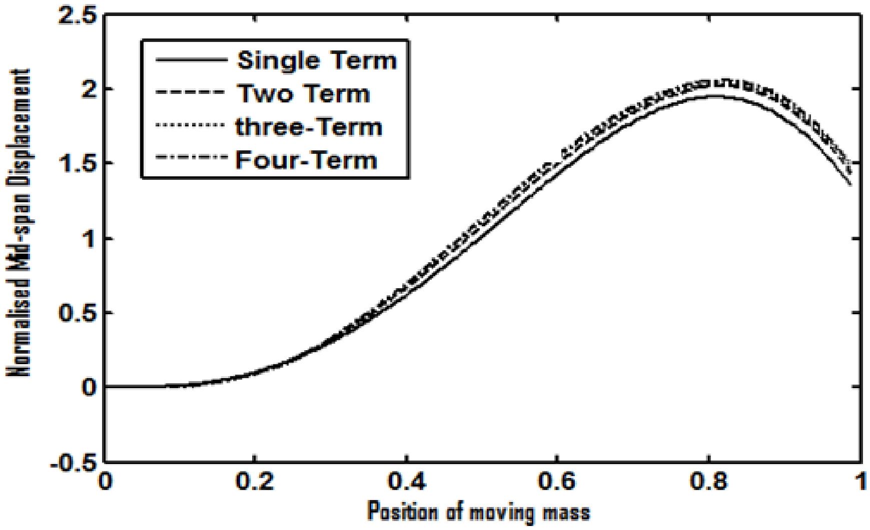


Fig. 3. Response computation with different modal truncation in Eq. (3).

with $Y_n(x)$ and integrating over the domain $(0,L)$, one obtains

$$\begin{aligned}
&\omega_n^2 q_n + \ddot{q}_n + \sum_{k=1}^{2} \alpha_k M_k \sum_{j=1}^{3} Y_j(ut-kd+d) Y_n(ut-kd+d) \ddot{q}_j \\
&+ 2u \sum_{k=1}^{2} \alpha_k M_k \sum_{j=1}^{3} \frac{dY_j(ut-kd+d)}{dx} Y_n(ut-kd+d) q_j \\
&+ u^2 \sum_{k=1}^{2} \alpha_k M_k \sum_{j=1}^{3} \frac{d^2Y_j(ut-kd+d)}{dx^2} Y_n(ut-kd+d) q_j \\
&= \sum_{k=1}^{2} \alpha_k M_k g Y_n(ut-kd+d) \qquad (6)
\end{aligned}$$

In the above equation, ω_n is the nth natural frequency of flexural vibration of the beam. If the moving mass inertia effects are neglected, then this equation simplifies to the following moving load equation:

$$\omega_n^2 q_n + \ddot{q}_n = \sum_{k=1}^{2} \alpha_k M_k g Y_n(ut-kd+d) \qquad (7)$$

Equation (7) can be detailed in three phases of traversal duration as

$$\omega_n^2 q_n + \ddot{q}_n = M_1 g Y_n(ut) \quad \text{for } 0 < t < d/u \qquad (8a)$$

$$\omega_n^2 q_n + \ddot{q}_n = M_1 g Y_n(ut) + M_2 g Y_n(ut-d) \quad \text{for } d/u < t < L/u \qquad (8b)$$

$$\omega_n^2 q_n + \ddot{q}_n = M_2 g Y_n(ut-d) \quad \text{for } L/u < t < (L+d)/u \qquad (8c)$$

Similarly, Eq. (6) with moving mass effects can also be split into three phases for the numerical integration.

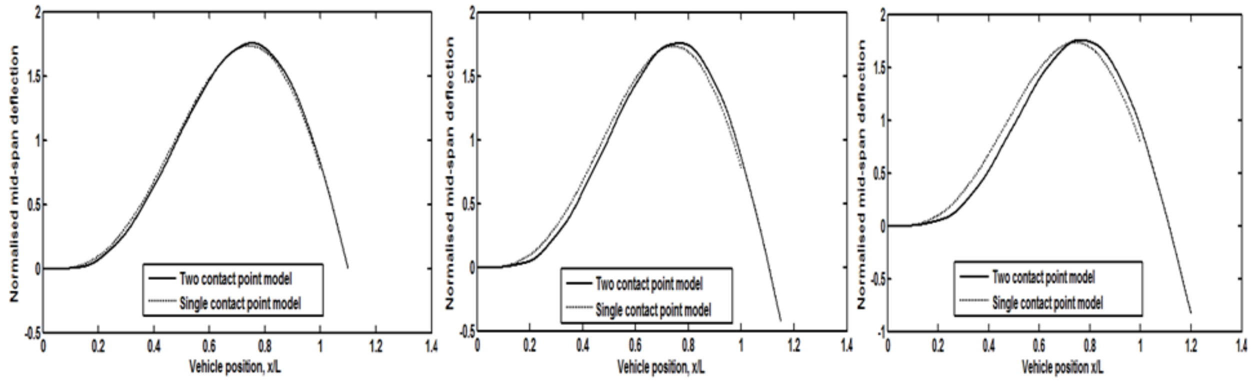


Fig. 4. Bridge mid-span deflection under moving load conditions (velocity ratio = 0.6).

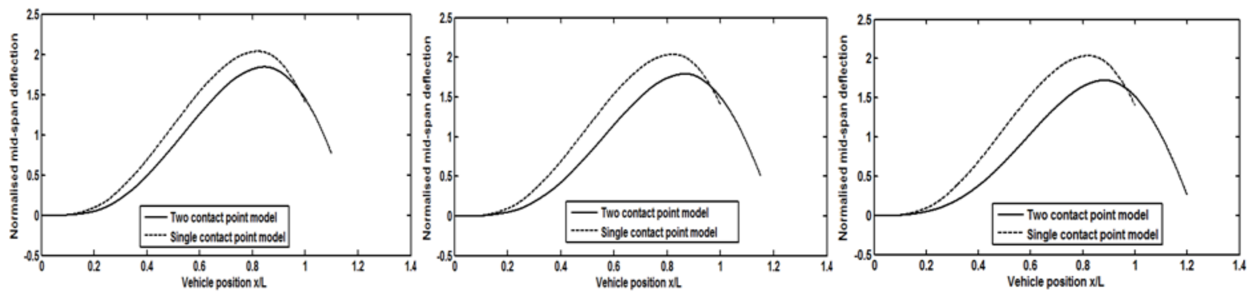


Fig. 5. Bridge mid-span deflection under moving mass conditions (velocity ratio = 0.6).

3. COMPARISON BETWEEN MOVING LOAD AND MOVING MASS RESPONSE COMPUTATIONS

Equation (8) for the moving load and Eq. (6) for the moving mass are solved for the response using a fourth order Runge–Kutta numerical integration algorithm, successively in the three traversal zones such that the final values of the first phase become the initial values for the second phase and similarly final values of the second phase become the initial values for the third phase. Here two simulation parameters are varied. The first is the mass ratio of the vehicle mass M to the bridge mass (ρAL) and the second one is the velocity ratio defined as $r = u/u_{\text{crit}}$, where $u_{\text{crit}} = L\omega_1/\pi$. For a simply supported beam, the governing equations would contain harmonic terms with frequencies $\Omega_n = n\pi u/L$, which shows that there is a critical velocity, u_{crit}, for which $\Omega_1 = \omega_1$.

For the simulation, we consider a simply supported beam with length $L = 20$ m, width $b = 2$ m and thickness $h = 0.2$ m with the following material properties: $E = 2.06 \times 10^{11}$ N/m^2 and $\rho = 7{,}800$ kg/m^3. This gives $u_{\text{crit}} = 46.61$ m/s $= 167.80$ km/h. Thus, a vehicle velocity of 100 km/hr will have the velocity ratio close to 0.6. The mass of the beam turns out to be 62,400 kg which gives a mass ratio of 0.1 for a vehicle of 7 tonne capacity. For the sake of simplicity, this mass is equally distributed ($M_1 = M_2 = M/2$) on the front and rear wheels. The wheel base is taken as 2, 3 and 4 m depending on the type of vehicles. The dynamic deflection of the bridge is computed at its mid-span in normalised form; as a ratio of dynamic deflection divided by mid-span deflection under static vehicle weight. The mid-span deflection is computed from Eq. (3) as

$$y(L/2,t) = \sum \sin\left(\frac{n\pi}{2}\right) * q_j(t) \tag{9}$$

Figures 4 and 5 show the bridge mid-span deflection under moving load and moving mass condition

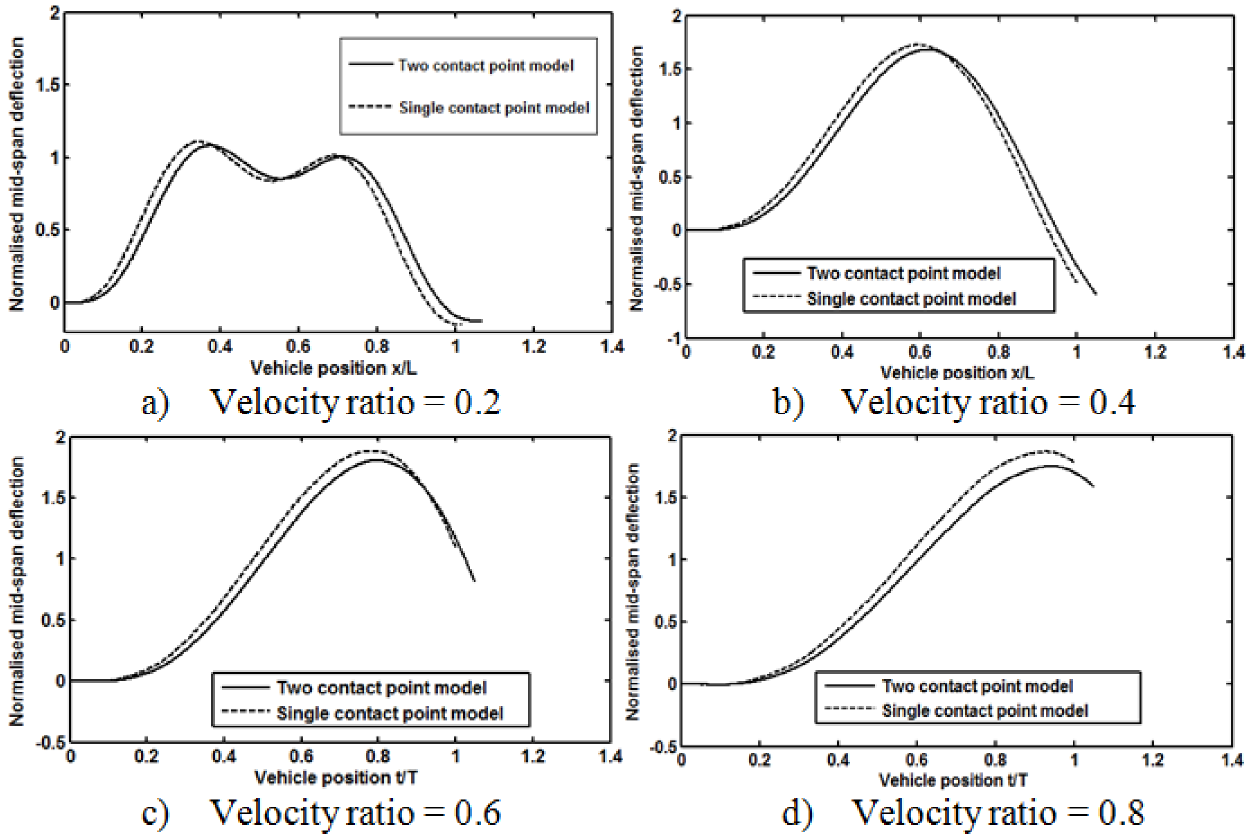


Fig. 6. Bridge mid-span deflection under moving mass condition, $d = 1$ m ($d/L = 0.05$).

respectively during the traversal of the vehicle, for both the models for a typical velocity ratio of 0.6 and vehicle mass ratio of 0.2.

Thus it can be seen that the response under a two contact point model differs significantly from the one under the single contact point model under moving mass condition. This difference is less visible in moving load responses. In the following section, simulation results are presented for a two contact point model of the vehicle considering it as a moving mass.

4. NUMERICAL SIMULATION FOR MOVING MASS CONDITION

From the discussion and observation of the previous section, two important things can be noted. First, the bridge dynamic response under a two contact point model can be significantly different from that obtained using single contact point model at a certain wheelbase to bridge length ratio. Second, the moving load response gives a partial picture of the dynamic response and hence the moving mass inertia effect must be included in the mathematical model. Accordingly, the simulation is carried out based on moving mass model Eq. (6) for different velocity ratios and different d/L ratios for two mass ratio parameters, 0.1 and 0.2.

4.1. Simulation with Mass Ratio 0.1

The mid-span deflection response for a vehicle to a bridge mass ratio of 0.1 is considered here.Numerical simulation is carried out for velocity ratios of 0.2, 0.4, 0.6 and 0.8 for three different wheel base distances of 1, 2 and 3 m. The results are shown in Figs. 6–8.

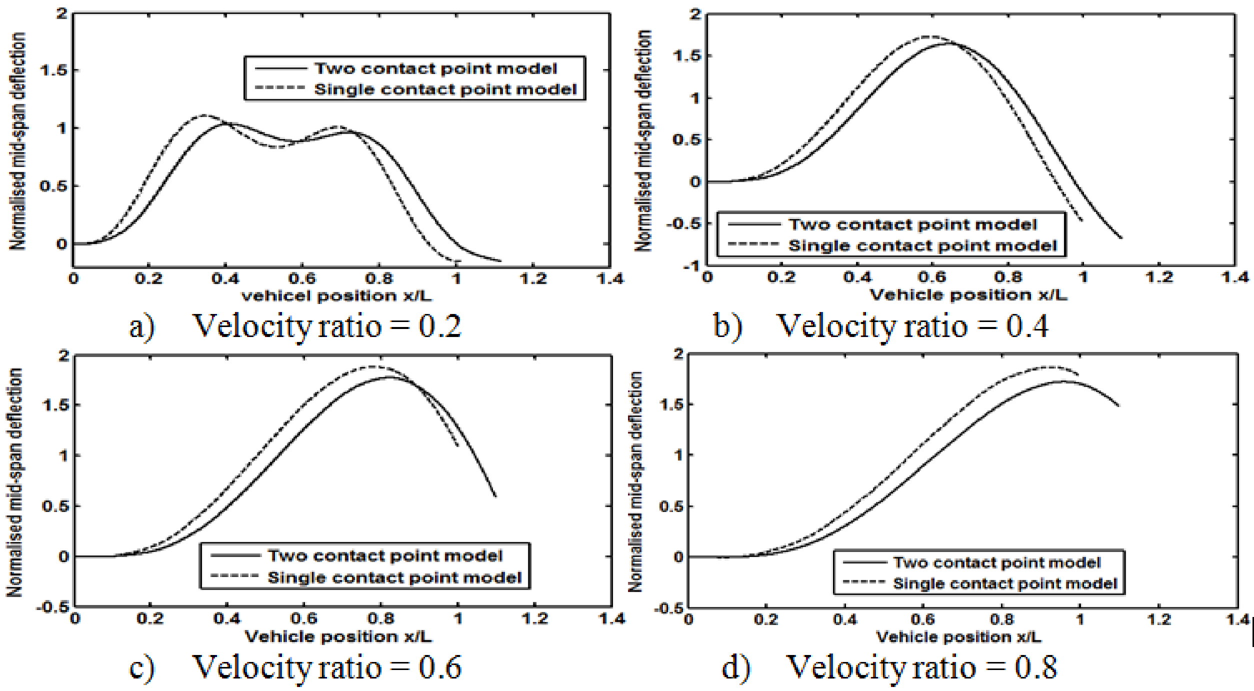


Fig. 7. Bridge mid-span deflection under moving mass condition, $d = 2$ m ($d/L = 0.1$).

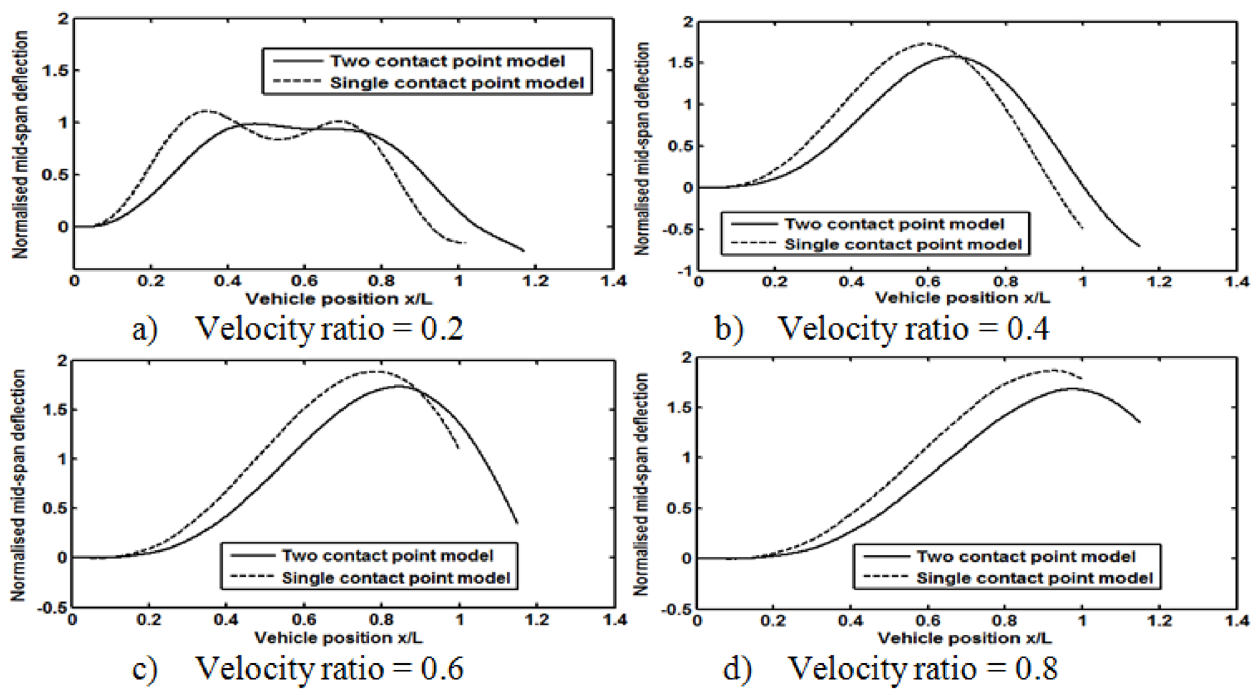


Fig. 8. Bridge mid-span deflection under moving mass condition, $d = 3$ m ($d/L = 0.15$).

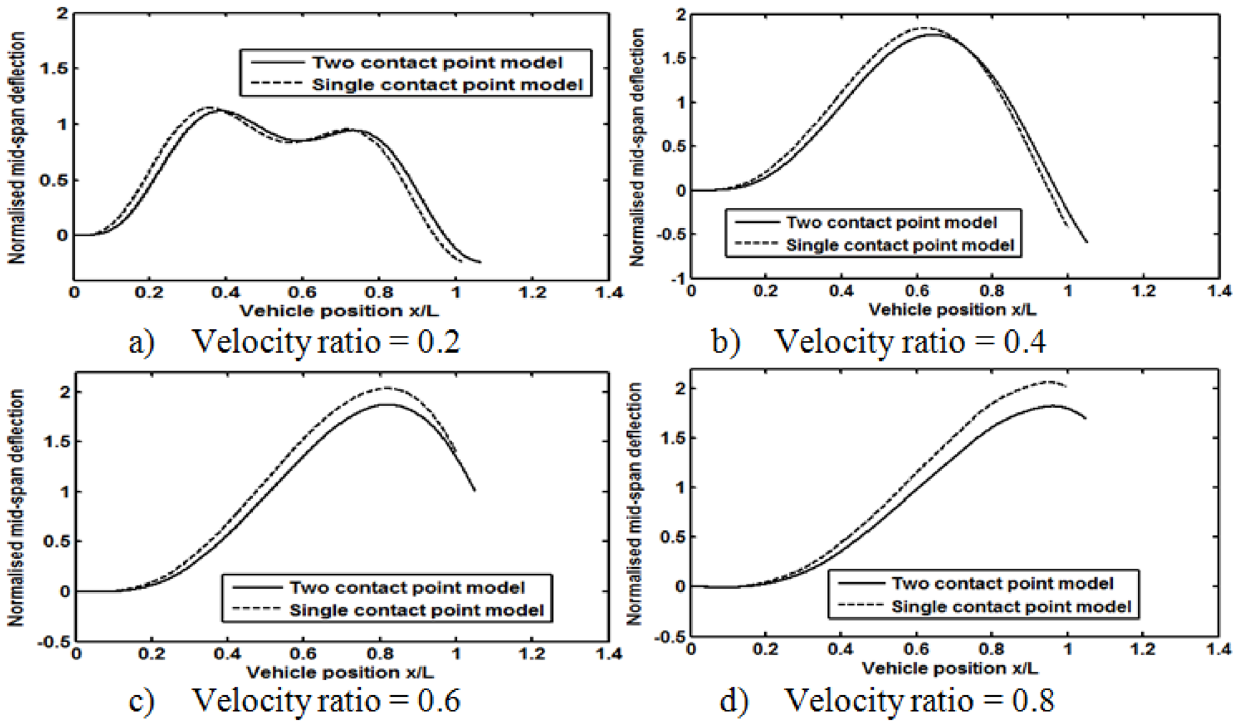


Fig. 9. Bridge mid-span deflection under moving mass condition, $d = 1$ m ($d/L = 0.05$).

The following observations can be noted from Figs. 6–8:

- The maximum deflection computed from the two contact point model is less than the one computed from single contact point model.
- The vehicle location corresponding to maximum deflection shifts towards the right in the two contact point model.
- The difference in maximum values between the two models increases as the velocity ratio increases. For $d = 1$ m, the percentage differences are found to be 2.35, 2.55, 4.10 and 6.44% respectively for velocity ratios of 0.2, 0.4, 0.6 and 0.8. Thus the difference becomes more significant for velocity ratio around 0.6 and greater.
- The difference in computed maximum mid-span deflection in the two models increases as vehicle wheel base increases (i.e., as d/L ratio increases). For velocity ratio of 0.6, the percentage differences are found to be 4.15, 5.64 and 8.09% respectively for wheelbase distances 1, 2 and 3 m.

4.2. Simulation with Mass Ratio 0.2

The numerical simulation is also carried out for heavier vehicles with mass ratio 0.2 to investigate effect of mass ratio on single contact point model and two contact point model.The responses are shown in Figs. 9, 10 and 11 respectively for wheelbase distances of 1, 2 and 3 m.

The observations from these figures can be summarised as:

- The difference between response maxima obtained from the single contact point model and the two contact point model increases as velocity ratio increases and also as the vehicle wheel base increases (i.e., as the d/L ratio increases).

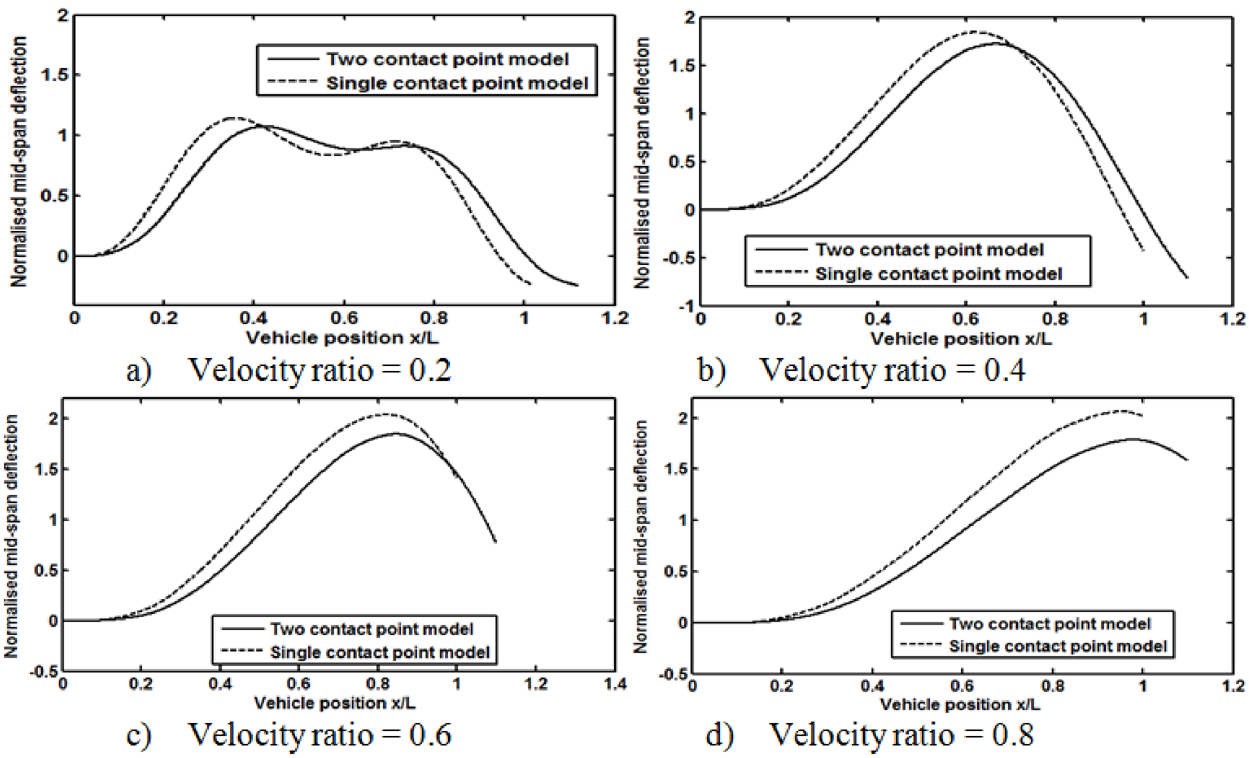


Fig. 10. Bridge mid-span deflection under moving mass condition, $d = 2$ m ($d/L = 0.1$).

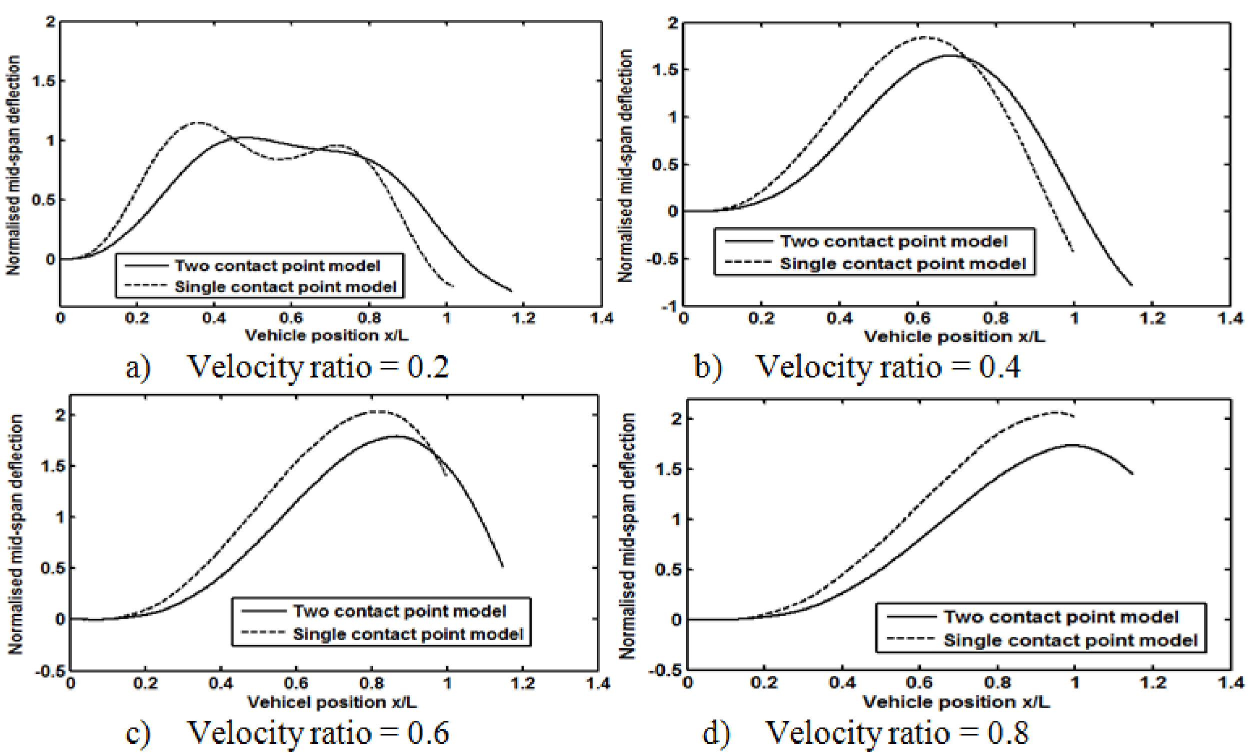
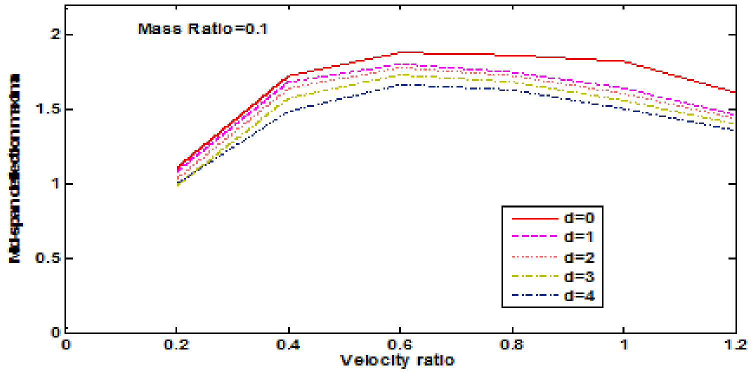
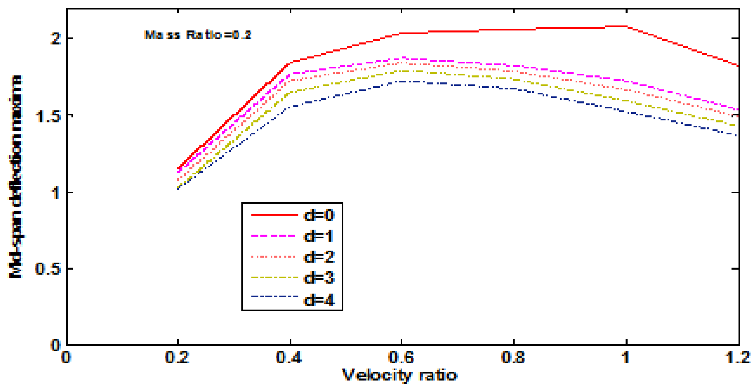


Fig. 11. Bridge mid-span deflection under moving mass condition, $d = 3$ m ($d/L = 0.15$).



a) Vehicle to bridge mass ratio ε = 0.1



b) Vehicle to bridge mass ratio ε = 0.2

Fig. 12. Mid-span deflection maxima for varying velocity ratio and wheel base distances.

- For $d = 1$ m, the percentage differences are found to be 6.29, 6.53, 9.49 and 13.29% respectively for velocity ratios of 0.2, 0.4, 0.6 and 0.8. Here also the two contact point model differs significantly for a velocity ratio of 0.6 and greater.

- For a velocity ratio of 0.6, the percentage differences are found to be 7.87, 9.49 and 12.00% respectively for wheelbase distances 1, 2 and 3 m.

Comparing Figs. 6–8 for a mass ratio equal to 0.1 with Figs. 9–11 for a mass ratio equal to 0.2, one can observe that

(i) The difference between a single contact point response and a two contact point response increases as the mass ratio increases from 0.1 to 0.2. This is self-explanatory from Eq. (5).

(ii) The peak response position on the beam shifts right as mass ratio increases and the shift is more for a higher d/L ratio. This can be explained by the fact that phase III vibration (Fig. 2c) contributes into the peak response and this contribution comes from half of the vehicle mass still remaining on the beam. As the mass ratio increases, this contribution increases and the duration is higher for a higher d/L ratio.

5. RESPONSE MAXIMA AS A FUNCTION OF VELOCITY RATIO AND WHEEL BASE DISTANCE

In the previous sections, it has been found that deflection maxima during vehicle traversal by two point model depends on the velocity of the vehicle as well as the wheel base distance.This dependence has been shown in Figs. 12a and 12b for mass ratios 0.1 and 0.2 and for a wheel base distance 1 to 4 m. The response characteristics will be the same as long as the mass ratio, velocity ratio and d/L ratio remain the same. Thus, a bridge mid-span deflection maximum depends on three non-dimensional parameters, i.e., the velocity ratio, the wheelbase to bridge length ratio and the vehicle to bridge mass ratio.

It is to be noted from this figure that for a single contact point model, the maximum deflection occurs close to a velocity ratio of 0.6 for mass ratio 0.1, whereas for a higher mass ratio such as 0.2, the maximum deflection occurs at a velocity ratio around 1.0. For the two contact point model, a maximum deflection occurs around the same velocity ratio but the value of the deflection maxima reduces with an increasing wheel base distance. This is very significant for heavier vehicles as the larger deflection maxima can be controlled or reduced by increasing the wheel base of the vehicle.

6. CONCLUSION

A mathematical model has been developed for two contact point vehicle-bridge interaction in the present work. This model is more representative than single contact point model mostly adopted in the previous research works. Mid-span deflection response is computed through numerical integration of the equations of motion in three phases and is characterised for different wheel base distances at different velocities of the vehicle. It is observed that for wheelbase to bridge span ratio greater than 0.1, two contact point model gives significantly different results particularly at higher velocity range (velocity ratio around or greater than 0.6). It is shown that for the same vehicle mass and velocity, increasing the wheel base distance reduces the deflection maxima. This aspect does not get highlighted in a single contact point model. The present concept of two contact points can be easily extended for three contact points for tractor-trailer type of vehicles.

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